Research of Dynamic Characteristic of Belt Conveyor

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Abstract. Aiming at the dynamic question of the belt conveyor, based on the Kelvin-Voigt viscoelastic constitutive law the longitudinal dynamic governing equation of the viscoelastic conveyor belt was established. The generalized coordinate method was adopted to solve dynamic displacement and dynamic tension of the weight tension belt conveyor at the end. According to the longitudinal dynamic model the natural frequency and the dynamic tension were investigated by the research of digital simulation. The results show that the natural frequency of the belt conveyor will become small when the throughput increases. Moreover, increasing the value of the starting time will lead to a deceasing in dynamic tension and then reduce the design safety coefficient and the cost of production.

Introduction

Belt conveyor is a viscoelastic body with driving mechanism and bearing mechanism as a whole that the longitudinal length has about ten kilometers. In the design of belt conveyor, we usually ignore vibration and impact in the belt, but to take static design into account. In order to ensure the safety of the operation of the conveyor, we often increase the safety factor of its design, which also increases the production costs of conveyor. To reduce production cost and optimize performance, at home and abroad, many research institutions have carried out the dynamic analysis of large belt conveyor [1-2]. In the early 1960s, the Soviet Union began to study the dynamics of the conveyor. The starting characteristics of the constant acceleration or AC motor series resistance are studied by using the impulse principle [3]. Based on theory of travelling wave, which was found by Hannover in Germany in 1973, and the dynamic characteristics of the conveyor was studied [4-5]. The lateral vibration of the belt conveyor was studied in Shandong University of Science and Technology, Xi'an University of Science And Technology and Taiyuan University of Science and Technology. The relationship between the lateral vibration and the speed and tension of the belt conveyor is obtained, which can provide a theoretical basis for the development of belt conveyor [10-11].

The study above is based on the discrete conveyor model. Although its analysis is aimed at viscoelastic model, in the process of analysis, it is simplified to the elastic model, which is not able to carry out more practical overall dynamic research. In this paper, a continuous dynamic equation on tail hammer tensioning of the belt conveyor is established on the basis of Kelvin-Voigt model. The viscoelastic continuity equation is solved by using the generalized coordinate method. Analysis of the natural frequency of the conveyor and the longitudinal dynamic tension of the conveyor, provides a more accurate method for the dynamic characteristic of belt conveyor.

Nomenclature

$\sigma(t)$	stress of conveyor belt
E	elastic modulus of belt
ε	strain
μ	viscous hysteresis coefficient
A	cross section area of belt
c	elastic wave propagation velocity

q	<i>equivalent mass of unit length of belt</i>
$u(x, t)$	<i>dynamic displacement</i>
M	<i>quality of hammer</i>
α	<i>load weight ratio</i>
ω_m	<i>natural frequency of belt</i>
ξ	<i>damping coefficient of belt</i>
ω_m	<i>damping frequency of belt</i>
K	<i>kinetic energy</i>
H	<i>potential energy</i>
R	<i>loss function</i>
Qm	<i>generalized force</i>
B	<i>belt width</i>
L	<i>belt length</i>
v	<i>belt speed</i>
Q	<i>transport capacity</i>
δ	<i>inclined angle</i>
qB	<i>quality of the transport belt per meter</i>
qG	<i>the quality of the transport material per meter</i>
qRO	<i>the quality of the bearing section of idlers per meter</i>
qRU	<i>the quality of the return section of idlers t per meter</i>

Continuous Dynamic Equation of Belt Conveyor

In this paper, the inertia, elasticity and damping of the conveyor belt with viscoelastic characteristics are set to be the continuous. The movement of the conveyor system is described by the partial differential equation. Fig.1 is a diagram of belt conveyor. In order to establish the mathematical model of conveyor, the following hypothesis is made: Neglect transverse vibration impact because it is small by comparing with longitudinal vibration; Neglect shear and bending stress of conveyor; Neglect the change of belt length which is caused by the longitudinal degree of the conveyor belt; Neglect lateral deformation which is caused by longitudinal tension of the belt.

Figure 1. Schematic diagram of the belt conveyor

The conveyor belt is made up of rubber and fabric core, which has both elastic and viscous characteristics. In this paper, Kelvin-Voigt as the dynamic model is used to study and the constitutive law is

$$\sigma(t) = E(1 + \mu \frac{\partial}{\partial t})\epsilon \quad (1)$$

Where $\sigma(t)$ —Stress of belt;

E —Elastic modulus of belt;

ϵ —strain;

μ —Viscous hysteresis coefficient.

