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A Novel of Improved algorithm adaptive of NURBS curve

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Keywords: NURBS curve; improved algorithm adaptive of NURBS; interpolation algorithm; simulation.

Abstract. In order to solve the problems of a improved algorithm adaptive of NURBS curve,Such as interpolation time bigger, calculation more complicated, and NURBS curve step error are not easy changed and so on. This paper proposed a study on the algorithm for improved algorithm adaptive of NURBS curve and simulation . We can use improved algorithm adaptive of NURBS curve that calculate (x_i, y_i, z_i). Simulation results show that the proposed NURBS curve interpolator meet the high-speed and high-accuracy interpolation requirements of CNC systems. The interpolation of NURBS curve should be finished. The simulation results show that the algorithm is correct; it is consistent with a NURBS curve interpolation requirements.

1. Introduction

Modern CNC manufacturing systems, NURBS(Non-Uniform Rational B-Spline) has become a mathematical tool used in the field FMS/CIMS. It has many character[1-3]: NURBS can give a unified mathematical representation for surfaces and curves , NURBS can change the shape by modifying weight vector and control point ,etc. But, NURBS has a shortcoming: interpolation time bigger, calculation more complicated, and NURBS curve step error are not easy changed. Lanzhou University of technology and Lanzhou Industry and Equipment Co. Ltd.researchers [4-6] proposed an NURBS algorithms which based on real-time interpolation and adaptive interpolation. Literature [7-9] give some NURBS interpolation algorithms, which makes NC programming complicated and interpolation calculate complicated . Shpitalni et al. [9] derived the same interpolation algorithm by using Taylor's expansion. Houng and Yang [10] were given Cubic spline curve interpolator by using Euler algorithm. Lo and Chung[11-22] proposed the error interpolation algorithm which error calculations changed by curve chord.

On the basis of the research above, a improved algorithm adaptive of NURBS and simulation is presented in this paper. Furthermore, this interpolation algorithm through actual processing of simulation are discussed. The simulation results show that the algorithm is consistent with a NURBS curve interpolation requirements. This interpolation algorithm can meet the high-speed and high-accuracy NURBS curves interpolation requirements.

2. NURBS Interpolator

In this paper, NURBS curve is used to represent a parametric of a Improved algorithm adaptive of NURBS curve, and it is introduced first. Supposed $p(u)$ can be represented a Improved algorithm adaptive of NURBS curve. While NURBS [3] are parametrically mathematical definition by the following Eq.(1):

$$P(u) = \frac{\sum_{i=0}^{n} \omega_i d_i \mathrm{N}_{i,k}(u)}{\sum_{i=0}^{n} \omega_i \mathrm{N}_{i,k}(u)} \tag{1}$$

Where u is cubic time a Improved algorithm adaptive of NURBS curve each parameter, k the order of a Improved algorithm adaptive of NURBS curve . p_i is the control points, ω_i is the weight vector , $N_{i,k}(u)$ is the blending function .

$$\begin{cases} N_{i,0} = \begin{cases} 1 \cdots\cdots\cdots u_i \le u \le u_{i+1} \\ 0 \cdots\cdots\cdots\cdots\cdots \text{other} \end{cases} \\ N_{i,k}(u) = \dfrac{u-u_i}{u_{i+k}-u_i} N_{i,k-1}(u) + \dfrac{u_{i+k+1}-u}{u_{i+k+1}-u_{i+1}} N_{i+1,k-1}(u) \\ \text{define} \dfrac{0}{0} = 0 \end{cases} \tag{2}$$

Where the knot vector belong to $U = [u_i, \cdots, u_{i+k+1}]$. Based on Eq. (1) and (2), a Improved algorithm adaptive of NURBS curve can be defined when ω_i, d_i, k and knot vector are given certain values. a Improved algorithm adaptive of NURBS curve is defined by three types of parameters: Locus of control, Weighted factor and Knot vector. $N_{i,k}$ is shown in Figure1.

