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A Backstepping Control Method for Mobile Robot Path Tracking

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Abstract. In this paper, the kinematic model of the mobile robot is built. Based on Backstepping time-varying state feedback method and Lyapunov theory, the time-varying feedback control law were designed and the global asymptotic stability is proved. Numerical simulations were conducted to show the effectiveness of the proposed algorithms. Critical errors about the mobile robot tracks different trajectories which makes the control algorithm does not converge were got.

Introduction

Control of wheeled mobile robots has attracted the attention of many researchers in robotic control because of their difficulties in control design and implementation[1].

Wheeled mobile robots with differential drive are typical nonholonomic systems[2,14]. Due to the non-integrity of the robot, the robots do not satisfy the Brockett's smooth stabilization condition, which makes the robot's tracking and attitude stabilization control (position, direction) very hard. Currently, most works are mainly based on searching for linearization, discontinuous control law, time-varying, dynamic feedback control law or mixed control law etc. However, real experimental research is still rare, these theoretical control algorithm applied to the actual robot control, still need to carry out systematic experimental study[3,12,13].

Establish the kinematic model of the wheeled mobile robot. A simple virtual feedback variable is constructed by the integral Backstepping[11] idea, and the time-varying feedback control rate is designed by using the Lyapunov direct method, and it is proven that the control effect can reach the global asymptotic stability. Discussed the influence of the errors in the kinematic model and errors in the localization of mobile robot on the convergence of the control algorithm. The simulation results show that the robot can track the desired trajectory quickly and efficiently under the control law. Through the simulation, critical errors about the mobile robot tracks different trajectories which makes the control algorithm does not converge were got.

Kinematic Model of Mobile Robot

The robot used in this study is a differential drived wheeled mobile robot. The two drive wheels are independent of each other and can be rotated forward or reverse. The center point C of the two driving wheels of the mobile robot is regarded as the center of its movement[4-6]. The moving robot has a linear velocity v and an angular velocity ω.

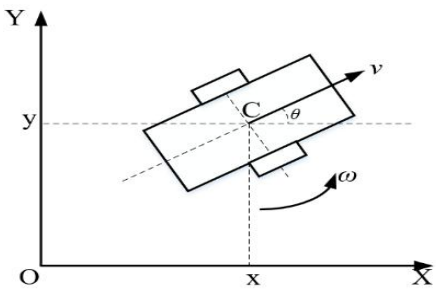


Figure 1. Kinematic analysis of mobile robot

As shown in Figure 1, x and y are the coordinates of the center of mass of the platform, θ is the angle between the heading direction and the OX axis specifying the orientation of the local platform with respect to the inertial frame. It is assumed that the movement of the mobile robot is an ideal motion, and there is no slipping phenomenon. Kinematic equations of the two-wheeled mobile robot are:

$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

Discrete (1):

$$\begin{aligned} x_{k+1} &= x_k + v_k\cos(\theta_k)\times\Delta t \\ y_{k+1} &= y_k + v_k\sin(\theta_k)\times\Delta t \\ \theta_{k+1} &= \theta_k + \omega\times\Delta t \end{aligned} \tag{2}$$

Where Δt is the sampling time interval, it is considered that the motion parameters of the mobile robot in the sampling time interval are invariable.

Trajectory Tracking Algorithm

The pose of the mobile robot in two-dimensional coordinate system is expressed by vector $p=(x,y,\theta)^T$, the moving robot who has a velocity $q=(v,\omega)^T$ tracking a reference car with velocity $q_r=(v_r,\omega_r)^T$ and pose $p_r=(x_r,y_r,\theta_r)^T$ as shown in Figure 2.

