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Mathematical model for solving the relationship of shadow length changing over time through solar elevation

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Abstract. In reconnaissance work, shadow positioning can often play a breakthrough role. To determine the shadow length of a straight rod at a fixed time and location, taking the solar elevation as the intermediate variable, and a model of the length of the shadow with multiple geometric parameters can be established through trigonometric function relationship.

Introduction

Firstly, through trigonometric function relationship, the mathematical relationship of the shadow length with solar elevation and the shadow length of the rod can be obtained. Thanks to the known location of the rod, we can know its latitude φ. Currently, in the paper of *Study on Mathematical Model of Shadow Length Change*, the formula of solar elevation has been proposed by Jun-Shuang Huo et al. To simplify the calculation, we omit a few of items which have a small effect on the outcome, and then a simplified formula is obtained. Combined with the known date, the solar declination δ and the local time angle Ω can be solved, then the solar elevation angle can be calculated. Finally, taking the solar elevation as the intermediate variable, the relationship between the shadow length and time is obtained, after that, the curve of the shadow length related to the measurement time can be plotted.

Model establishment process

In order to determine the shadow length of an existing rod at a fixed time and location, taking the solar elevation as the intermediate variable, and the mathematical model for the relationship of shadow length changing with multiple geographic parameters is established. These parameters include latitude and longitude of the observation point, solar elevation, solar declination and the local time angle. The relationship between solar elevation angle and observation time can be obtained by these parameters, and the functional relationship of the shadow length varied with time can be obtained after eliminate the intermediate variable.

The solar elevation angle is the elevation angle of the sun. It is numerically equal to the angle between the direction of the sun and the idealized horizon. In this paper, solar elevation is represented as h. The solar azimuth angle is the azimuth angle of the sun. It defines in which direction the sun is, whereas the solar zenith angle or its complementary angle solar elevation defines how high the sun is. And it can be approximated as the angle of the shadow of the rod on the ground with the local south direction. In this paper, solar azimuth angle is represented as A[1][2].

View the point of the rod intersecting with the ground as the coordinate origin, and the Dimensional Cartesian coordinate system can be established as below. In this coordinate system, the x-axis represents the east direction and the y-axis represents the north direction of the earth. Accordingly, the reverse direction of the x-axis represents the west direction and the reverse direction of the y-axis represents the south direction. And the z-axis represents the straight line in which the straight rod is located. In the following picture, h represents solar elevation, A represents solar azimuth, l represents the length of the rod, and r represents the length of the shadow.

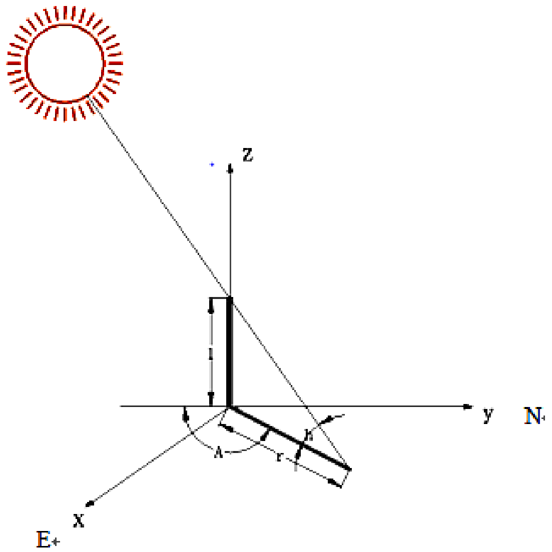

Fig. 1 Geometric Representation of Solar Altitude Angle *h* and Solar Azimuth *A*

Taking the rod as the researching subject, according to the geometric relationship, the solar elevation satisfies the following trigonometric function relationship that

$$\tan h = \frac{l}{r} \qquad (1)$$

Owe to the length of the rod is known, as long as the solar elevation is solved, the length of the rod shallow can be easily calculated.

The solar elevation in one place is related to the solar declination of the day and the local time angle Ω of the local place. In the paper issued by Jun-Shuang Huo et al., we know that the solar declination δ satisfies the following relationship that[3]

$$\delta=0.3723+23.2567\sin\theta+0.1149\sin2\theta-0.1712\sin3\theta-0.758\cos\theta+0.3656\cos2\theta+0.0201\cos3\theta \qquad (2)$$

Where θ is called the day angle, and it can be calculated through that

$$\theta = \frac{2\pi t}{365.2422} \qquad (3)$$

Where t is composed of two parts that

$$t = N - N_0 \qquad (4)$$

In this formula, N is called the accumulation date, which indicates sequence number in a year. For example, in a year, the accumulation date of January 1st is 1. In the average year, the accumulation date of December 31st is 365, however, in the leap year, it is 366. N_0 can be represented that

$$N_0 = 79.6764 + 0.2422 \times (Y - 1985) - INT[\frac{Y-1985}{4}] \qquad (5)$$

Where INT means rounding function, Y means the year.