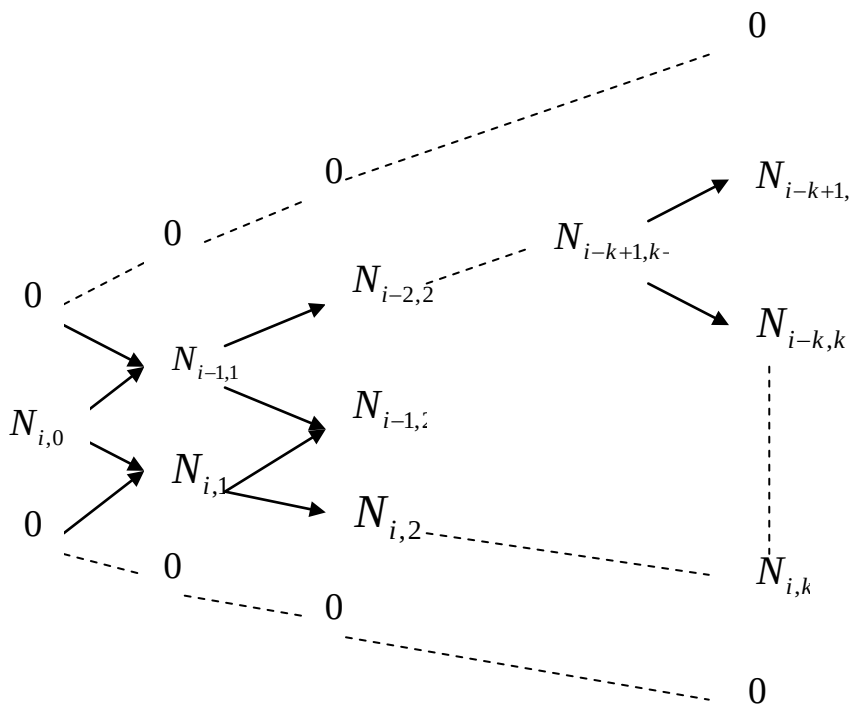

Fig .1: Figure of $N_{i,k}$ formula

Mathematical expression

$$\text{Formula:}\begin{cases} x(u)=\dfrac{\sum_{i=0}^{n} w_i x_i N_{i,k}(u)}{\sum_{i=0}^{n} w_i N_{i,k}(u)} \\ y(u)=\dfrac{\sum_{i=0}^{n} w_i y_i N_{i,k}(u)}{\sum_{i=0}^{n} w_i N_{i,k}(u)} \\ z(u)=\dfrac{\sum_{i=0}^{n} w_i z_i N_{i,k}(u)}{\sum_{i=0}^{n} w_i N_{i,k}(u)} \end{cases} \tag{3}$$

where $N_{i,k}{}'(u)$ Ni,k(u) is a blending function defined by the recursive

$$N_{i,k}{}'(u)=k\left[\frac{N_{i,k-1}(u)}{u_{i+k}-u_i}-\frac{N_{i+1,k-1}(u)}{u_{i+k+1}-u_{i+1}}\right] \tag{4}$$

Where, $N_{i,k}{}''(u)$ can be represented as follows:

$$N_{i,k}{}''(u)=k(k-1)\left\{\left[\frac{N_{i,k-2}(u)}{(u_{i+k}-u_i)(u_{i+k-1}-u_i)}-\frac{N_{i+1,k-2}(u)}{(u_{i+k}-u_i)(u_{i+k}-u_{i+1})}\right]-\right.$$
$$\left.\left[\frac{N_{i+1,k-2}(u)}{(u_{i+k+1}-u_{i+1})(u_{i+k}-u_{i+1})}-\frac{N_{i+2,k-2}(u)}{(u_{i+k+1}-u_{i+1})(u_{i+k+1}-u_{i+2})}\right]\right\} \tag{5}$$

Supposed $P^*(u)=\sum_{i=0}^{n} w_i d_i N_{i,k}(u)$, $W(u)=\sum_{i=0}^{n} w_i N_{i,k}(u)$, (6)