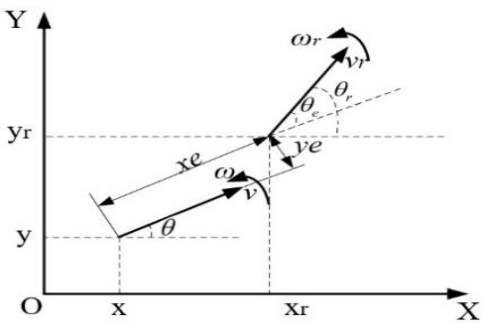


Figure 2. Position error of mobile robot

The pose error is defined as $p_e=(x_e,y_e,\theta e)^T$ in the local coordinate system of mobile robot. So the pose error equation is[7][8]:

$$p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}(p_r - p) = T_e(p_r - p) \tag{3}$$

Where T_e is the transfer matrix, taking the derivative of equation(3), the following differential equation[9] are got:

$$\dot{p}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v_r\cos(\theta_e) \\ -\omega x_e + v_r\sin(\theta_e) \\ \omega_r - \omega \end{bmatrix} \tag{4}$$

The tracking problem[10]of mobile robot based on kinematic model is to determine the velocity $q=(v,\omega)^T$ of the mobile robot, and make the robot follow the reference position $p_r=(x_r,y_r,\theta_r)^T$ and the

reference velocity $q_r=(v_r,\omega_r)^T$ under the initial error and make p_e bounded and $\lim_{t\to\infty}\|(x_e,y_e,\theta_e)^T\|=0$.

The structure of trajectory tracking control system of mobile robot is shown in Fig 3.

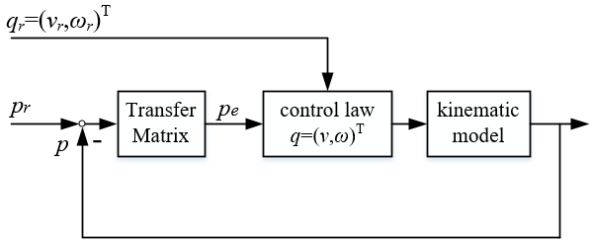


Figure 3. Mobile Robot Path Tracking Control System

According to the backstepping[11] idea, the error component x_e in the robot system is regarded as a virtual control variable, $a_1\sin(\arctan(\omega))y_e$ as a virtual feedback, so, the new virtual error variable is:

$$\bar{x}_e = x_e - a_1\sin(\arctan(\omega))y_e \tag{5}$$

Where a_1 is a constant greater than 0. If the control action makes the error component $x_e\to a_1\sin(\arctan(\omega))y_e$, and makes $\theta_e\to 0$, according to equation (4), then $\dot{y}_e\to -a_1\omega\sin(\arctan(\omega))y_e$ can be guaranteed. Simultaneously for Lyapunov function $V_y=\frac{1}{2}y_e^2$, find the time derivative, obtained $\dot{V}_y = y_e\dot{y}_e$, because $\omega\sin(\arctan(\omega))\geq 0$. So according to barbalat theorem[11], it's known when $t\to\infty$, y_e converges to 0, as the sequel, y_e is indirectly controlled. In summary, the essence of the control law is to seek the input control $q=(v,\omega)^T$, when $t\to\infty$, $x_e\to a_1\sin(\arctan(\omega))y_e$, $\theta_e\to 0$. The Lyapunov function is constructed according to the above idea:

$$V=\frac{1}{2}\bar{x}_e^2+\frac{1}{2}y_e^2+\frac{2}{a_3}(1-\cos(\frac{\theta_e}{2})) \tag{6}$$

where a_3 is a constant greater than 0. Because $a_3>0$, it is clear that $V\geq 0$, only if when $(\bar{x}_e,y_e,\theta_e)^T=0$, V=0. Equation (5) is converted to the following equation:

$$\begin{aligned}\dot{\bar{x}}_e &= x_e - a_1\cos(\arctan(\omega))\frac{1}{1+\omega^2}\dot{\omega}y_e \\ &\quad - a_1\sin(\arctan(\omega))\dot{y}_e\end{aligned} \tag{7}$$

