Quality Evaluation of World's Top 40 Universities Based on Multivariate Analysis

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Abstract. University comprehensive quality evaluation is an important method to evaluate the quality of World University. Scoring on a university via the study and research of all aspects of a university, is an ideal way to show the whole picture of a university, that is, its quality level is built on the comprehensive evaluation of all aspects. Sampling on 40 world universities' quality evaluation data in 2005, with six indicators as variables, including Score on Alumni (equivalent to the number of Nobel prize and the Fields prize awarded alumni), Score on Award(equivalent to the number of Nobel prize and the Fields prize for teachers), Score on HiCi (The number of faculty who have been cited the highest in each subject area), Score on N&S (The number of papers published in the magazine), Score on SCI (Number of papers indexed by the Science Citation Index and the Social Science Citation Index), and Score on Size (the teachers' mean value of above five indicators), this study takes use of SAS statistical software for cluster analysis. Based on its classification result, this software is used again to carry out discriminant analysis for deeper evaluation of the comprehensive quality of these universities and generate typical correlation analysis to evaluate the reasonability of the indicators.

Keywords: University Quality, Cluster Analysis, Discriminant Analysis, Typical Correlation Analysis,

1. Ranking of world's top 40 universities

The comprehensive quality evaluation data of the world's top 40 universities in 2005 is shown in table 1:

Code	University	Score on Alumni	Score on Award	Score on HiCi	Score on N&S	Score on SCI	Score on Size
1	Harvard Univ	100	100	100	100	100	72.4
2	Univ Cambridge	99.8	93.4	53.3	56.6	70.9	66.9
3	Stanford Univ	41.1	72.2	88.5	70.9	72.3	65
4	Univ California - Berkeley	71.8	76	69.4	73.9	72.2	52.7
5	Massachusetts Inst Tech (MIT)	74	80.6	66.7	65.8	64.3	53
6	California Inst Tech	59.2	68.6	59.8	65.8	52.5	100

Table 1, cont.

7	Columbia Univ	79.4	60.6	56.1	54.2	69.5	45.4
8	Princeton Univ	63.4	76.8	60.9	48.7	48.5	59.1
9	Univ Chicago	75.6	81.9	50.3	44.7	56.4	42.2
10	Univ Oxford	64.3	59.1	48.4	55.6	68.4	53.2
11	Yale Univ	52.1	44.5	60.3	57.2	63.9	49.3
12	Cornell Univ	46.5	52.4	55	48.8	66.3	39.8
13	Univ California – San Diego	17.7	34.7	59.8	56.5	64.5	46.6
14	Univ California – Los Angeles	27.3	32.8	56.7	50.1	75.6	34.3
15	Univ Pennsylvania	35.5	35.1	56.7	42.9	71.8	39.1
16	Univ Wisconsin - Madison	43	36.3	52.1	46.3	68.7	29
17	Univ Washington - Seattle	28.8	32.4	53.9	47.1	73.8	27.2
18	Univ California – San Francisco	0	37.6	55.6	57.9	58.8	45.2
19	Johns Hopkins Univ	51.4	28.3	41.6	52.2	67.7	24.9
20	Tokyo Univ	36	14.4	38.5	52.1	86.5	34.7
21	Univ Michigan – Ann Arbor	43	0	61.9	43	76.5	30.9
22	Kyoto Univ	39.7	34.1	34.2	37	72.3	31.1
23	Imperial Coll London	20.8	38.1	40.8	38.2	64.6	40.3
24	Univ Toronto	28.1	19.7	39.3	38.9	76.7	41.9
25	Univ Illinois – Urbana Champaign	41.6	37.4	44.4	34.1	58	26
26	Univ Coll London	30.7	32.9	37.7	41.5	60.5	38.8
27	Swiss Fed Inst Tech - Zurich	40.2	37	35.1	41.1	43.4	52.4
28	Washington Univ - St. Louis	25.1	26.6	38.5	46.5	53.9	39.9
29	New York Univ	33.8	25	43	35.3	55.4	26.3
30	Rockefeller Univ	22.6	59.8	28.3	44.1	24	35.9
31	Northwestern Univ	21.7	19.3	44.4	33.8	57.6	36.2
32	Duke Univ	20.8	0	47.1	45.3	60.8	38.9
33	Univ Minnesota – Twin Cities	36	0	49.7	35.2	68.4	23.8
34	Univ California – Santa Barbara	0	36	42.3	39	44.1	35.8
35	Univ Colorado - Boulder	16.6	29.8	40.8	36.6	46.3	29.5
36	Univ Texas - Austin	21.7	17.1	49.1	30	54.8	21.7
37	Univ British Columbia	20.8	19.3	32.4	32.5	60.4	33.9