Supposed $P(u)=\dfrac{P^*(u)}{W(u)}$, we shuold $P^*(u)$

$$P^{\bullet}(u)=\frac{d}{du}P(u)=\frac{1}{W(u)}\left[P^{*\bullet}(u)-W^{\bullet}(u)P(u)\right] \tag{7}$$

where $x(u)$ The derivative of-u is $x'(u)$

$$x'(u)=\frac{\sum_{i=0}^{n} w_i N_{i,k}(u)\cdot\sum_{i=0}^{n} N_{i,k}{}'(u) w_i x_i-\sum_{i=0}^{n} w_i N_{i,k}{}'(u)\cdot\sum_{i=0}^{n} N_{i,k}(u) w_i x_i}{(\sum_{i=0}^{n} w_i N_{i,k}(u))^2} \tag{8}$$

Where $x'(u)$ The derivative of-u is $x''(u)$

$$x''(u)=\frac{\sum_{i=0}^{n} w_i N_{i,k}(u)\cdot\sum_{i=0}^{n} N_{i,k}{}''(u) w_i x_i-\sum_{i=0}^{n} w_i N_{i,k}{}''(u)\cdot\sum_{i=0}^{n} N_{i,k}(u) w_i x_i}{(\sum_{i=0}^{n} w_i N_{i,k}(u))^2}-$$
$$\frac{2\sum_{i=0}^{n} w_i N_{i,k}{}'(u)(\sum_{i=0}^{n} w_i N_{i,k}(u)\cdot\sum_{i=0}^{n} N_{i,k}{}'(u) w_i x_i-\sum_{i=0}^{n} N_{i,k}(u) w_i x_i\cdot\sum_{i=0}^{n} w_i N_{i,k}{}'(u))}{(\sum_{i=0}^{n} w_i N_{i,k}(u))^3} \tag{9}$$

Where $y(u)$ The derivative of-u is $y'(u)$

$$y'(u)=\frac{\sum_{i=0}^{n}w_iN_{i,k}(u)\cdot\sum_{i=0}^{n}N_{i,k}'(u)w_iy_i-\sum_{i=0}^{n}w_iN_{i,k}'(u)\cdot\sum_{i=0}^{n}N_{i,k}(u)w_iy_i}{(\sum_{i=0}^{n}w_iN_{i,k}(u))^2} \tag{10}$$

Where $y'(u)$ The derivative of-u is $y''(u)$

$$y''(u)=\frac{\sum_{i=0}^{n}w_iN_{i,k}(u)\cdot\sum_{i=0}^{n}N_{i,k}''(u)w_iy_i-\sum_{i=0}^{n}w_iN_{i,k}''(u)\cdot\sum_{i=0}^{n}N_{i,k}(u)w_iy_i}{(\sum_{i=0}^{n}w_iN_{i,k}(u))^2}-$$
$$\frac{2\sum_{i=0}^{n}w_iN_{i,k}'(u)(\sum_{i=0}^{n}w_iN_{i,k}(u)\cdot\sum_{i=0}^{n}N_{i,k}'(u)w_iy_i-\sum_{i=0}^{n}N_{i,k}(u)w_iy_i\cdot\sum_{i=0}^{n}w_iN_{i,k}'(u))}{(\sum_{i=0}^{n}w_iN_{i,k}(u))^3} \tag{11}$$

Where $z(u)$ The derivative of-u is $z'(u)$

$$z'(u)=\frac{\sum_{i=0}^{n}w_iN_{i,k}(u)\cdot\sum_{i=0}^{n}N_{i,k}'(u)w_iz_i-\sum_{i=0}^{n}w_iN_{i,k}'(u)\cdot\sum_{i=0}^{n}N_{i,k}(u)w_iz_i}{(\sum_{i=0}^{n}w_iN_{i,k}(u))^2} \tag{12}$$