Derivating the Lyapunov function and determines the control law:

$$\begin{aligned}\dot{V} &= \bar{x}_e\dot{\bar{x}}_e + y_e\dot{y}_e + \frac{1}{a_3}\sin(\frac{\theta_e}{2})\dot{\theta}_e \\ &= \bar{x}_e[\dot{x}_e - a_1\cos(\arctan(\omega))\frac{1}{1+\omega^2}\dot{\omega}y_e - a_1\sin(\arctan(\omega))\dot{y}_e] \\ &\quad + y_e(-\omega x_e + v_r\sin\theta_e) + \frac{1}{a_3}\sin(\frac{\theta_e}{2})(\omega_r-\omega) \\ &= \bar{x}_e[\omega y_e - v + v_r\cos(\theta_e) - a_1\cos(\arctan(\omega))\frac{1}{1+\omega^2}\dot{\omega}y_e \\ &\quad - a_1\sin(\arctan(\omega))\dot{y}_e] + y_e(-\omega(\bar{x}_e + a_1\sin(\arctan(\omega))y_e) \\ &\quad + v_r\sin\theta_e) + \frac{1}{a_3}\sin(\frac{\theta_e}{2})(\omega_r-\omega)\end{aligned} \tag{8}$$

$$
\begin{aligned}
&= \bar{x}_e[-v + v_r\cos(\theta_e) - a_1\cos(\arctan(\omega))\frac{1}{1+\omega^2}\dot{\omega}\, y_e \\
&\quad - a_1\sin(\arctan(\omega))(-\omega x_e + v_r\sin\theta_e)] - a_1 y_e^2\omega\sin(\arctan(\omega)) \\
&\quad + \frac{1}{a_3}\sin(\frac{\theta_e}{2})[-\omega + \omega_r + 2a_3 y_e v_r\cos(\frac{\theta_e}{2})]
\end{aligned}
$$

Take the control law of the system:

$$
\begin{aligned}
v &= v_r\cos\theta_e + a_1\sin[\arctan(\omega)\omega x_e \\
&\quad - a_1 v_r\sin(\arctan(\omega))\sin\theta_e \\
&\quad - a_1\cos(\arctan(\omega))]\frac{1}{1+\omega^2}\dot{\omega}\, y_e \\
&\quad + a_2[x_e - a_1\sin(\arctan(\omega))y_e] \\
\omega &= \omega_r + 2a_3 y_e v_r\cos(\frac{\theta_e}{2}) + a_4\sin(\frac{\theta_e}{2})
\end{aligned}
\tag{9}
$$

Where a_2, a_4 is a constant greater than 0. Equation (9) is taken into Equation (8):

$$
\dot{V} = -a_2\bar{x}_e^2 - a_1 y_e^2\omega\sin(\arctan(\omega)) - \frac{a_4}{a_3}\sin^2(\frac{\theta_e}{2}) \tag{10}
$$

Since a_1, a_2, a_3, a_4 are constants greater than 0, and $\omega\sin(\arctan(\omega)) \geq 0$, so $\dot{V} \leq 0$. It can be seen that V is a positive definite continuously differentiable function and bounded. $\dot{V}$ is a negative semidefinite continuous function, then according to Barbalat theorem we can know that when t converge to gigantic, $\dot{V}$ converge to 0, thus $\bar{x}_e^2$, $y_e^2\omega\sin(\arctan(\omega))$, $\sin^2(\theta_e/2)$ converge to 0 respectively, $\theta_e \to 0$, $x_e \to a_1\sin(\arctan(\omega))y_e$ can be obtained. From $t\to\infty$, v_r and ω_r do not converge to 0 at the same time ω does not converge to 0 can be know. And because $y_e^2\omega\sin(\arctan(\omega)) \to 0$, $y_e \to 0$ and $x_e \to a_1\sin(\arctan(\omega))y_e$ can be obtained, so $x_e \to 0$. Because when $t\to\infty$, $\sin^2(\theta_e/2) \to 0$, so $\theta_e \to 0$ is know. Based on the above analysis: the pose error of closed loop system is globally uniformly bounded, and when $t\to\infty$, $\|x_e(t)+y_e(t)+\theta_e(t)\| \to 0$ is concluded.

