

Study on the Adaptive Generalized Synchronization of Hetero Structure Chaotic Systems

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Keywords: Vibration system, Nonlinear dynamics, Parameter monitor, Attractor.

Abstract. A mathematical model of adaptive generalized chaotic synchronization (AGCS) was established to investigate the synchronization between a multi-degree freedom non-autonomous (nonlinear vibration) driving system and an autonomous chaotic (Lorenz) response system. Within the hetero structure synchronization model, the AGCS nonlinear adaptive controller is designed. Both the AGCS error curves and parameters identification error curves are proved to converge to zero by Lyapunov stability theory. Numerical investigations were carried out to obtain the synchronization characteristic pattern. Results show that these two systems can be synchronized by using AGCS technique, with both the synchronization error and the parameter identification error quickly converging to zero. The chaotic attractor is observed in phase plane, and the synchronization diagonal line is applied to show the existence of the synchronized behavior. The model established in this study can be used for condition monitoring and fault diagnosis of chaotic vibration systems in many engineering fields.

Introduction

Since 1990, when the first experimental study of chaos synchronization (CS) was conducted successfully by Pecora and Carroll¹, CS has been widely applied in many fields such as security communication, ultra-wideband wireless transmission and chaos anti-controlling.

In order to improve the performance of synchronization between two chaotic systems, many studies have been carried out focusing on improving the efficiency of controller²⁻¹⁴.

For two chaotic systems with same structure, Tan⁹ designed a new nonlinear feedback controller based on fractional Laplace transform theory. This controller can be used for generalized synchronization between fractional-order chaotic systems. A criterion for synchronizing two chaotic systems was also given. Niu¹⁰ investigated the self-synchronization of variable coefficients chaotic system, in which a controlling method with strictly positive real transfer function matrix was used. Zeng¹¹ studied the global asymptotic synchronization of chaotic systems with the same structures. The controller was designed based on radial basis function neural network.

A great number of studies were also conducted focusing on the synchronization of different structures chaotic systems. For example, Liu¹² investigated the active disturbance rejection controller for anti-synchronization between chaotic systems with different structures. Chen¹³ studied the adaptive generalized synchronization (AGS) between a Chen system and a multi-scroll chaotic system. Sun¹⁴ studied the complexity of generalized synchronization (GS) between two unidirectional coupled Lorenz systems under parametric excitation.

Although many researches have been conducted on the subject of chaotic synchronization, most of them were focusing on autonomous chaotic systems. No investigation of the adaptive generalized synchronization between multi-degree-freedom non-autonomous and autonomous systems has been carried out, though the non-autonomous systems are widely used in practice¹⁵⁻²⁰. It is very important to extend the study and application of AGCS to the non-autonomous systems.

In Sec.2, a mathematic model was established to investigate the chaos synchronization between a multi-degree freedom non-autonomous (nonlinear vibration) system and an autonomous chaotic (Lorenz) system. In Sec.3, an adaptive controller was designed to synchronize the two systems. The

convergences of both synchronization error and parameter identification error were also studied using the Lyapunov theory. In Sec.4, the AGCS between these two systems were simulated and the reliability of using this model to detect and monitor parameter variations is investigated. Conclusions were given in Sec.6.

Adaptive Generalized Synchronization Model for Chaotic Systems

An AGCS model was designed to synchronize a multi-degree freedom non-autonomous (nonlinear vibration) driving system and an autonomous chaotic (Lorenz) response system.

The driving system used in the model has two degree of freedoms, as seen in Fig. 1. It includes a nonlinear stiffness spring between an upper mass m_1 and the middle mass; a linear stiffness spring between a middle mass m_2 and a foundation. This system has been previously studied by Yu Xiang et al.²¹ In his paper where the characteristics of nonlinear stiffness was carefully investigated.

