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Finding the Overlapped Covered Sub-regions C^2, C^3 and the Maximum Covered Regions in WSN by Using Net Arcs (NA)Method

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***Abstract*-One of the fundamental issues in WSN is the design of energy efficient approaches that achieve full area coverage and maintain the connectivity of the whole network. Coverage reflects how well a sensor field is monitored. In order to monitor the area of interest in the most efficient way, finding the connected covered sub-regions allows the use of Sleep scheduling techniques that put cooperatively the nodes monitoring these areas into sleep-mode without increasing the response time of the network. In our work, we use the idea of graph theory and we propose a new method called Net Arcs (NA) that transform the graph model from theory state to the real time. We use some techniques to find the all overlapped covered sub-regions (2-Edges) and (3-Edges) and we ensure the connectivity of the network by finding the maximum covered regions. Finding these sub-regions increases WSN lifetime and provides improved coverage performance.**

Keywords-wireless sensor networks; wireless sensor network routing; sensor's groups; energy efficiency; network lifetime; net arcs

I. INTRODUCTION

Two of the most fundamental issues in WSN are sensing coverage and network connectivity.

In the recent years, the area coverage problem has been thoroughly studied. In [1,2] Ke et al. proved that the problem of fully covering critical grids using a minimum number of sensors (known also as critical grid coverage problem) is NP-Complete. The paper [3] presents fundamental studies on the sensing coverage and the network connectivity from mathematical modeling, theoretical analysis, and performance evaluation perspectives, by considering lattice WSNs that follow a pattern-based deployment strategy and random WSNs that follow a random deployment strategy in the aim to provide guidelines in selecting critical network parameters for WSN design and implementation in practice.It is more practical and efficient to monitor critical areas rather than common areas if the sensor field is large, or the available budget cannot provide enough sensors to fully cover the entire field. Most of the sensors deployment algorithms divides the sensor field into square grid cells and deploys the sensors on constrained locations such as grid points [4]. In [5] author proposed a protocol, called Distributed Lifetime Coverage Optimization protocol (DiLCO), which maintains the coverage and improves the lifetime of a wireless sensor network by partitioning the area of interest into sub-regions using a classical divide-and-conquer method then combining two effective techniques. Simulation results proved that each set built ensure the coverage at a low energy cost, due the optimization of the network lifetime.

In addition, keeping the WSN connected is also important because sensing data may need to be sent to the data center[6].The paper [7]proposed an overlapping-node Removal Algorithm (ODRA) that utilizes the backup node to detect the nodes in WSN sensing coverage overlapping node.

It converts the node to the atmosphere node and prevents unnecessary redundancy data and improves the network lifetime of the total network that the sensing area is overlapped through the network assessment in order to remove this redundancy data. The author in [8] utilizes the number of maximum coverage set as the upper limit of coverage node set division. On the basis of this maximum, it takes number of minimum layer overlapping subfields, which satisfy division condition, as node's utility function. By using the game theory model, it puts forward a distribution algorithm to get optimal strategy by iteration. Simulation results show that it realizes maximization of network lifetime when ensuring area overall coverage.

The organization of this paper is as follows: Section 2 gives some preliminary definitions about the intersections points of sensors. In Section 3, graph model of our WSN, in Section 4 we present our new method called Net Arcs (NA) to transform the graph model from the theory state to the real state as it is in real time, In Section 5 we find overlapped covered the sub-regions (2-edges) ,(3-edges) and the maximum covered regions ,and we show that these regions are always connected in order to maintain the connectivity of the network. Finally, Section 6 is the conclusion and the further work.

II. PROBLEM STATE

Let's have a set S of n partially overlapped sensors in the area, we suppose that the n sensors are homogenous and the sensing area is a disk. Each sensor is defined by a center which is the location of the node (Sensor) (x_{s_j}, y_{s_j}), and the radius$r$.

Assuming there is no fully overlapped between sensors.

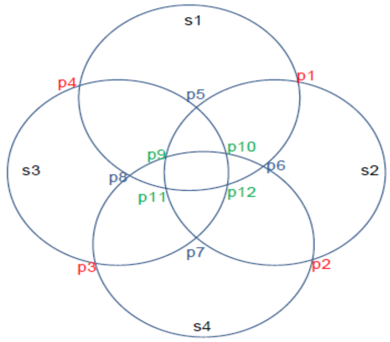

Figure 1. WSN with 4 sensors.

Definition 1Sensor Neighbor List:

For any node $s_a \in S$, its neighbor list is defined as a set of nodes $nl(s_a) = \left\{ s_b | s_b \in S \text{ and } \sqrt{(x_{s_a} - x_{s_b})^2 + (y_{s_a} - y_{s_b})^2} \le r_a + r_b \right\}$, where r_a and r_b are the sensing ranges for s_a and s_b respectively$\in S$ is the set for all sensor nodes in the network. The $nl(S) = \{nl(s_i) | s_i \in S\}$ is an array of neighbor Lists.