Table 1, cont.

8	Univ Texas Southwestern Med Center	24.3	33.9	31.4	38.2	37.9	31
39	Pennsylvania State Univ - Univ Park	14	0	45.8	37.9	59.9	24
40	Vanderbilt Univ	12.5	30.2	34.2	24.5	49.2	35.6

Table 1 Data Sheet of Comprehensive Quality Evaluation of 40 Universities in the World in 2005

2. Clustering analysis

2.1. Group-average cluster method
2.1.1. Root mean square of the data

The sample size is 40, so the distance among the various universities is $C_{40}^2 = 780$, distance root mean square = 72.42767. There are 38 clusters in this method, and the number of universities in each category varies from 2-40.

2.1.2. Pseudo-F statistic

$$F = \frac{(W - P_m)/(m-1)}{P_m/(n-m)}$$

(1)

Pseudo-F is used to evaluate clustering effects of m clusters. The larger the value of the pseudo-F statistic, the more distinctive n samples can be divided into m clusters. Pseudo-F statistics can be used as a useful indicator to determine the number of clusters. In this example, the normalized Euclidean distance between the first clusters is the smallest, and the maximum of pseudo-F statistic is 51.4, and the pseudo-F is the minimum in last clustering.

2.1.3. Pseudo-t^2 statistics

$$Pseudot^2 = \frac{D_{KL}^2}{(W_K - W_L)/(n_K + n_L - 2)}$$

(2)

In this formula $D_{KL}^2 = W_M - W_K - W_L$ is the new cluster G_M increment of Sum of Squared Deviations produced by the merging of combines Cluster G_K with Combine

Cluster G_L. Pseudo-t^2 is used to evaluate the effect of merging clusters G_K and G_L.

The larger the t^2, indicating after merging into new cluster G_M by combining G_K and G_L, the increment D^2_{KL} is larger than the original sum of squared deviations of G_K and G_L, which means that the two clusters are separated well, that is, the effect of the last cluster is good.

Pseudo t^2 statistics are useful indicators for determining the number of clusters. Pseudo t^2 is minimum in the first clustering. Analogy accordingly, the final clustering is the maximum clustering formed by the 39th universities' clustering with the first group clustering, and the standardized root mean square distance between them is the largest, and the pseudo t^2 is maximum, equals to 42.8. The arborescence of the average clustering method is shown in figure 1.
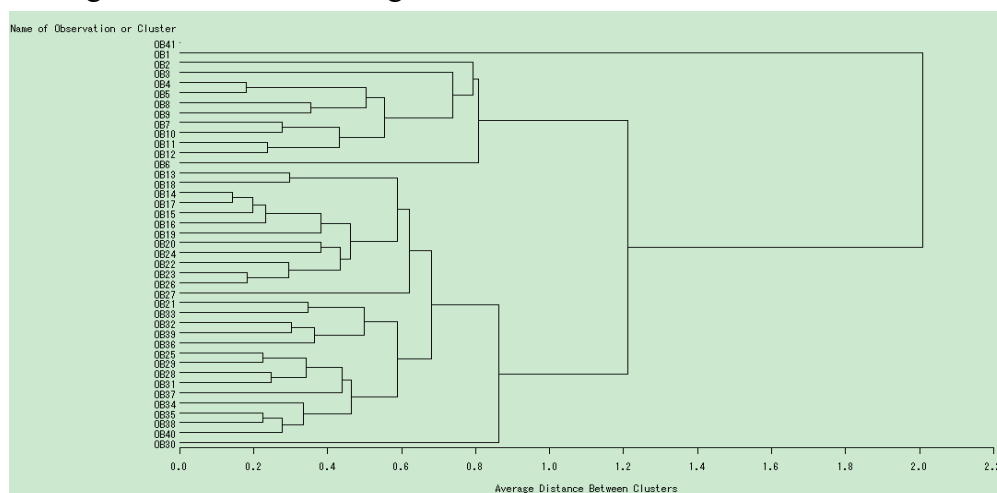