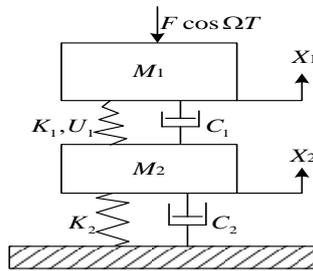


Fig.1. Two-degree-freedom nonlinear vibration system

The dynamic equation of the system can be expressed as:

$$\begin{aligned}
 M_1 \ddot{X}_1 + C_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + U_1(X_1 - X_2)^3 \\
 = F \cos(\Omega T) + G_0 \\
 M_2 \ddot{X}_2 - C_1(\dot{X}_1 - \dot{X}_2) - K_1(X_1 - X_2) - U_1(X_1 - X_2)^3 \\
 + C_2 \dot{X}_2 + K_2 X_2 = 0
 \end{aligned} \tag{1}$$

where, M_1, M_2 are upper and lower mass blocks respectively; X_1, X_2 are vibration displacement; C_1, C_2 are system damping; K_1, K_2 are linear stiffness; U_1 is the nonlinear stiffness; F is excitation force amplitude; Ω is the excitation frequency; G_0 is the gravity of upper mass block and

$$\ddot{X}_i = d^2 X_i / dT^2, \dot{X}_i = dX_i / dT \quad (i=1,2)$$

The dimensionless equation is derived from Eq.(1) and gets

$$\begin{aligned}
 \ddot{x}_1 + \xi_1(\dot{x}_1 - \dot{x}_2) + (x_1 - x_2) + (x_1 - x_2)^3 \\
 = f \cos(\omega t) + G \\
 \ddot{x}_2 - \mu \xi_1(\dot{x}_1 - \dot{x}_2) - \mu(x_1 - x_2) - \mu(x_1 - x_2)^3 \\
 + \xi_2 \dot{x}_2 + \mu K x_2 = 0
 \end{aligned} \tag{2}$$

where $x_1 = X_1 / \sqrt{K_1 / U_1}$; $x_2 = X_2 / \sqrt{K_1 / U_1}$; $t = \Omega_0 T$; $\Omega_0 = \sqrt{K_1 / M_1}$; $f = F / K_1 \sqrt{K_1 / U_1}$; $\omega = \Omega / \Omega_0$; $G = G_0 / K_1 \sqrt{K_1 / U_1}$; $\xi_1 = C_1 / \sqrt{K_1 M_1}$; $\xi_2 = C_2 \mu / \sqrt{K_1 M_1}$; $K = K_2 / K_1$; $\mu = M_2 / M_1$; $\ddot{x}_i = d^2 x_i / dt^2$; $\dot{x}_i = dx_i / dt$ ($i=1,2$). To simplify the analysis, let $\mu = 2$ in Eq.(2), and yields

$$\begin{aligned}
 & \ddot{x}_1 + \xi_1(\dot{x}_1 - \dot{x}_2) + (x_1 - x_2) + (x_1 - x_2)^3 \\
 & = f \cos(\omega t) + G \\
 & \ddot{x}_2 - 2\xi_1(\dot{x}_1 - \dot{x}_2) - 2(x_1 - x_2) - 2(x_1 - x_2)^3 \\
 & + \xi_2\dot{x}_2 + 2Kx_2 = 0
 \end{aligned} \tag{3}$$

In order to better analyze the states of variables, Eq. (3) can be re-written as first-order state equations:

$$\begin{aligned}
 \dot{x}_1 & = x_3 \\
 \dot{x}_2 & = x_4 \\
 \dot{x}_3 & = -\xi_1(x_3 - x_4) - (x_1 - x_2) \\
 & \quad - (x_1 - x_2)^3 + f \cos(\omega t) + G \\
 \dot{x}_4 & = 2\xi_1(x_3 - x_4) + 2(x_1 - x_2) \\
 & \quad + 2(x_1 - x_2)^3 - \xi_2x_4 - 2Kx_2
 \end{aligned} \tag{4}$$

The equations for response system (Lorenz system) with relevant control components are given as

$$\begin{aligned}
 \dot{y}_1 & = a(y_2 - y_1) + u_1 \\
 \dot{y}_2 & = cy_1 - y_2 - y_1y_3 + u_2 \\
 \dot{y}_3 & = y_1y_2 - by_3 + u_3
 \end{aligned} \tag{5}$$

where, y_1, y_2, y_3 are system outputs; a, b, c are Lorenz system parameters and u_1, u_2, u_3 are control components which will be further discuss in Sec.3.