Further Analysis

From the analysis of the previous section, the control law of trajectory tracking of mobile robot can be obtained and the pose error of closed loop system is globally uniformly bounded and when $t\to\infty$, $\|x_e(t)+y_e(t)+\theta_e(t)\| \to 0$ is concluded. But in fact, there is a structural error in the mobile robot and there is a wheel slip and idling in the movement, which lead to errors in the kinematic model of the mobile robot. And there is an error in the sensor measurement which makes errors in the localization of mobile robot. From above analysis we have a question how large the kinematic model error and localization error are, the control algorithm does not converge. For this situation, we can add the noise to the real position of the mobile robot and kinematic model to get the critical error.

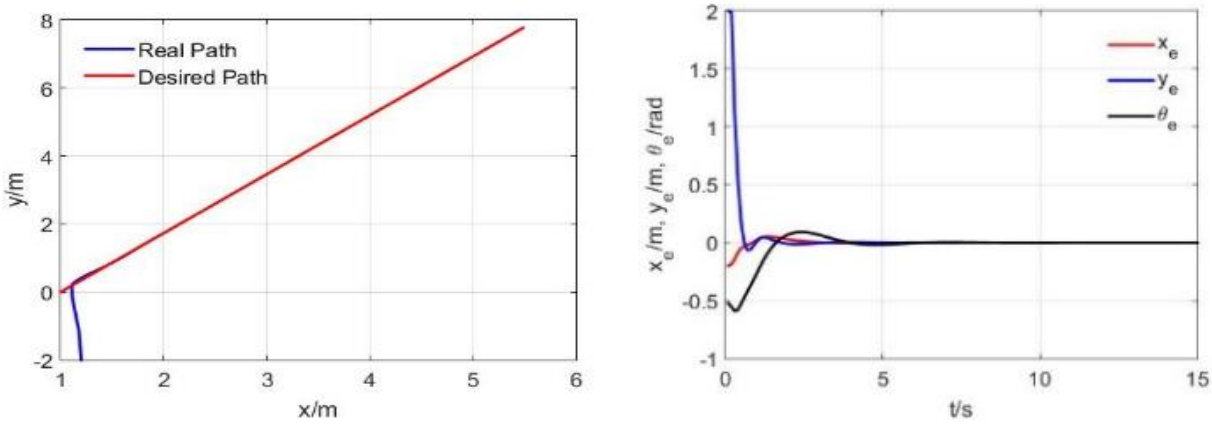
Simulation

In the Matlab environment, the effectiveness of the above algorithm is verified by controlling the linear trajectory tracking, sinusoidal trajectory and arbitrary trajectory of the mobile robot. Critical errors about the mobile robot tracks different trajectories which makes the control algorithm does not converge were got.

Linear Trajectory

The linear velocity and the angular velocity of the reference robot are v_r=0.3 m/s and ω_r = 0, the

initial pose in the global coordinate system are $x_r(0)$ =1m, $y_r(0)$ = 0, $\theta_r(0)$ = π/3rad. At the same time, the initial position of the mobile robot in the global coordinate system are $x(0)$ = 1.2m, $y(0)$ = -2m, $\theta(0)$ = π/2rad. Take the control parameters a_1 = 3, a_2 = 4, a_3 = 11, a_4 = 4. Linear trajectory effect is shown in Figure 4.