Where $z'(u)$ The derivative of-u is $z''(u)$

$$z''(u)=\frac{\sum_{i=0}^{n}w_iN_{i,k}(u)\cdot\sum_{i=0}^{n}N_{i,k}''(u)w_iz_i-\sum_{i=0}^{n}w_iN_{i,k}''(u)\cdot\sum_{i=0}^{n}N_{i,k}(u)w_iz_i}{(\sum_{i=0}^{n}w_iN_{i,k}(u))^2}-$$
$$\frac{2\sum_{i=0}^{n}w_iN_{i,k}'(u)(\sum_{i=0}^{n}w_iN_{i,k}(u)\cdot\sum_{i=0}^{n}N_{i,k}'(u)w_iz_i-\sum_{i=0}^{n}N_{i,k}(u)w_iz_i\cdot\sum_{i=0}^{n}w_iN_{i,k}'(u))}{(\sum_{i=0}^{n}w_iN_{i,k}(u))^3} \tag{13}$$

The Taylor expansion of parameter u to time t, the corresponding approximated algorithm can be obtained.

$$u_{i+1}=u_i+\frac{du}{dt}\Big|_{t=t_i}T+\frac{1}{2}\frac{d^2u}{dt^2}\Big|_{t=t_i}T^2+H.O.T. \tag{14}$$

The second-order expansion of Taylor formula

$$u_{i+1}=u_i+\frac{VT}{\sqrt{(x')^2+(y')^2+(z')^2}}+\frac{(VT)^2\times\left(x'x''+y'y''+z'z''\right)}{2((x')^2+(y')^2+(z')^2)^2} \tag{15}$$

From Eq.(10), Eq.(11), Eq.(12), Eq.(13),Eq.(14), Eq.(15) we should get (x_i,y_i,z_i), a Improved algorithm adaptive of NURBS curve should be finished.

4. **A Improved algorithm of flow chart adaptive of NURBS curve**

Figure 2 for Flowchart of algorithm. a Improved algorithm adaptive of NURBS curve is explained as follow:

Step1: Input NURBS curve paramter, such as NURBS curve control points, weight vector and so on.

Step2: Calculate Taylor's expansion.

Step3: Calculate $x', x'', y', y'', z', z''$.

Step4: Get NURBS curve position (x_i, y_i, z_i) can be calculated by using m Improved algorithm adaptive of NURBS curve.

Step5: NURBS curve interpolation is finished.

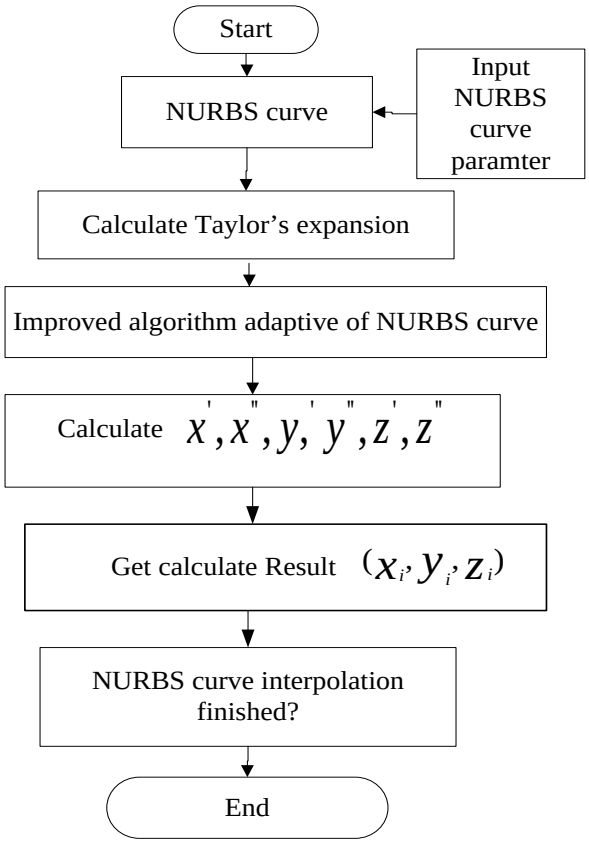

Fig .2: Flowchart of algorithm

5. Experiment simulation and data analysis

In this simulation ,this interpolation scheme is realized on the motion controller developed by our own lab, based on DSP TMS20C543,Development environment is a PC with AMD Sempron 2,800+2.1Ghz CPU,2GB RAM, and main frequency is 1.44MHz,machine tool is machine center. Machining parameters and dynamics parameters are shown in Table 1.