After the integration of equations (2) to (5), through ignoring items which have less impact on the results, the simplified formula of solar declination δ can be obtained that

$$\delta = 23.45\sin\frac{2\pi(284+n)}{365} \qquad (6)$$

Where n is the number of days from January 1st to the observed day.

Taking the provisions of the solar declination and geographical latitude which are in the north latitude as positive, and the south latitude is negative. The local time angle of which the rod located is denoted by Ω, and it can be calculated from that

$$\Omega = 15(t_b - 12) \qquad (7)$$

Where t_b represents the observation time corresponding to Beijing time.

In all, after the solar declination δ, the observation latitude φ and the local time angle Ω is calculated, the solar elevation h can be represented that

$$\sin h = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos\Omega \qquad (8)$$

Finally, through trigonometric function relationship that

$$1+\tan^2 h=\sec^2 h=1/\cos^2 h=1/(1-\sin^2 h)$$

A formula about trigonometric function relationship can be derivated as below

$$\tan h = \frac{\sin h}{\sqrt{1-\sin^2 h}} \qquad (9)$$

From the formula (6) (7) (8) we know that the solar elevation h meets that

$$\sin h = \sin\varphi \sin\{23.45\,[\sin 2\pi\,(284+n)/365]\} + \cos\varphi \cos\delta \cos\{15*(t_b-12)\}$$

Substituting it into Eq. (9), and the shadow length be can finally calculated by that

$$r = \frac{l}{\tan h} \qquad (10)$$

Where l means the length of the pole, h means the solar elevation, and r means the length of the shallow. Through this formula, the relationship of shadow length changing over time can be obtained.

Using MATLAB to draw the functional curve

To verify the accuracy of the model, this model is used to study a specific time and place. As below, the model is used to study the shadow length of a 3-metre-long rod in the Tiananmen Square changed with time from 9 to 15 on October 22, 2015.

The latitude φ of the observation is 39 degrees 54 minutes and 26 seconds, and n is 295. Through MATLAB software, the function curve can be drawn as below.

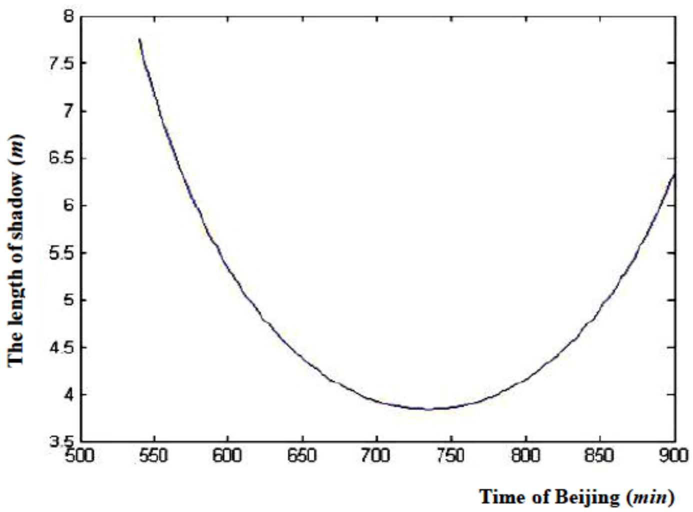

Fig. 2 The functional curve of the shadow length changed with time

From the figure above we can see that in Beijing time between 9: 00-15: 00, the shadow length of the straight rod and the time have a parabolic relationship. From 9: 00 to 12: 14, with the passage of time, the solar elevation gradually increased, and the shadow length gradually decreased. Correspondingly, at about 12:14 the solar elevation reaches the peak value, and the corresponding shadow length has a minimum value. However, in the period of 12: 14 to 15: 00, the solar elevation gradually decreased in the opposite direction, and the shadow length gradually increased. The whole curve is approximately symmetrical, and the symmetry axis is at about noon time. This result satisfies the objective law of temporal variation in the autumn of the northern hemisphere.

Summary

In this paper, in order to determine the shadow length of a straight rod at a fixed time and location, we take the solar elevation as the intermediate variable, through trigonometric function relationship, the relationship of shadow length changed over time can be obtained. Finally, the functional curve of the shadow length changed over time can be drawn through MATLAB software. And the result satisfies our common sense.

Reference

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