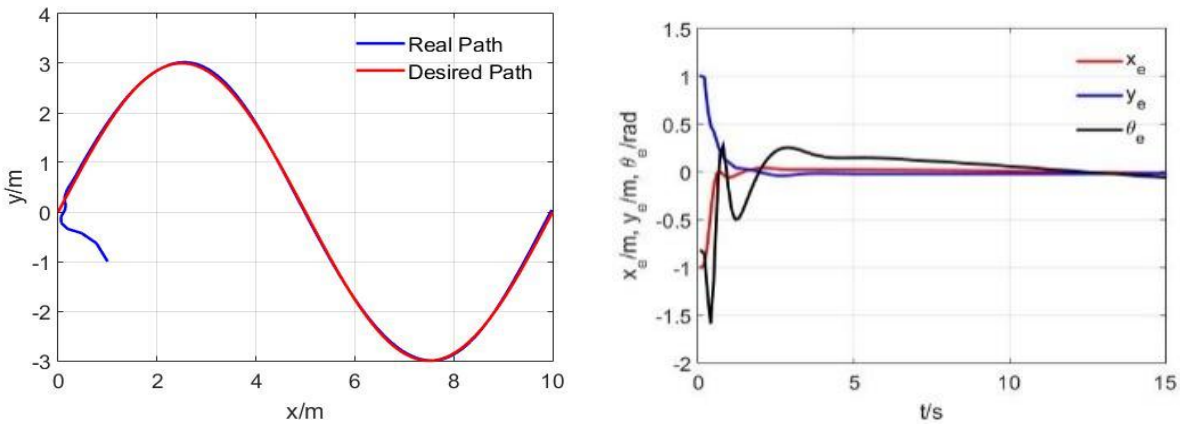


(a) Linear tracking effect (b) Pose error time response curve

Figure 4. Linear trajectory

Sinusoidal Curves Trajectory

The initial position of the sine desired trajectory is y_r =3sin(2πx_r/10). $x_r(0)$ = 0, $y_r(0)$ = 0, $\theta_r(0)$ = 3π/5rad. At the same time, the initial position of the mobile robot in the global coordinate system are $x(0)$ = 1m, $y(0)$ = -1m, $\theta(0)$ = 2π/3rad. Take the control parameters a_1 = 0.2, a_2 = 2, a_3 = 45, a_4 = 5. Sinusoidal curves trajectory effect is shown in Figure 5.

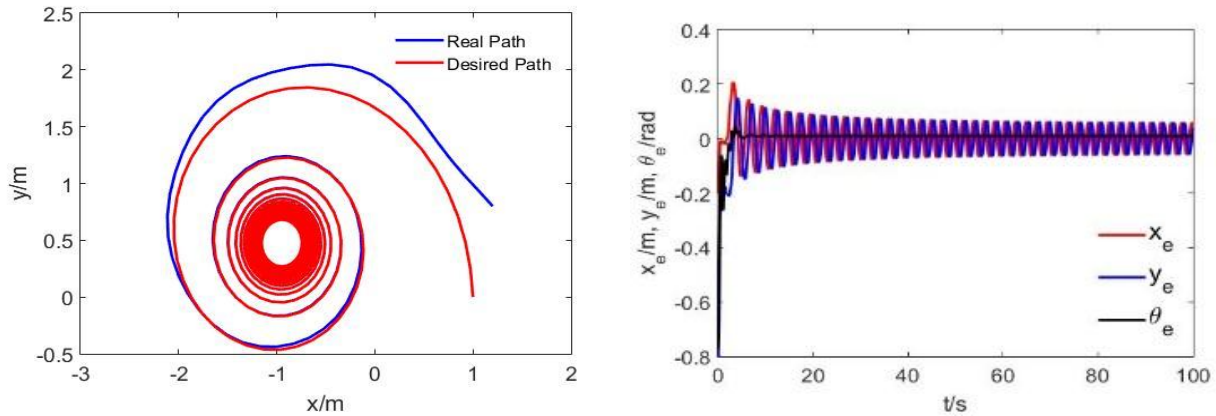


(a) Tracking effect (b) Pose error time response curve

Figure 5. Sinusoidal curves trajectory

Arbitrary Trajectory

The initial position of the arbitrary trajectory are $x_r(0)$ = 1m, $y_r(0)$ = 0, $\theta_r(0)$ = π/2rad. At the same time, the initial position of the mobile robot in the global coordinate system are $x(0)$ = 1.2m, $y(0)$ = 0.8m, $\theta(0)$ = 3π/4rad. Take the control parameters a_1 = a_2 = 0.01, a_3 = 1.5, a_4 = 15.2. Sinusoidal curves trajectory effect is shown in Figure 6.