To synchronize these two systems, a target function which consists of the state variables in Eq.(4) is given as:

$$\varphi = (\varphi_1, \varphi_2, \varphi_3) = (x_1 + x_3, x_2 + x_4, x_1 + x_2)^T \tag{6}$$

Therefore synchronization errors can be expressed as

$$e_1 = y_1 - \varphi_1, \quad e_2 = y_2 - \varphi_2, \quad e_3 = y_3 - \varphi_3 \tag{7}$$

The total synchronization error of AGCS is given as $e = (e_1^2 + e_2^2 + e_3^2)^{1/2}$. The adaptive generalized synchronization between system(4) and system (5) can be achieved when $\lim_{t \rightarrow +\infty} e(t) = 0$.

Design of Controller

The control components in Eq.(5) are designed to be nonlinear and adaptive, as follows:

$$\begin{aligned}
 u_1 & = (a-1)y_1 - ay_2 + x_1 + (2 - \hat{\xi}_1)x_3 \\
 & \quad + \hat{\xi}_1x_4 - (x_1 - x_2)^3 + \hat{f} \cos(\omega t) + \hat{G} \\
 u_2 & = -cy_1 + y_1y_3 + 2x_1 - (1 + 2\hat{K})x_2 \\
 & \quad + 2\hat{\xi}_1x_3 + (2 - 2\hat{\xi}_1 - \hat{\xi}_2)x_4 + 2(x_1 - x_2)^3 \\
 u_3 & = (b-1)y_3 - y_1y_2 + x_1 + x_2 + x_3 + x_4
 \end{aligned} \tag{8}$$

where $\hat{\xi}_1, \hat{\xi}_2, \hat{f}, \hat{G}, \hat{K}$ corresponding to those in the driven system are unknown parameters need to be identified. The identification errors are expressed as

$$\begin{aligned}\bar{\xi}_1 &= \hat{\xi}_1 - \xi_1, \bar{\xi}_2 = \hat{\xi}_2 - \xi_2, \\ \bar{f} &= \hat{f} - f, \bar{G} = \hat{G} - G, \bar{K} = \hat{K} - K\end{aligned}\quad (9)$$

Coefficients of the controller are actively adjusted by the synchronization errors. The adaptive laws for parameter identification are given as

$$\begin{aligned}\dot{\hat{\xi}}_1 &= v(e_1 - 2e_2)(x_3 - x_4) \\ \dot{\hat{\xi}}_2 &= ve_2x_4 \\ \dot{\hat{f}} &= -ve_1 \cos(\omega t) \\ \dot{\hat{G}} &= -ve_1 \\ \dot{\hat{K}} &= 2vx_2e_2\end{aligned}\quad (10)$$

where v is the control coefficient used to adjust converging velocity.

To prove the adaptive generalized synchronization can be achieved between these two systems, the Lyapunov function was used to investigate the convergence of errors. Take the Lyapunov function as

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) + \frac{1}{2v}(\bar{\xi}_1^2 + \bar{\xi}_2^2 + \bar{f}^2 + \bar{G}^2 + \bar{K}^2)\quad (11)$$

Combining Eq.(9) with Eq.(11), and thus,

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &+ \frac{1}{v}(\bar{\xi}_1\dot{\hat{\xi}}_1 + \bar{\xi}_2\dot{\hat{\xi}}_2 + \bar{f}\dot{\hat{f}} + \bar{G}\dot{\hat{G}} + \bar{K}\dot{\hat{K}})\end{aligned}\quad (12)$$

By differentiating Eq.(7) and substituting into Eq. (12), we get:

$$\begin{aligned}\dot{V} &= e_1(\dot{y}_1 - \dot{\phi}_1) + e_2(\dot{y}_2 - \dot{\phi}_2) + e_3(\dot{y}_3 - \dot{\phi}_3) \\ &+ \frac{1}{v}(\bar{\xi}_1\dot{\hat{\xi}}_1 + \bar{\xi}_2\dot{\hat{\xi}}_2 + \bar{f}\dot{\hat{f}} + \bar{G}\dot{\hat{G}} + \bar{K}\dot{\hat{K}})\end{aligned}\quad (13)$$