Figure 1 The arborescence produced under the average clustering method

2.2 The shortest distance clustering method
The shortest distance clustering method is similar to the group-average cluster method, while we just offer a brief analysis here.
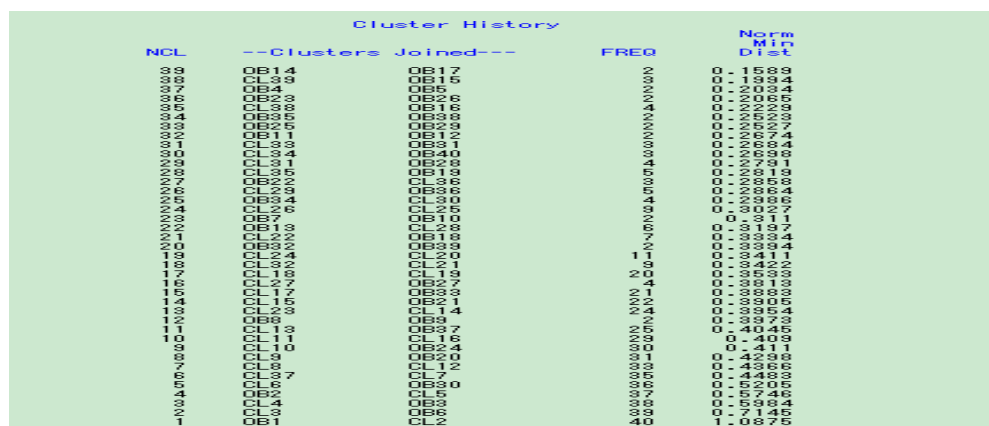
2.2.1. The average distance among observed values.
From SAS analysis, the average distance of observed value between these 40 observations is 64.40822. There are 39 clusters in this approach, with the number of universities in each group ranging from 2 to 40.

2.2.2. Normal Min Distance (Norm Min Dist.).
It can be seen from figure 2 that the shortest distance from the top to the bottom

increases from 0.1589 to 1.0875.

Figure 2 The normal minimum distance of the universities

The arborescence under the shortest distance clustering method is shown in Figure 3.

Figure 3 The arborescence under the shortest distance clustering method

2.3 Conclusion

According to the clustering arborescence under the group-average cluster method, we divided 40 universities into two types: the two universities ranked as the last two in the Excel table belong to Type 2, while the remaining 38 universities are Type 1.

3. The principal component analysis

Principal component analysis (PCA) is a statistical analysis method which reduces several indicators into a few comprehensive indexes by using dimension reduction technology. By using of this method, we analyzed the quality of the 40 universities in the world in 2005 and summed up the six variables into one or two comprehensive variables to help us analyze the data more easily. From the SAS operating results, we can gain the following information via analyzing.

3.1. The extraction of principal components

3.1.1. Basic information

a. The correlation matrix between 10 variables

Figure 4 The correlation matrix between 10 variables

b. Eigenvalues and eigenvectors

Eigenvectors	**T1**	**T2**	**T3**
Score on Alumni	0.393410	-.493462	-.340283
Score on Award	0.431409	0.276845	0.238879
Score on HiCi	0.448854	0.080797	0.194581
Score on N&S	0.336267	0.725076	-.156377
Score on SCI	0.405146	-.356288	0.628191
Score on Size	-.424767	0.143696	0.608445
Eigenvalues	4.58763836	0.83401292	0.27802842
Contribution rate	0.7646	0.1390	0.0463
Accumulative contribution rate	0.7646	0.9036	0.9499

Table 3 The first three eigenvalues and eigenvectors

We proceed from the correlation matrix for principal component analysis. It can be seen from Table 4-1 that the cumulative contribution rate of the first two principal components has reached 90.36%, and the cumulative contribution rate of the first three principal components is 94.99%, indicating that the first two principal components have contented most of the information, hence the first two principal components are considered to be taken, since they can be a good summary of this set of data.