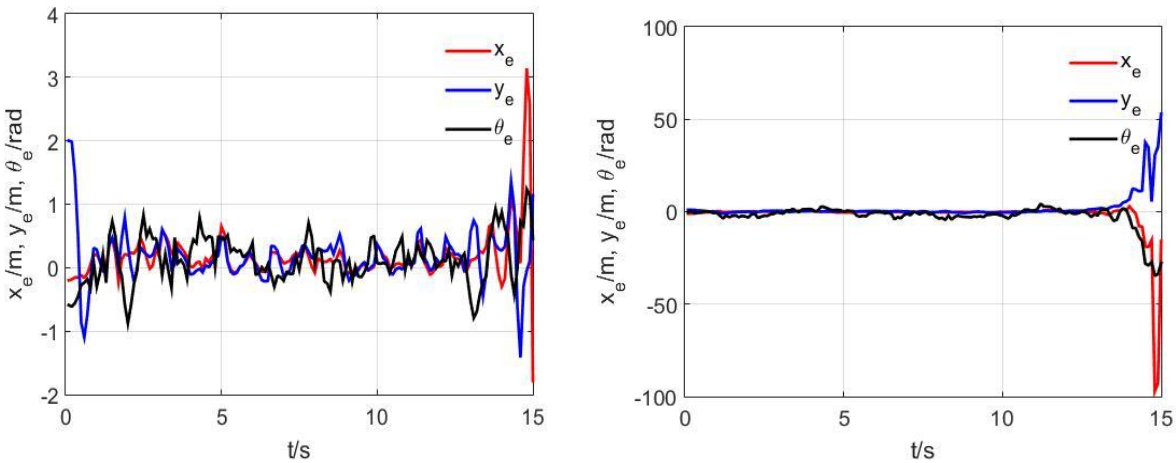


(a) Tracking effect (b) Pose error time response curve

Figure 6. Arbitrary curve trajectory

Tracking with Errors in Localization

Let the mobile robot track the same trajectory in 5.1 and 5.3, gradually increase the localization noise of the mobile robot. Linear trajectory and Sinusoidal Curves Trajectory pose error time response curve is shown in Figure 7. When localization errors reached Δx = 0.15m, Δy = 0.15m, $\Delta\theta$ = π/11rad , the control algorithm does not converge for linear trajectory. When localization errors reached Δx = 0.3m, Δy = 0.3m, $\Delta\theta$ = π/9rad , the control algorithm does not converge for sinusoidal curves trajectory.