In the paper, the interpolation of improved algorithm adaptive of NURBS is utilized as an example to the Newton-Rapson iterative algorithm. The control points ,weight vector, and knot vector of NURBS for the provided example are assigned as follows:

The control pointsare $P = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1.5 \\ 2.5 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 2.5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3.5 \end{bmatrix} \begin{bmatrix} 3 \\ 1.15 \\ 3.5 \end{bmatrix}$ (unit: mm) ,

The weight vector is $\omega_i = \{0 \quad 0 \quad 0.405 \quad 0.0636 \quad 0.742 \quad 0.742\}$,

and the knot vector is $u = [1.6 \quad 1.7 \quad 0.7 \quad 1.25 \quad 0.6 \quad 0.85 \quad 1 \quad 1.2 \quad 1.25]$.

Tab. 1 Machining parameters and dynamics parameters

$J(\mathrm{mm \cdot s^{-2}})$	$F(\mathrm{mm \cdot s^{-1}})$	$T(\mathrm{ms})$	$\delta_{\max}(\mu\mathrm{m})$	$v_{x\max}(\mathrm{mm \cdot s^{-1}})$	$v_{y\max}(\mathrm{mm \cdot s^{-1}})$	$v_{z\max}(\mathrm{mm \cdot s^{-1}})$
38100	240	1.15	1.015	60	85	55

J the allowable acceleration and jerk, F command interpolation federate, T the interpolation period, $\delta_{\max}$ the maximum value of the chord error, $v_{x\max}$ the max feed rate vale of x-axis, $v_{y\max}$,the max feed rate vale of y-axis, $v_{z\max}$ the max feed rate vale of z-axis. By Using computer soft in NURBS curve interpolation, as shown in Tab.2, Figure 3,Figure 4 and Figure 5.

G code

```
G01 X16.549000 Y0.010000 Z-14.222000
G01 X16.447000 Y0.739000 Z-14.096000
G01 X16.349000 Y1.375000 Z-13.986000
G01 X16.170000 Y2.386000 Z-13.829000
G01 X16.003000 Y3.213000 Z-13.728000
G01 X15.892000 Y3.714000 Z-13.678000
G01 X13.913000 Y12.634000 Z-15.078000
G01 X13.882000 Y13.136000 Z-15.309000
G01 X13.876000 Y13.444000 Z-15.446000
G01 X13.869000 Y13.560000 Z-15.502000
G01 X13.862000 Y13.703000 Z-15.564000
G01 X13.855000 Y13.846000 Z-15.626000
```

NURBS Code ↓

```
NURBS p4 {0.000000, 0.000000, 0.000000, 0.000000,
0.385549} X16.549000 Y0.010000 Z- 14.222000 w1.000000
k0.502421 X16.323334 Y1.827069 Z-13.890654 w1.000000
k0.606291 X15.814100 Y4.190536 Z-13.515593 w1.000000
k0.711405 X15.082927 Y7.029935 Z-13.619467 w1.000000
k0.810409 X14.692049 Y8.546189 Z-13.792276 w1.000000
k1.000000 X14.349083 Y9.960615 Z-14.103859 w1.000000
k1.000000 X13.975937 Y11.749510 Z-14.652294 w1.000000
k1.000000 X13.871343 Y13.008857 Z-15.255747 w1.000000
```

The FANUC CNC system has a large share in the current CNC system market, and they make these parameters as part of the NC program command parameters, Interpolation G code shown in Figure 1:

```
G06.2 K3 U0 X0 Y0 Z1 W1.6 F18
U0 X25 Y30 Z6 W1.7
U0 X50 Y50 Z16 W0.7
U0 X65 Y60 Z22 W1.25
U0.4531 X77 Y70 Z30 W0.6
U0.5485 X105 Y57 Z32 W0.85
U0.6306 X132 Y40 Z41 W1
U0.7426 X142 Y30 Z51 W1.2
U1 X152 Y10 Z21 W1.2
U1
U1
```