In this research, the first principal component has approximately equal positive payloads on Score on Score on Alumni x1、Score on Award x2、Score on HiCi x3、Score on N&S x4、Score on SCI x5、Score on Size x6, and thus can be considered as a measurement on the five indicators (except Score on Size) of the universities. The second principal component has a high positive load on the variable Score on N & S (x4) and a high negative load on the variable Score on Alumni (x1). This principal component can be considered to be suitable for measuring the proportion of the

university quality parameters.

3.2. The analysis of principal components

First of all, 40 universities have been ranked based on the first principal component and the second principal component. The 39th university is -2.29007 after the first principal component scores are ranked in descending order, and the university has all five indicators with scores below 50, in particular its Score on Alumni is 0 point, so the university is the worst in terms of the quality among 40 universities.

On the contrary, the first university has the highest score of 6.11023, the university's comprehensive level is ranked the first, the evaluation indicators are also outstanding, so the quality evaluation result of this University is excellent. Similarly, it is possible to analyze the case which is sorted by the second principal component.

Scatterplot of the first and second principal components output is shown as in figure 11:
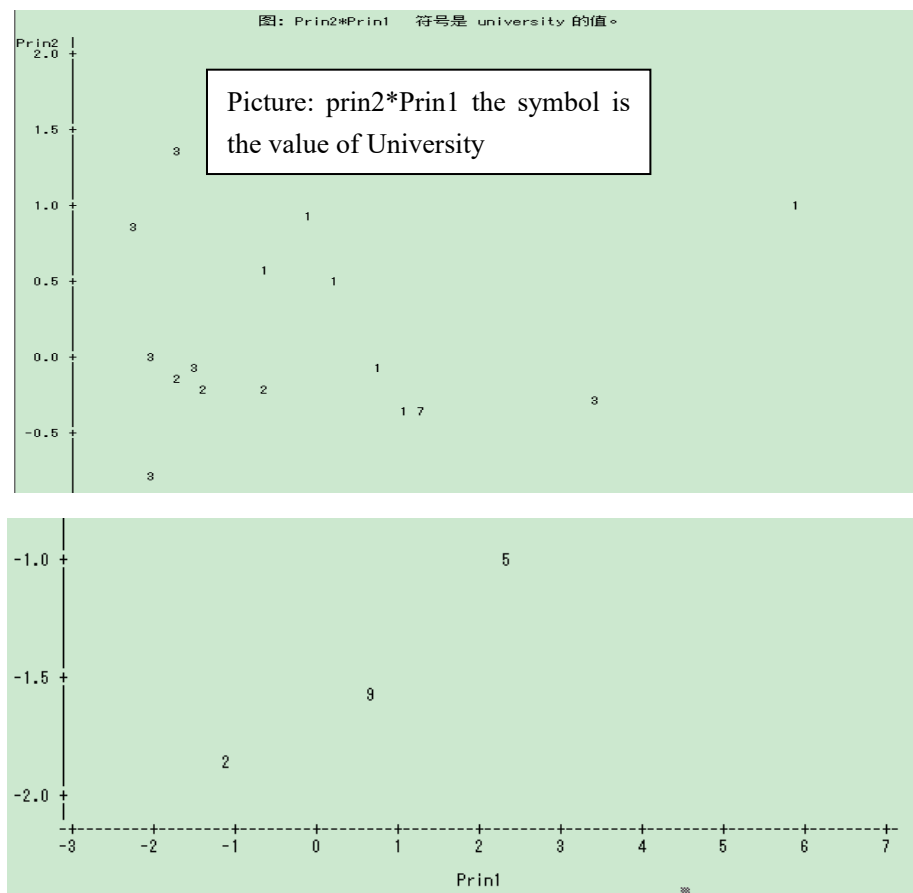

Figure 5 Scatterplot of the first and second principal components output

The scatterplot has a more intuitive description for the N & S and Alumni of University. Each scatter point has been output by tens of observations, and the duplication of characters can be identified by contracting against SAS output table.