Tension of the belt at x is

$$S(x, t) = EA \left(1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial x} \quad (2)$$

Where A—Cross section area of belt.

Under unstable conditions, especially the starting and braking process, the belt isn't only effected by static tension, but also effected by dynamic tension which is caused by speed change of the conveyor. Dynamic displacement $u=u(x, t)$ is due to dynamic tension. In this paper, we mainly study the dynamic characteristics of the conveyor, and ignore the static displacement caused by the static tension.

The longitudinal vibration of the conveyor can be neglected, and the dynamic equation of the viscoelastic longitudinal vibration of the conveyor is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial x^2} - a(t) \quad (3)$$

Where c —Elastic wave propagation velocity,
 q —Equivalent mass of unit length of belt.

The boundary and initial conditions of tail hammer tension is following.

A boundary condition is

$$u(0, t) = 0 \quad \frac{M}{2} \frac{\partial^2 u(l, t)}{\partial t^2} = -EA \left(1 + \mu \frac{\partial}{\partial t}\right) \frac{\partial u(l, t)}{\partial x} \quad (4)$$

Where M —The quality of the hammer.

B initial condition is

$$u(x, 0) = 0 \quad \frac{\partial u(x, 0)}{\partial t} = 0 \quad (5)$$

When the conveyor starts up, displacement and speed of the conveyor are zero in the whole belt.

Viscoelastic Longitudinal Vibration Analysis

Homogeneous Equation

Eq.(3) is a non homogeneous viscoelastic vibration equation. In order to get its analytical solution, and it needs to be homogeneous to obtain the mode function. $u(x, t)$ can be solved by the homogeneous and separation of variables.

set up

$$u(x, t) = X(x)T(t) \quad (6)$$

Where $X(x)$ is a function of x coordinates, $T(t)$ is a function of t coordinates. Set Eq. (6) into Eq. (3) which can be obtained by the homogeneous equation

$$\frac{\ddot{T}}{c^2 \left(T + \mu \dot{T}\right)} = \frac{X''}{X} = -\lambda^2 \quad (7)$$

By the boundary condition of the longitudinal vibration and the normalized condition, the vibration mode function is

$$X_m = b \sin \lambda_m x = b \sin \frac{\eta_m x}{l}$$

Where a —The equivalent mass of belt and half of hammer mass ratio (load weight ratio)

$$\alpha = \frac{2ql}{M} = \eta \tan \eta$$

b —Constant, determined by the initial condition.

Where $\alpha = \eta \tan \eta$ is transcendental equation, which has an infinite number of solutions, Table 1 shows value of η corresponding to different α .

Table 1. Value of η corresponding to different α

α	η_1	η_2
0.01	0.10	3.15
0.4	0.60	3.26
0.8	0.79	3.37
1.0	0.86	3.43
2.0	1.08	3.64
4.0	1.27	3.94
10	1.43	4.31
100	1.56	4.67
1000	1.57	4.71

By Eq. (7), we can conclude

$$T = e^{-\xi \omega_m t} (D_1 \cos \omega_m t + D_2 \sin \omega_m t) \quad (8)$$

Where ω_m —Natural frequency of belt ; $\omega_m = \frac{c \eta_m}{l}$;

ξ —Damping coefficient of belt, $\xi = \frac{\mu \omega_m}{2}$;

ω_m —Damping vibration frequency of belt, $\omega_m = \omega_m \sqrt{1 - \xi^2}$

From ω_m and table 1, it shows that the natural frequency of the conveyor increases with α value, but the damping frequency and α do not have the linear relationship.

The subscript m of ω_m means that for an arbitrary natural number, a corresponding natural frequency is obtained, and it is the natural frequency. It can be seen from ω_m that the natural frequency of conveyor is not unique, but an infinite number of discrete values. The natural frequency does not exist between two arbitrary discrete values. While $m=1$, natural frequency is the fundamental frequency. The fundamental frequency vibration is the dominant position in the system of free vibration and forced vibration. When a device starts up, the fundamental frequency is the first frequency of resonance. Fundamental frequency is the primary task on the vibration of belt conveyor.

Non Homogeneous Equation

The vibration of belt is caused by the unstable condition. In this paper, the generalized coordinate method is used to solve the Eq.(3). The displacement at any cross section of belt under unstable condition is

$$u(x, t) = \sum_{m=1}^{\infty} X_m(x) q_m(t) \quad (9)$$

From the above, $X(x)$ is mode of vibration which is solved by homogeneous equation, that is $X_m(x) = \sin \frac{\eta_m x}{l}$. In order to solve generalized coordinates $q_m(t)$, the second Lagrange differential equation must be used.