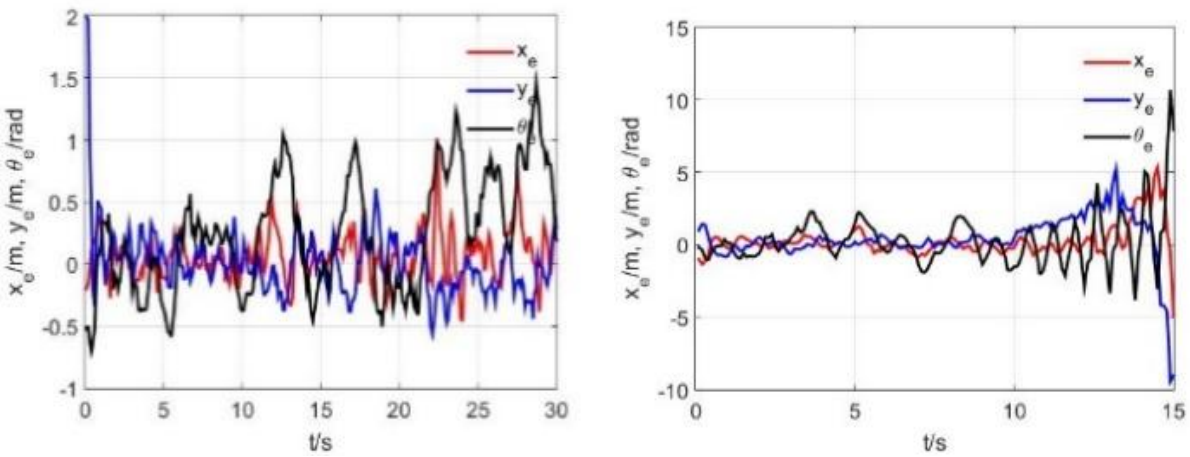


(a) Linear trajectory (b) Sinusoidal curves trajectory

Figure 7. Tracking with errors in localization pose error time response curve

Tracking with Errors in Kinematic Model

Let the mobile robot track the same trajectory in 5.1 and 5.2, gradually increase the kinematic model noise of the mobile robot. Linear trajectory and Sinusoidal Curves Trajectory pose error time response curve is shown in Figure 8. When kinematic model errors reached Δx = 0.08m, Δy = 0.08m, $\Delta\theta$ = π/18rad, the control algorithm does not converge for Linear trajectory. When kinematic model errors reached Δx = 0.2m, Δy = 0.2m, $\Delta\theta$ = π/16rad, the control algorithm does not converge for sinusoidal curves trajectory.