As it can be seen in the scatterplot, among all observations, the 1st university is on the far right, indicating that the N & S quality of this university is high. While observing other data shows that the university also has strong comprehensive strength. Thought the observed value 1 is no longer ranks first in the primal component 2, but it still ranks high, indicating that the Alumni quality of this University is lower than N & S quality, but it is still in leader place compared with other universities. Observations of the 39th University is at the far left, indicating that it has the lowest N & S quality in the current year compared to other universities. Similarly, all observed values in the table can be analyzed.

4. Typical correlation analysis

Canonical correlation analysis is a statistical method to study the correlation between two groups of variables, which can truly reflect the mutual linear dependency between two groups of variables.

In this part, the data of 40 universities' evaluation indexes in the world are divided into two groups. Score on Alumni, Score on Award, and Score on HiCi are used as a set of quality indicators. Score on N & S, Score on SCI, Score on Size are selected as another set of quality indicators, and then conduct a typical correlation analysis between the two groups of quality indicators.

4.1 Typical correlation variables

In the SAS output of the canonical correlation analysis of data, we can get the mean and standard deviation of score variables of each university group. The correlation coefficient matrix of the two quality indexes and the correlation coefficient matrix between the two groups are as follows:

$$R_{xx} = \begin{Bmatrix} 1.0000 & 0.7498 & 0.5476 \\ 0.7498 & 1.0000 & 0.5449 \\ 0.5476 & 0.5449 & 1.0000 \end{Bmatrix} \quad (3)$$

$$R_{yy} = \begin{Bmatrix} 1.0000 & 0.5058 & 0.7091 \\ 0.5058 & 1.0000 & 0.1365 \\ 0.7091 & 0.1365 & 1.0000 \end{Bmatrix} \quad (4)$$

$$R_{xy} = \begin{Bmatrix} 0.6392 & 0.4304 & 0.5627 \\ 0.6896 & 0.0867 & 0.7210 \\ 0.8222 & 0.5736 & 0.5552 \end{Bmatrix} \quad (5)$$

Eigenvalue of $R_{xx}^{-1}R_{xy}R_{yy}^{-1}R_{yx}$ is $\lambda_1 = 3.5231$, $\lambda_2 = 0.9049$, $\lambda_3 = 0.0470$.

The corresponding raw coefficients are shown in figure 19.

Figure 6 The raw coefficients

From this we can get the first pair of typical variables:

$$u_1 = \hat{a}_1^T X = 0.007151X_1 + 0.010666X_2 + 0.047937X_3 \tag{6}$$

$$v_1 = \hat{b}_1^T Y = 0.054266Y_1 + 0.011004Y_2 + 0.013528Y_3 \tag{7}$$

Similarly, the second and third pairs of typical variables can be obtained. Typical canonical correlation coefficients are shown in figure 20:

Figure 7 Typical canonical correlation coefficients

We can get the first pair of typical variables:

$$u_1^* = \hat{a}_1^{*T} X^* = 0.1730X_1^* + 0.2735X_2^* - 0.6963X_3^* \tag{8}$$

$$v_1^* = \hat{b}_1^{*T} Y^* = 0.7632Y_1^* + 0.1491Y_2^* + 0.2102Y_3^* \tag{9}$$

Similarly, the second and third pairs of typical variables can be obtained.