Substitute Eqs.(4),(5),(6),(9) and Eq.(10) into Eq.(13), and the Eq.(14) is deduced,

$$\begin{aligned}\dot{V} &= e_1[x_1 - y_1 + x_3 + (\xi_1 - \hat{\xi}_1)(x_3 - x_4) \\ &+ (\hat{f} - f) \cos(\omega t) + (\hat{G} - G)] + e_2[x_2 + x_4 - y_2 \\ &+ (\xi_2 - \hat{\xi}_2)x_4 + 2(K - \hat{K})x_2 - 2(\xi_1 - \hat{\xi}_1)x_3] \\ &+ e_3(x_1 + x_2 - y_3) - (\xi_1 - \hat{\xi}_1)(e_1 - 2e_2)(x_3 - x_4) \\ &- (\xi_2 - \hat{\xi}_2)e_2x_4 - (\hat{f} - f)e_1 \cos(\omega t) \\ &- e_1(\hat{G} - G) - 2e_2(K - \hat{K})x_2 \\ &= e_1(x_1 - y_1 + x_3) + e_2(x_2 + x_4 - y_2) \\ &+ e_3(x_1 + x_2 - y_3) \\ &= -(e_1^2 + e_2^2 + e_3^2) \\ &= -e^2 \leq 0\end{aligned}\quad (14)$$

Since $\dot{V} \leq 0$, and

$$\int_0^t e^2 dt = \int_0^t (-\dot{V}) dt = V(0) - V(t) \leq V(0) \quad (15-1)$$

According to Barbalat lemma¹⁶, the limitation of $e(t)$ is $\lim_{t \rightarrow +\infty} e(t) = 0$. Then, the synchronization between systems (4) and (5) can be achieved.

Furthermore, the parameters identification errors need to converge to ensure the parameters given in Eq.(4) can be identified effectively. Considering Eq.(7), we can get

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - \dot{\phi}_1 \\ \dot{e}_2 &= \dot{y}_2 - \dot{\phi}_2 \\ \dot{e}_3 &= \dot{y}_3 - \dot{\phi}_3 \end{aligned} \quad (15-2)$$

Substitute Eqs.(5),(6) into Eq.(15), and obtain

$$\begin{aligned} \dot{e}_1 &= a(y_2 - y_1) + u_1 - \dot{x}_1 - \dot{x}_3 \\ \dot{e}_2 &= cy_1 - y_2 - y_1y_3 + u_2 - \dot{x}_2 - \dot{x}_4 \\ \dot{e}_3 &= y_1y_2 - by_3 + u_3 - \dot{x}_1 - \dot{x}_2 \end{aligned} \quad (16)$$

Then Eq.(17) is derived by substitute Eqs.(4),(8) into Eq.(16),

$$\begin{aligned} \dot{e}_1 &= -e_1 + (\xi_1 - \hat{\xi}_1)(x_3 - x_4) \\ &\quad + (\hat{f} - f) \cos(\omega t) + (\hat{G} - G) \\ \dot{e}_2 &= -e_2 + (\xi_2 - \hat{\xi}_2)x_4 \\ &\quad + 2(K - \hat{K})x_2 - 2(\xi_1 - \hat{\xi}_1)x_3 \\ \dot{e}_3 &= x_1 + x_2 - y_3 = -e_3 \end{aligned} \quad (17)$$

when $t \rightarrow \infty$, considering the limitation of error $e(t) \rightarrow 0$, thus the corresponding components $e_1(t) \rightarrow 0$, $e_2(t) \rightarrow 0$, $e_3(t) \rightarrow 0$. Since $e_1(t), e_2(t), e_3(t)$ are smooth functions, the derivative of errors are satisfied $\dot{e}_1(t) \rightarrow 0$, $\dot{e}_2(t) \rightarrow 0$, $\dot{e}_3(t) \rightarrow 0$.