U1

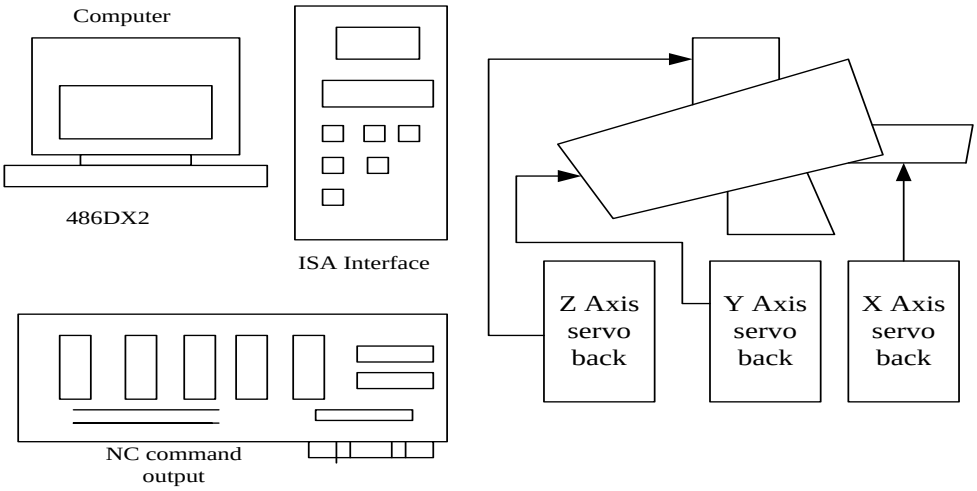

Fig .3: The experimental setup

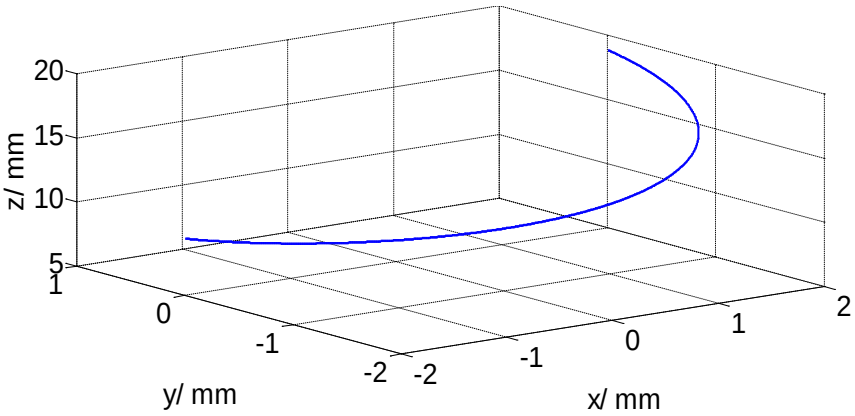

Fig. 4 : NURBS curve interpolation

Table 2 Partial interpolation results of NURBS curve

Number of steps	u	X(mm)	Y(mm)	Z(m)	△L(mm)
420	0.51	0.634955	0.90286	18.4648	0.297
421	0.514	0.637206	0.59191	18.5699	0.3
422	0.5181	0.639471	0.593553	18.6782	0.3
423	0.5223	0.641723	0.595198	18.7886	0.3
424	0.5264	0.643961	0.596847	18.9012	0.3
425	0.5306	0.646186	0.5985	18.0159	0.2665
426	0.5342	0.638152	0.599974	18.1198	0.2387
427	0.5373	0.649895	0.601291	18.3139	0.2192
428	0.54.2	0.651494	0.602507	19.3017	0.2062
429	0.5428	0.65229	0.603653	19.3853	0.1967
430	0.5452	0.654412	0.604749	19.4657	0.1894
431	0.5475	0.655775	0.605805	20.5439	0.1845
432	0. 5497	0. 657097	0. 606836	20. 6207	0. 1851
433	0. 5518	0. 658421	0. 607872	20. 6983	0. 1861
434	0. 5539	0. 659749	0. 608912	21. 7767	0. 1872
435	0. 556	0. 661084	0. 609956	21. 856	0. 1883

