A Backstepping Control Method for Mobile Robot Path Tracking
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Abstract. In this paper, the kinematic model of the mobile robot is built. Based on Backstepping time-varying state feedback method and Lyapunov theory, the time-varying feedback control law were designed and the global asymptotic stability is proved. Numerical simulations were conducted to show the effectiveness of the proposed algorithms. Critical errors about the mobile robot tracks different trajectories which makes the control algorithm does not converge were got.

Introduction
Control of wheeled mobile robots has attracted the attention of many researchers in robotic control because of their difficulties in control design and implementation[1]. Wheeled mobile robots with differential drive are typical nonholonomic systems[2,14]. Due to the non-integrity of the robot, the robots do not satisfy the Brockett's smooth stabilization condition, which makes the robot's tracking and attitude stabilization control (position, direction) very hard. Currently, most works are mainly based on searching for linearization, discontinuous control law, time-varying, dynamic feedback control law or mixed control law etc. However, real experimental research is still rare, these theoretical control algorithm applied to the actual robot control, still need to carry out systematic experimental study[3,12,13].

Establish the kinematic model of the wheeled mobile robot. A simple virtual feedback variable is constructed by the integral Backstepping[11] idea, and the time-varying feedback control rate is designed by using the Lyapunov direct method, and it is proven that the control effect can reach the global asymptotic stability. Discussed the influence of the errors in the kinematic model and errors in the localization of mobile robot on the convergence of the control algorithm. The simulation results show that the robot can track the desired trajectory quickly and efficiently under the control law. Through the simulation, critical errors about the mobile robot tracks different trajectories which makes the control algorithm does not converge were got.

Kinematic Model of Mobile Robot
The robot used in this study is a differential drived wheeled mobile robot. The two drive wheels are independent of each other and can be rotated forward or reverse. The center point $C$ of the two driving wheels of the mobile robot is regarded as the center of its movement[4-6]. The moving robot has a linear velocity $v$ and an angular velocity $\omega$.

![Figure 1. Kinematic analysis of mobile robot](image-url)
As shown in Figure 1, \(x \) and \(y \) are the coordinates of the center of mass of the platform, \(\theta \) is the angle between the heading direction and the OX axis specifying the orientation of the local platform with respect to the inertial frame. It is assumed that the movement of the mobile robot is an ideal motion, and there is no slipping phenomenon. Kinematic equations of the two-wheeled mobile robot are:

\[
\begin{align*}
\dot{x} &= v \cos(\theta) \\
\dot{y} &= v \sin(\theta) \\
\dot{\theta} &= \omega
\end{align*}
\]

Discrete (1):

\[
\begin{align*}
x_{k+1} &= x_k + v_k \cos(\theta_k) \times \Delta t \\
y_{k+1} &= y_k + v_k \sin(\theta_k) \times \Delta t \\
\theta_{k+1} &= \theta_k + \omega \times \Delta t
\end{align*}
\]

Where \(\Delta t\) is the sampling time interval, it is considered that the motion parameters of the mobile robot in the sampling time interval are invariable.

**Trajectory Tracking Algorithm**

The pose of the mobile robot in two-dimensional coordinate system is expressed by vector \(p=(x,y,\theta)^T\), the moving robot who has a velocity \(q=(v,\omega)^T\) tracking a reference car with velocity \(q_r=(v_r,\omega_r)^T\) and pose \(p_r=(x_r,y_r,\theta_r)^T\) as shown in Figure 2.

![Figure 2. Position error of mobile robot](image)

The pose error is defined as \(p_e=(x_e,y_e,\theta_e)^T\) in the local coordinate system of mobile robot. So the pose error equation is\[7][8]:

\[
p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} (p_r - p) = T_e (p_r - p)
\]

Where \(T_e\) is the transfer matrix, taking the derivative of equation(3), the following differential equation\[9\] are got:

\[
p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v \cos(\theta) \\ -\omega x_e + v \sin(\theta) \\ \omega - \omega \end{bmatrix}
\]

The tracking problem\[10\]of mobile robot based on kinematic model is to determine the velocity \(q=(v,\omega)^T\) of the mobile robot, and make the robot follow the reference position \(p_r=(x_r,y_r,\theta_r)^T\) and the
reference velocity \( q = (v, \omega)^T \) under the initial error and make \( p_e \) bounded and \( \lim_{t \to \infty} \| (x, y, \theta) \| = 0 \).