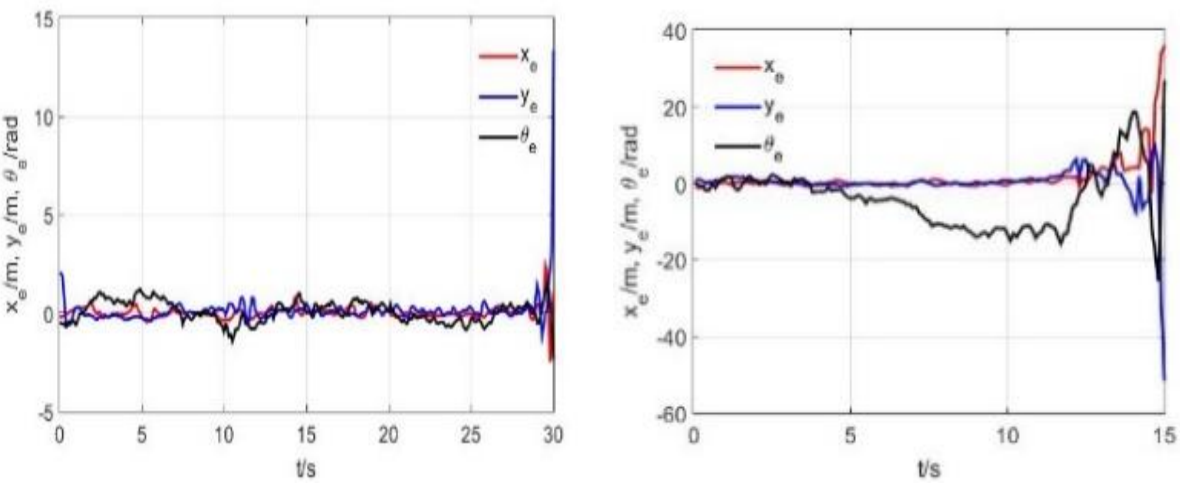


(a) Linear trajectory (b) Sinusoidal curves trajectory

Figure 8. Tracking with errors in kinematic model pose error time response curve

Tracking with Errors in Localization and Kinematic Model

Let the mobile robot track the same trajectory in 5.1 and 5.2, gradually increase the kinematic model noise and localization noise of the mobile robot. Linear trajectory and Sinusoidal Curves Trajectory pose error time response curve is shown in Figure 9. When localization errors reached Δx = 0.1m, Δy = 0.1m, $\Delta\theta = \pi/11$rad, and kinematic model errors reached Δx = 0.05m, Δy = 0.05m, $\Delta\theta = \pi/18$rad, the control algorithm does not converge for linear trajectory. When localization errors reached Δx = 0.13m, Δy = 0.13m, $\Delta\theta = \pi/25$rad, and kinematic model errors reached Δx = 0.2m, Δy = 0.2m, $\Delta\theta = \pi/15$rad, the control algorithm does not converge for sinusoidal curves trajectory.



(a) Linear Trajectory (b) Sinusoidal Curves Trajectory

Figure 9. Tracking with errors in localization and kinematic model pose error time response curve

Summary

In this paper, the trajectory tracking of wheeled mobile robot with nonholonomic constraints is discussed. And the global trajectory tracking controller is constructed by backstepping method. The simulation results show a good performance using backstepping method to let the mobile robot track different trajectories.

The simulation results also show that the influence of the errors in the kinematic model and errors in the localization of mobile robot on the convergence of the control algorithm.

References

1. Zidani G, Drid S, Chrifi-Alaoui L, et al. Backstepping controller for a wheeled mobile robot[C]. 2015 4th International Conference on Systems and Control (ICSC). IEEE, 2015: 443-448.

2. Kolmanovsky I, McClamroch N H. Developments in nonholonomic control problems[J]. IEEE Control Systems, 1995,15(6): 20-36.

3. Kawka P A, Alleyne A G. Stability and feedback control of wireless networked systems[C]. Proceedings of the 2005, American Control Conference, 2005. IEEE, 2005: 2953-2959.

4. Do K D. Bounded controllers for global path tracking control of unicycle-type mobile robots[J]. Robotics and Autonomous Systems, 2013, 61(8): 775-784.

5. Pourboghrat F, Karlsson M P. Adaptive control of dynamic mobile robots with nonholonomic constraints[J]. Computers & Electrical Engineering, 2002, 28(4): 241-253.

6. Mohareri O, Dhaouadi R, Rad A B. Indirect adaptive tracking control of a nonholonomic mobile robot via neural networks[J]. Neurocomputing, 2012, 88: 54-66.

7. Dierks T, Jagannathan S. Control of nonholonomic mobile robot formations: backstepping kinematics into dynamics[C]. 2007 IEEE International Conference on Control Applications. IEEE, 2007: 94-99.

8. Yun X, Yamamoto Y. Internal dynamics of a wheeled mobile robot[C]. Intelligent Robots and Systems'93, IROS'93. Proceedings of the 1993 IEEE/RSJ International Conference on. IEEE, 1993, 2: 1288-1294.

9. Kanayama Y, Kimura Y, Miyazaki F, et al. A stable tracking control method for an autonomous mobile robot[C]. Robotics and Automation, 1990. Proceedings 1990 IEEE International Conference on. IEEE, 1990: 384-389.

10. Velagic J, Lacevic B, Osmic N. Nonlinear motion control of mobile robot dynamic model[M]. INTECH Open Access Publisher, 2008.

11. Wang L S, Chang F R, Tsai P S. Continuation method of backstepping tracking control for car-like mobile robots[C]. 2006 IEEE International Conference on Systems, Man and Cybernetics. IEEE, 2006, 3: 2018-2023.

12. Fukao T, Nakagawa H, Adachi N. Adaptive tracking control of a nonholonomic mobile robot[J]. IEEE transactions on Robotics and Automation, 2000, 16(5): 609-615.

13. Ploplys N J, Kawka P A, Alleyne A G. Closed-loop control over wireless networks[J]. IEEE control systems, 2004, 24(3): 58-71.

14. Siegwart R, Nourbakhsh I R, Scaramuzza D. Introduction to autonomous mobile robots[M]. MIT press, 2011.