Therefore, as t approaches infinite, we can get Eq.(18) from Eq.(17)

$$\begin{aligned} (\xi_1 - \hat{\xi}_1)(x_3 - x_4) + (\hat{f} - f) \cos(\omega t) + (\hat{G} - G) &= 0 \\ (\xi_2 - \hat{\xi}_2)x_4 + 2(K - \hat{K})x_2 - 2(\xi_1 - \hat{\xi}_1)x_3 &= 0 \end{aligned} \quad (18)$$

Substitute Eq. (9) into Eq. (18), and thus

$$\bar{f} \cos(\omega t) - \bar{\xi}_1(x_3 - x_4) + \bar{G} = 0 \quad (19)$$

$$2\bar{\xi}_1x_3 - \bar{\xi}_2x_4 - 2\bar{K}x_2 = 0 \quad (20)$$

As the state variables of the system (4), x_2, x_3, x_4 are not identically equal to zero. To ensure the Eq. (20) can be hold under all the states, following conditions must be satisfied, $\bar{\xi}_1 = \bar{\xi}_2 = \bar{K} = 0$

$$\hat{\xi}_1 \rightarrow \xi_1, \hat{\xi}_2 \rightarrow \xi_2, \hat{K} \rightarrow K$$

Then the Eq. (19) can be expressed as

$$\bar{f} \cos(\omega t) + \bar{G} = 0 \quad (21)$$

Since value $\cos(\omega t)$ is not identically equal to zero, to ensure the Eq.(21) is tenable, following conditions must be satisfied, $\bar{f} = \bar{G} = 0$, and thus, $\hat{f} \rightarrow f, \hat{G} \rightarrow G$.

In summary, if the controller (8) and adaptive law (10) are adopted, the adaptive generalized synchronization can be achieved. Both the synchronization errors and the parameter identification errors converge to zero.

Numerical Simulation of AGCS

When parameters $\xi_1 = 0.02, \xi_2 = 0.2, K=100, f=8.8, \omega = 3.7, G=96$, coefficient $v=15$, giving the initial values of the two-degree freedom non-autonomous chaotic system (4) as

$$[x_1(0), x_2(0), x_3(0), x_4(0)] = [1, 1, 1, 1]$$

a chaotic behavior occurs. Relevant x_1, x_2, x_3, x_4 can be obtained by using Runge-Kutta algorithm and the phase diagrams of x_1, x_3 and x_2, x_4 are shown in Fig.2.