The equation is

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{q}_m} - \frac{\partial K}{\partial q_m} + \frac{\partial R}{\partial \dot{q}_m} + \frac{\partial H}{\partial q} = Q_m \quad (10)$$

Where K—kinetic energy;

H—potential energy;

R—Loss function (to determine mechanical energy reduction), $R = -\frac{1}{2} \frac{d}{dt} (K + H)$;

Q_m —Generalized force corresponding to generalized coordinates.

Dot from q_m denotes differentiation of the generalized coordinates with respect to time. Meanwhile, generalized velocity is the time derivative of generalized coordinates.

The kinetic energy of the conveyor system is equal to kinetic energy of the conveyor belt's equivalent mass plus kinetic energy of load. Kinetic energy of conveyor system is

$$K = \frac{q}{2} \int_0^l X^2 \sum_{n=1}^{\infty} \dot{q}_m^2 dx + \frac{M}{2} \left[\sum_{n=1}^{\infty} X^2 \dot{q}_m^2 \right] \quad (11)$$

Potential energy of conveyor system is

$$H = \frac{EA}{2} \int_0^l \left[\sum_{n=1}^{\infty} X'(x) q_m(t) \right]^2 dx \quad (12)$$

The loss function depends on the internal resistance of conveyor belt. Because the belt is viscoelastic, it can be gotten by the orthogonality.

$$R = \frac{\mu EA}{2} \sum_{n=1}^{\infty} \dot{q}_m^2 \int_0^l X'^2 dx \quad (13)$$

Set Eq. (11), Eq.(12) and Eq.(13) into Eq. (10), and rearrange as

$$\begin{aligned} \ddot{q}_m + \frac{\mu c^2 \int_0^l X'^2 dx}{\int_0^l X^2 dx + \frac{M}{q} X^2(l)} \dot{q}_m + \frac{c^2 \int_0^l X'^2 dx}{\int_0^l X^2 dx + \frac{M}{q} X^2(l)} q_m \\ = \frac{Q_m}{q \int_0^l X^2 dx + M X^2(l)} \end{aligned} \quad (14)$$

Simplifying Eq. (7) and the boundary condition, we have

$$\ddot{q}_m + \mu c^2 \lambda^2 \dot{q}_m + c^2 \lambda^2 q_m = - \frac{\lambda^2 Q_m}{q \int_0^l X'^2 dx} \quad (15)$$

Eq.(15) is suitable for various boundary conditions (such as fixed winch, constant tension). But under different boundary conditions, $X(x)$ has different modes.

Deadweight and Hammer force act on a micro unit dx . We can conclude from principle of virtual work

$$Q_m = M a(t) X_m(l) + \int_0^l q a(t) X_m dx = \frac{\alpha M}{\eta_m} a(t) \quad (16)$$

Set Eq.(16) into Eq. (15), to give

$$u(x, t) = \frac{q l^2}{EA} H_m \sum_{m=1}^{\infty} \frac{\sin \frac{\eta_m x}{l}}{\eta_m (1 - \xi^2)} A(t) \quad (17)$$

Where $H_m = \frac{4}{\eta_m (2\eta_m + \sin 2\eta_m)}$

Setting (17) into the Eq. (2), then dynamic tension of belt is

$$\begin{aligned} A(t) = \omega_m \int_0^t e^{-\xi \omega_m (t-\tau)} a(\tau) \sin \omega_m (t-\tau) d\tau \\ s(x, t) = q l \sum_{m=1}^{\infty} \frac{H_m}{1 - \xi^2} c o s \frac{\eta_m x}{l} \left[A(t) + \mu \dot{A}(t) \right] \end{aligned} \quad (18)$$

From Eq. (17) and Eq. (18), we can conclude that the maximum displacement of belt takes place on the tension roller, and the maximum dynamic tension occurs at the input point of driving roller.

Effect of Load Ratio on Natural Frequency

From Eq.(8) and Table 1, we can conclude that, with the increase of load ratio, the natural

frequency of the conveyor is increasing but there is no linear relationship. There are two kinds of limit conditions for the natural frequency of the conveyor.

(1) When the hammer quality is smaller than the equivalent mass of conveyor belt, it means as $\alpha = \infty$, then $\eta = \frac{2n-1}{2} \pi (n=1,2,\dots)$. Based on this, we can conclude $\nu_m = \frac{c}{l} \frac{2n-1}{2} \pi$. At this time, the vibration frequency of the system is the largest.