The structure of trajectory tracking control system of mobile robot is shown in Fig 3.

\[
q = (v, \omega)^T
\]

![Figure 3. Mobile Robot Path Tracking Control System](image)

According to the backstepping idea, the error component \( x_e \) in the robot system is regarded as a virtual control variable, \( a_1 \sin(\arctan(\omega)) y_e \) as a virtual feedback, so, the new virtual error variable is:

\[
\dot{x}_e = x_e - a_1 \sin(\arctan(\omega)) y_e
\]

(5)

Where \( a_1 \) is a constant greater than 0. If the control action makes the error component \( x_e \to a_1 \sin(\arctan(\omega)) y_e \), and \( \theta_e \to 0 \), according to equation (4), then \( y_e \to a_1 \cos(\arctan(\omega)) y_e \) can be guaranteed. Simultaneously for Lyapunov function \( V = \frac{1}{2} y_e^2 \), find the time derivative, obtained \( \dot{V} = y_e \dot{y}_e \), because \( \cos(\arctan(\omega)) \geq 0 \). So according to barbalat theorem, it’s known when \( t \to \infty \), \( y_e \) converges to 0, as the sequel, \( y_e \) is indirectly controlled. In summary, the essence of the control law is to seek the input control \( q = (v, \omega)^T \), when \( t \to \infty \), \( x_e \to a_1 \sin(\arctan(\omega)) y_e \), \( \theta_e \to 0 \). The Lyapunov function is constructed according to the above idea:

\[
V = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 + \frac{2}{a_3} (1 - \cos(\frac{\omega}{2}))
\]

(6)

where \( a_3 \) is a constant greater than 0. Because \( a_3 > 0 \), it is clear that \( V \geq 0 \), only if when \( (x_e, y_e, \theta_e)^T = 0 \), \( V = 0 \). Equation (5) is converted to the following equation:

\[
\dot{x}_e = x_e - a_1 \cos(\arctan(\omega)) \frac{1}{1 + \omega^2} \omega y_e
\]

(7)

Derivating the Lyapunov function and determines the control law:

\[
\dot{V} = x_e \dot{x}_e + y_e \dot{y}_e + \frac{1}{a_3} \sin(\frac{\omega}{2}) \dot{\theta}_e
\]

\[
= x_e \left[ x_e - a_1 \cos(\arctan(\omega)) \frac{1}{1 + \omega^2} \omega y_e - a_1 \sin(\arctan(\omega)) y_e \right]
\]

\[
+ y_e \left[ -\omega x_e + v_i \sin(\frac{\omega}{2}) (\omega y_e - \omega) + \frac{1}{a_3} \sin(\frac{\omega}{2}) (\omega y_e - \omega) \right]
\]

\[
= x_e \left[ a_1 \cos(\arctan(\omega)) \frac{1}{1 + \omega^2} \omega y_e \right]
\]

\[
- a_1 \sin(\arctan(\omega)) y_e \right] + y_e \left[ -\omega x_e + a_1 \sin(\arctan(\omega)) y_e \right]
\]

\[
+ v_i \sin(\frac{\omega}{2}) (\omega y_e - \omega)
\]

(8)
\[ x = \frac{1}{1 + \omega^2} \omega y \]

\[-a_2 \sin(\arctan(\omega))(-a \omega x + v \sin(\theta)) - a_3 y_2 \sin(\arctan(\omega))
+ \frac{1}{a_4} \sin(\frac{\theta}{2})[-\omega + \omega y + 2a_1 y_2 \cos(\frac{\theta}{2})] \]