4.2 The test of the typical correlation coefficient significance

Next, we perform a canonical correlation analysis:

Establish the null hypothesis H_0: $\rho_1 = \rho_2 = ... = \rho_m = 0$, (10) that the alternative

assumes H_1: $\rho_1, \rho_2, ..., \rho_m$, and at least one of them is not zero. Construct likelihood

ratio statistics $\Lambda = \prod_{i=1}^{m}(1 - r_i^2)$, (11) for sufficient large n, when H_0 established, the

statistic $Q = -\left[n - \frac{1}{2}(p + q + 3)\right]\ln \Lambda$ (12

) nearly obeyed the distribution of χ^2 of pq . At a given level of significance = 0.05,

if $Q \geq \chi_\partial^2(pq)$, the original hypothesis is rejected, the correlation of the representative

variables u_1 and v_1 is significant, otherwise the typical correlation coefficient is not

significant.

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                    Canonical Correlation Analysis

                            Adjusted    Approximate        Squared
                 Canonical  Canonical    Standard         Canonical
                Correlation Correlation    Error         Correlation

             1   0.882561   0.869909     0.035402         0.778913
             2   0.689237   0.680027     0.084060         0.475048
             3   0.211906       .        0.152938         0.044904

                                       Test of H0: The canonical correlations in the
              Eigenvalues of Inv(E)*H      current row and all that follow are zero
               = CanRsq/(1-CanRsq)
                                         Likelihood Approximate
      Eigenvalue Difference Proportion Cumulative  Ratio    F Value Num DF Den DF Pr > F

   1    3.5231    2.6182     0.7873     0.7873  0.11084837   13.53    9  82.898 <.0001
   2    0.9049    0.8579     0.2022     0.9895  0.50137928    7.21    4     70  <.0001
   3    0.0470               0.0105     1.0000  0.95509574    1.69    1     36  0.2015
```

Figure8 Results of canonical correlation analysis

As it can be seen from figure 21, v1 and u1, u2, u3 of the typical correlation coefficients are not 0, they are respectively 0.882561, 0.689237 and 0.211906.

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                Multivariate Statistics and F Approximations

                      S=3    M=-0.5    N=16

     Statistic                   Value    F Value   Num DF   Den DF   Pr > F

     Wilks' Lambda             0.11084837   13.53      9     82.898   <.0001
     Pillai's Trace            1.29886555    9.16      9       108    <.0001
     Hotelling-Lawley Trace    4.47506124   16.57      9      50.34   <.0001
     Roy's Greatest Root       3.52310918   42.28      3        36    <.0001

          NOTE: F Statistic for Roy's Greatest Root is an upper bound.
```

Figure 9 Multivariate statistics and F approximations

As shown in figure 22, p values tested by Wilks' Lambda, Pillai's Trace, Hotelling-Lawley Trace and Roy's Greatest Root are less than 0.0001. Thus, the correlation of the visible indicators is significant based on the above analysis. As a result, we can find that it is necessary to evaluate Alumni, Award, HiCi, N & S, SCI, Size as the quality indicators.

4.3 Brief Summary

According to the results of Canonical Correlation Analysis and the actual situation of each university, there is a strong correlation among Alumni, Award, HiCi, N & S, SCI and Size indicators which represent the overall quality of the university. Therefore, in the process of perfecting themselves, the university should not only focus on their single-level of their own access to the honor or the size. And for people to choose the school, besides the focus of whether the school is of a certain level of excellency or deficiency, the comprehensive aspects of the certain university should also be concerned. By doing this, on the one hand, each university can better balance the development of all directions when planning the future development. On the other hand, students can objectively choose the school with excellent comprehensive strength.

5. Conclusion

After multivariate analysis, we can find that university quality assessment plays an important role in measuring university comprehensive level. Although the selection of this sample size represents only index score of a year, it reflects the quality of scientific assessment of the University to a certain extent.

We are able to analyze the quality of the universities in each group, and we can classify the two groups into two types, namely, the excellent type and the general type, so as to judge the comprehensive quality of the university objectively. In addition, it is important to choose principal indicators for judgment, and to conduct the extreme value analysis, by which to comprehensively assess the quality of the university. One thing need extra attention that although these indicators are designed very scientifically, in discriminant analysis, we still found wrong classification of the universities, hence this quality evaluation system is worth to be further improved, so as to provide a more reasonable comprehensive quality reflection of the University.

6. Reference

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