(2) When the hammer quality is larger than the equivalent mass of conveyor belt, it means as $\alpha = 0$, then we can conclude $\nu_m = \frac{c}{l} \sqrt{\alpha}$. At this time, the free vibration frequency of the conveyor is almost zero. This situation almost does not exist, so that the vibration of the conveyor will always exist.

Case Analysis

Main parameters of belt conveyor are shown in Table. 2.

Table 2. Main parameters of belt conveyor

Nomenclature	parameter
Belt length L/m	1504
Belt width B/m	0.8
Belt speed $v/(m/s)$	3.15
Transport capacity $Q/(t/h)$	257
Inclined angle $\delta/(^\circ)$	0
Type	ST1250
Belt thickness/ m	0.017
Quality of the transport belt per meter $q_B/(kg/m)$	24.7
Quality of the transport material per meter $q_G/(kg/m)$	22.68
Quality of the bearing section of idlers per meter $q_{RO}/(kg/m)$	8.8
Quality of the return section of idlers t per meter $/(kg/m)$	2.93
Hammer quality(kg)	1750

Natural Frequency of the System

Eq.(8) shows that there is a close relationship between the natural frequency and hammer quality. By the above parameters and Eq.(8), full load of bearing strand, empty load of bearing strand and the first three order natural frequency of return strand are shown in Tab. 3. It is known that the natural frequency of full load is the smallest, so the natural frequency of the bearing load strand should be avoided in resonance analysis.

Table.3 First three order natural frequency of the carrying strand and return strand

Nomenclature	ν_1	ν_2	ν_3
Full load of bearing strand	1.48	4.45	7.45
Empty load of bearing strand	1.82	5.51	9.32
Empty load of return strand	1.97	6.0	10.21

Tension Response Characteristics of Different Starting Time

Eq.(18) shows that at input point ($x=0$) of driving roller, dynamic tension has the maximum value. In the case of the above parameters, this paper analyzes the dynamic tension input point ($x=0$) of driving roller, dynamic load coefficient and strain curve respectively corresponding to starting time of 60s, 120s and 180s. Dynamic load coefficient defines as dynamic tension plus static tension, and then divided by static tension.

As can be seen from Fig.2, when the starting time is 60s, 120s, and 180s, tension peak at input

point of driving roller respective is $6.4 \times 10^4 \text{N}$, $6.0 \times 10^4 \text{N}$ and $5.8 \times 10^4 \text{N}$. Stable value is about $5.5 \times 10^4 \text{N}$. Moreover, dynamic tension respectively reduces 16.4%, 9.1% and 5.5%. It can be known that increasing the starting time that is to reduce the starting acceleration can effectively reduce dynamic tension. It provides a theoretical basis for reducing design safety factor. As shown in Fig. 2, the tension respectively reaches the first peak at 19s, 60s and 30s. It is because the start time is shorter, the input speed is the greater. The large speed wave can cause that the influence of running resistance on speed wave is smaller. Therefore, when the starting time is 60s, the time of the first peak is shortest.

Figure 2. Tension at the meeting point of drive drum at different starting time

Fig. 3 and Fig. 4 respectively correspond to dynamic loading coefficient curve K and strain curve at different starting time. When the dynamic load coefficient is 1.16, 1.05 and 1.09 respectively, and the maximum strain is respective 0.0032, 0.00289 and 0.0030. The attenuation trend of the curve and the attenuation trend of the tension are basically consistent, and the peak time is also basically consistent. It further confirms that increasing start time can effectively reduce starting dynamic tension.

Fig.3 Dynamic loading coefficient K at the meeting point of drive drum at different starting time

Fig.4 Strain at the meeting point of drive drum at different starting time

From the above analysis, it is known that with starting time extending, the dynamic tension of belt is reduced by 9.375% which is equal to the reduction of the dynamic tension.

Conclusion

(1) Based on Kelvin-Vogit model, the partial differential equation of the belt conveyor is established. Moreover, dynamic equation of the longitudinal vibration of the conveyor is obtained accurately.

(2) The analytical solution of dynamic displacement and dynamic tension of belt is derived by using the generalized coordinate method. The relationship between the natural frequency of conveyor and the load ratio is established. With the load ratio increasing that is constantly increasing transport capacity, the natural frequency of conveyor is gradually reduced.

(3) Simulation shows that increasing the starting time, that is to reduce the starting acceleration, can effectively reduce the starting dynamic tension and dynamic load coefficient, so as to provide a theoretical basis for the dynamic design of the conveyor.

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