Take the control law of the system:

\[ v = v_r \cos(\theta_r) + a_1 \sin(\arctan(\omega))x_r \]

\[-a_2 v_r \sin(\arctan(\omega)) \sin(\theta_r)
- a_1 \cos(\arctan(\omega)) \frac{1}{1 + \omega^2} \omega y_r \]

\[ + a_3 [x_r - a_1 \sin(\arctan(\omega)) y_r] \]

\[ \omega = \omega_r + 2a_1 y_2 v_r \cos(\frac{\theta_r}{2}) + a_4 \sin(\frac{\theta_r}{2}) \]

Where \( a_2, a_4 \) is a constant greater than 0. Equation (9) is taken into Equation (8):

\[ V = -a_2 x_2^2 - a_3 y_2 \sin(\arctan(\omega)) - \frac{a_4}{a_3} \sin^2(\frac{\theta_r}{2}) \]

Since \( a_1, a_2, a_3, a_4 \) are constants greater than 0, and \( \omega \sin(\arctan(\omega)) \geq 0 \), so \( V \leq 0 \). It can be seen that \( V \) is a positive definite continuously differentiable function and bounded. \( V \) is a negative semidefinite continuous function, then according to Barbalat theorem we can know that when \( t \) converge to gigantic, \( V \) converge to 0, thus \( x_2^2, y_2^2 \sin(\arctan(\omega)), \sin^2(\theta_r / 2) \) converge to 0 respectively, \( \theta_r \rightarrow 0, x_e \rightarrow a_1 \sin(\arctan(\omega))y_e \) can be obtained. From \( t \rightarrow \infty \), \( v_r \) and \( \omega_r \) do not converge to 0 at the same time \( \omega \) does not converge to 0 can be know. And because \( y_2^2 \sin(\arctan(\omega)) \rightarrow 0 \), \( y_e \rightarrow 0 \) and \( x_e \rightarrow a_1 \sin(\arctan(\omega))y_e \) can be obtained, so \( x_e \rightarrow 0 \). Because when \( t \rightarrow \infty \), \( \sin^2(\theta_r / 2) \rightarrow 0 \), so \( \theta_e \rightarrow 0 \) is know. Based on the above analysis: the pose error of closed loop system is globally uniformly bounded, and when \( t \rightarrow \infty \), \( ||x_e(t) + y_e(t) + \theta_e(t)|| \rightarrow 0 \) is concluded.

Further Analysis

From the analysis of the previous section, the control law of trajectory tracking of mobile robot can be obtained and the pose error of closed loop system is globally uniformly bounded and when \( t \rightarrow \infty \), \( ||x_e(t) + y_e(t) + \theta_e(t)|| \rightarrow 0 \) is concluded. But in fact, there is a structural error in the mobile robot and there is a wheel slip in the movement, which lead to errors in the kinematic model of the mobile robot. And there is an error in the sensor measurement which makes errors in the localization of mobile robot. From above analysis we have a question how large the kinematic model error and localization error are, the control algorithm does not converge. For this situation, we can add the noise to the real position of the mobile robot and kinematic model to get the critical error.

Simulation

In the Matlab environment, the effectiveness of the above algorithm is verified by controlling the linear trajectory tracking, sinusoidal trajectory and arbitrary trajectory of the mobile robot. Critical errors about the mobile robot tracks different trajectories which makes the control algorithm does not converge were got.

Linear Trajectory

The linear velocity and the angular velocity of the reference robot are \( v_r = 0.3 \text{ m/s} \) and \( \omega_r = 0 \), the
initial pose in the global coordinate system are \( x_r(0) = 1 \text{m}, y_r(0) = 0, \theta_r(0) = \pi/3 \text{rad} \). At the same time, the initial position of the mobile robot in the global coordinate system are \( x(0) = 1.2 \text{m}, y(0) = -2 \text{m}, \theta(0) = \pi/2 \text{rad} \). Take the control parameters \( a_1 = 3, a_2 = 4, a_3 = 11, a_4 = 4 \). Linear trajectory effect is shown in Figure 4.

\[ \text{Figure 4. Linear trajectory} \]