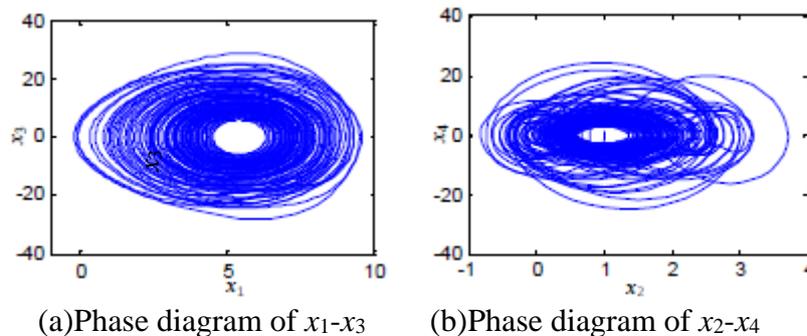
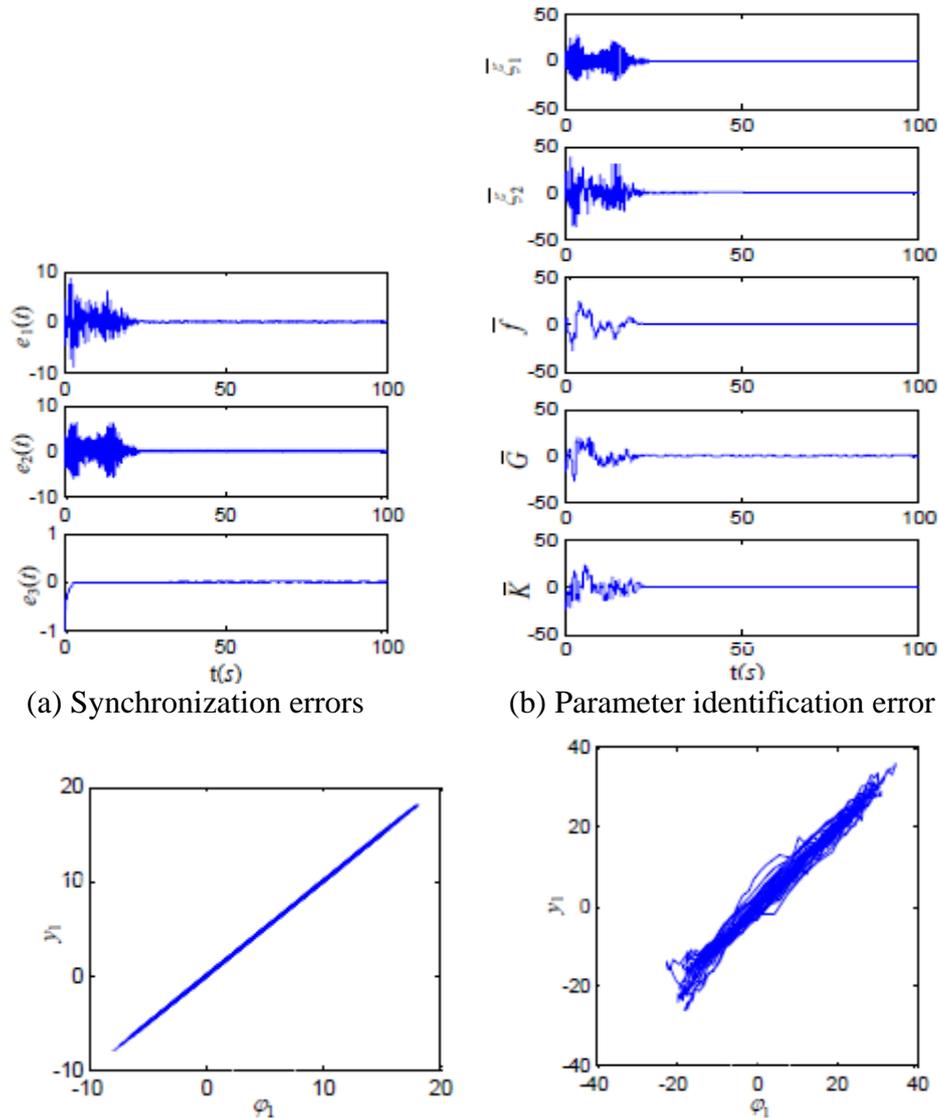


Fig.2. Phase diagram of two-degree freedom on-autonomous chaotic system

The initial values of Lorenz response system, Eq. (5), are given as $[y_1(0), y_2(0), y_3(0)] = [1, 1, 1]$ and relevant parameters as $a=10, b=28, c=8/3$. The adaptive controller introduced in Sec.3 is used to synchronize these two systems. To achieve the synchronization, the synchronization errors, e_1, e_2, e_3 , and identification errors, $\bar{\xi}_1, \bar{\xi}_2, \bar{f}, \bar{G}, \bar{K}$ should all converge to zero. Results are shown in Fig.3.

From Fig.3, it is clear that all the errors converge to zero after about 20s, when the AGS of these two systems were achieved, as shown in Figs.3 (a) and (b). Under the synchronized status, the AGS synchronization pattern between two systems (in Fig.3.c) stays in a diagonal line.



(C) Synchronization pattern between two systems (d) Non-synchronization between two systems

Fig.3.Synchronization process between two systems

Conclusions

An adaptive generalized chaotic synchronization (AGCS) technique is successfully used to synchronize a two-degree-freedom non-autonomous chaotic (nonlinear vibration) system and an autonomous (Lorenz) system, where the first system is the driving system, and the second one is the response system. The AGCS synchronization frame is given and the nonlinear adaptive controller is designed to achieve the adaptive generalized synchronization. Both the AGCS error curves and parameters identification error curves are proved to converge to zero by Lyapunov stability theory. A numerical simulation was carried out to obtain the synchronization characteristic pattern. It is observed that the AGCS is achieved when errors converge to zero, and the synchronization diagonal line is applied to show the existence of the synchronized behavior. If the diagonal line with slope 1.0 exists, the two systems work in a synchronization state.

Acknowledgement

Project is supported by Project 425517K143 and the National Natural Science Foundation of China (Grant No. 51579242, 51509253).

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