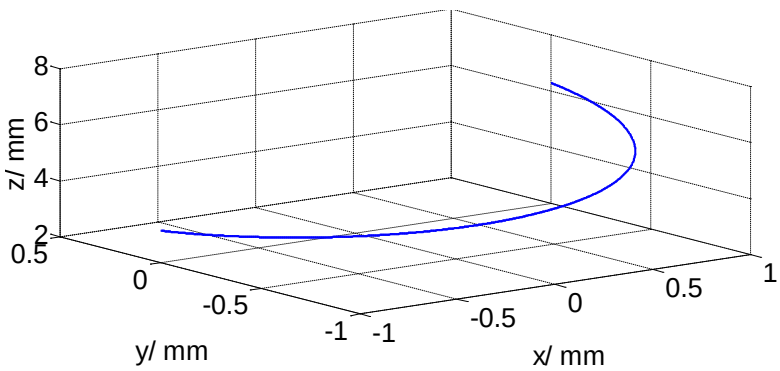

Fig. 5 : Improved algorithm adaptive of NURBS

Table 3 Partial interpolation results of Improved algorithm adaptive of NURBS

Number of steps	u	X(mm)	Y(mm)	Z(m)	△L(mm)
420	0.515	0.6349558	0.5902868	6.4648	0.2975
421	0.5145	0.6372068	0.5919177	6.5699	0.355
422	0.51815	0.6394718	0.593553	6.6782	0.366
423	0.52235	0.6417236	0.5951986	6.7886	0.322
424	0.52645	0.6439617	0.5968475	6.9012	0.3224
425	0.53065	0.6461864	0.598544	6.0159	0.26651
426	0.53425	0.6381525	0.5999744	6.1198	0.23871
427	0.53735	0.6498952	0.6012913	6.3139	0.21922
428	0.54.25	0.6514943	0.6025071	6.3017	0.20623
429	0.54285	0.652291	0.6036532	6.3853	0.19674
430	0.54525	0.65441222	0.6047494	6.4657	0.18944
431	0.54755	0.65577533	0.6058053	6.5439	0.18455
432	0.54975	0.6570973	0.6068365	6.6207	0.18513
433	0.55185	0.6584214	0.6078724	6.6983	0.18611
434	0.55396	0.6597495	0.6089128	6.7767	0.18721
435	0.5566	0.6610846	0.6099567	6.856	0.18831

Tab.4 Table analysis of interpolation of NURBS curve results

Parameters algorithms	Interpolation time(s)	Max step error (mm)	Min step error (mm)
NURBS interpolation	17.84	9.0801	0.712
Newton-Rapson iterative interpolation	15.5	7.9801	0.46

As can be seen from Fig. 4, Fig.5 and table 1 and table 2 , in the process of the interpolation, interpolation time reduced , Max step error deceased, Max step error value is 7.980, which meet the expected to interpolation, i.e. to reduce the compensation error and interpolation step chord error. To verify the high efficiency and reliability of this improved algorithms adaptive of NURBS are applied in the experiments to make a comparison .It can be seen that mproved algorithm adaptive of NURBS is feasible and efficent.

In a word, the algorithm for improved algorithm adaptive of NURBS presented in this paper could satisfy high-speed and high-accuracy interpolation requirements ,which can be used for actual interpolation processing.

6. Conclusions

In the paper,Study on improved algorithm adaptive of NURBS and simulation is introduced. We can use Improved algorithm adaptive of NURBS that calculate (x_i, y_i, z_i). Simulation results show that the proposed NURBS curve interpolator meet the high-speed and high-accuracy interpolation requirements of CNC systems. The interpolation of NURBS curve should be finished. Simulation results show that the proposed NURBS curve interpolator meet the high-speed and high-accuracy interpolation requirements of CNC systems, it is consistent with a NURBS curve interpolation requirements. In addition ,NC machining time can be reduced. Implementation on NC machine has proven the feasibility of a developed interpolation algorithm.

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