Sinusoidal Curves Trajectory

The initial position of the sine desired trajectory is \( y_r = 3 \sin(2 \pi x_r/10), x_r(0) = 0, y_r(0) = 0, \theta_r(0) = 3\pi/5 \text{rad} \). At the same time, the initial position of the mobile robot in the global coordinate system are \( x(0) = 1 \text{m}, y(0) = -1 \text{m}, \theta(0) = 2\pi/3 \text{rad} \). Take the control parameters \( a_1 = 0.2, a_2 = 2, a_3 = 45, a_4 = 5 \). Sinusoidal curves trajectory effect is shown in Figure 5.

\[ \text{Figure 5. Sinusoidal curves trajectory} \]

Arbitrary Trajectory

The initial position of the arbitrary trajectory are \( x_r(0) = 1 \text{m}, y_r(0) = 0, \theta_r(0) = \pi/2 \text{rad} \). At the same time, the initial position of the mobile robot in the global coordinate system are \( x(0) = 1.2 \text{m}, y(0) = 0.8 \text{m}, \theta(0) = 3\pi/4 \text{rad} \). Take the control parameters \( a_1 = a_2 = 0.01, a_3 = 1.5, a_4 = 15.2 \). Sinusoidal curves trajectory effect is shown in Figure 6.
Let the mobile robot track the same trajectory in 5.1 and 5.3, gradually increase the localization noise of the mobile robot. Linear trajectory and Sinusoidal Curves Trajectory pose error time response curve is shown in Figure 7. When localization errors reached $\Delta x = 0.15m$, $\Delta y = 0.15m$, $\Delta \theta = \pi/11rad$, the control algorithm does not converge for linear trajectory. When localization errors reached $\Delta x = 0.3m$, $\Delta y = 0.3m$, $\Delta \theta = \pi/9rad$, the control algorithm does not converge for sinusoidal curves trajectory.

Let the mobile robot track the same trajectory in 5.1 and 5.2, gradually increase the kinematic model noise of the mobile robot. Linear trajectory and Sinusoidal Curves Trajectory pose error time response curve is shown in Figure 8. When kinematic model errors reached $\Delta x = 0.08m$, $\Delta y = 0.08m$, $\Delta \theta = \pi/18rad$, the control algorithm does not converge for Linear trajectory. When kinematic model errors reached $\Delta x = 0.2m$, $\Delta y = 0.2m$, $\Delta \theta = \pi/16rad$, the control algorithm does not converge for sinusoidal curves trajectory.
Tracking with Errors in Localization and Kinematic Model

Let the mobile robot track the same trajectory in 5.1 and 5.2, gradually increase the kinematic model noise and localization noise of the mobile robot. Linear trajectory and Sinusoidal Curves Trajectory pose error time response curve is shown in Figure 9. When localization errors reached $\Delta x = 0.1\, \text{m}$, $\Delta y = 0.1\, \text{m}$, $\Delta \theta = \pi/11\, \text{rad}$, and kinematic model errors reached $\Delta x = 0.05\, \text{m}$, $\Delta y = 0.05\, \text{m}$, $\Delta \theta = \pi/18\, \text{rad}$, the control algorithm does not converge for linear trajectory. When localization errors reached $\Delta x = 0.13\, \text{m}$, $\Delta y = 0.13\, \text{m}$, $\Delta \theta = \pi/25\, \text{rad}$, and kinematic model errors reached $\Delta x = 0.2\, \text{m}$, $\Delta y = 0.2\, \text{m}$, $\Delta \theta = \pi/15\, \text{rad}$, the control algorithm does not converge for sinusoidal curves trajectory.

Summary

In this paper, the trajectory tracking of wheeled mobile robot with nonholonomic constraints is discussed. And the global trajectory tracking controller is constructed by backstepping method. The simulation results show a good performance using backstepping method to let the mobile robot track different trajectories. The simulation results also show that the influence of the errors in the kinematic model and errors in the localization of mobile robot on the convergence of the control algorithm.
References