From the figure (1) we have:

$$nl(s_1) = \{s_1, s_2, s_3, s_4\}, nl(s_2) = \{s_1, s_2, s_3, s_4\}$$
$$nl(s_3) = \{s_1, s_2, s_3, s_4\}, nl(s_4) = \{s_1, s_2, s_3, s_4\}$$
$$nl(S) = \{nl(s_1), nl(s_2), nl(s_3), nl(s_4)\}$$

Definition 2 Intersection points:

For any node $s_a \in S$, its intersection points set is defined as a set of points$\Delta(s_a) = \bigcup_{s_b \in nl(s_a)} (s_a \cap s_b) = \bigcup_{s_b \in nl(s_a)} \{p_j^{(s_a, s_b)}(x_j, y_j), p_m^{(s_a, s_b)}(x_m, y_m)\} / j, m \in \mathbb{N}$.

See case (a) and case (b)

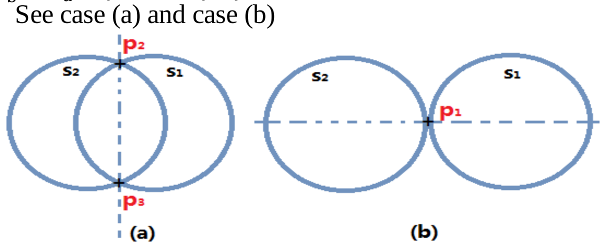

Figure 2. The case (a) shows the case that the intersection of two sensorss_1, $s_2$are two distinct points p_2 and p_3, the case (b) shows that the two sensors intersect in only one redundant points p_1.

From the figure (1) we will get:

$$\begin{cases} \Delta(s_1) = \{p_1(s_1, s_2), p_{11}(s_1, s_2), p_4(s_1, s_3), p_{12}(s_1, s_3), p_6(s_1, s_4), p_8(s_1, s_4)\} \\ \Delta(s_2) = \{p_1(s_1, s_2), p_{p11}(s_1, s_2), p_5(s_2, s_3), p_7(s_2, s_3), p_2(s_2, s_4), p_9(s_2, s_4)\} \\ \Delta(s_3) = \{p_4(s_1, s_3), p_{p12}(s_1, s_3), p_5(s_2, s_3), p_7(s_2, s_3), p_3(s_3, s_4), p_{10}(s_3, s_4)\} \\ \Delta(s_4) = \{p_6(s_1, s_4), p_8(s_1, s_4), p_2(s_2, s_4), p_9(s_2, s_4), p_3(s_3, s_4), p_{10}(s_3, s_4)\} \end{cases}$$

Definition 3 System intersection points:

Let'sρ be the set of intersection points in S, defined as:

$$\rho = \bigcup_{s_a \in S} \Delta(s_a)$$

$$\rho = \Delta(s_1) \cup \Delta(s_2) \cup \Delta(s_3) \cup \Delta(s_4)$$
$$= \{p_1(s_1,s_2), p_{11}(s_1,s_2), p_4(s_1,s_3), p_{12}(s_1,s_3), p_6(s_1,s_4), p_8(s_1,s_4)\} \cup$$
$$\{p_1(s_1,s_2), p_{p11}(s_1,s_2), p_5(s_2,s_3), p_7(s_2,s_3), p_2(s_2,s_4), p_9(s_2,s_4)\} \cup$$
$$\{p_4(s_1,s_3), p_{p12}(s_1,s_3), p_5(s_2,s_3), p_7(s_2,s_3), p_3(s_3,s_4), p_{10}(s_3,s_4)\} \cup$$
$$\{p_6(s_1,s_4), p_8(s_1,s_4), p_2(s_2,s_4), p_9(s_2,s_4), p_3(s_3,s_4), p_{10}(s_3,s_4)\}.$$
$$\rho = \{p_1(s_1,s_2), p_2(s_2,s_4), p_3(s_3,s_4), p_4(s_1,s_3),$$
$$p_5(s_2,s_3), p_6(s_1,s_4), p_7(s_2,s_3), p_8(s_1,s_4), p_9(s_2,s_4),$$
$$p_{10}(s_3,s_4), p_{p11}(s_1,s_2), p_{12}(s_1,s_3)\}.$$

Definition 4 Inclusion points:

For each node $s_j \in S$, the inclusion points of s_j are defined bythe set

$$\rho^{s_j} = \left\{ p_i | p_i \in \rho \text{ and } \sqrt{(x_{p_i} - x_{s_j})^2 + (y_{p_i} - y_{s_j})^2} \le r_{s_j} \right\}$$

where r_{s_j} is the sensing radius of s_j. For example, in Figure (1), $\rho^{s_1} = \{p_1, p_4, p_5, p_6, p_8, p_9, p_{10}, p_{11}, p_{12}\}$. It is clearly that $\rho^{s_j} \subset \rho$.

Definition 5 Inclusion multiset:

The Inclusion multiset of the system S, $\rho^* = \{\rho^{s_1}, \rho^{s_2}, \rho^{s_3} \ldots\}$ is the collection of all inclusion points for each node in S. For example, in Figure1

$\rho^{s_1} = \{p_1, p_4, p_5, p_6, p_8, p_9, p_{10}, p_{11}, p_{12}\}$,

$\rho^{s_2} = \{p_1, p_2, p_5, p_6, p_7, p_9, p_{10}, p_{11}, p_{12}\}$,

$\rho^{s_3} = \{p_3, p_4, p_5, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}\}$ and

$\rho^{s_4} = \{p_2, p_3, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}\}$. Hence,

$$\rho^* = \left\{ \begin{matrix} p_1, p_4, p_5, p_6, p_8, p_9, p_{10}, p_{11}, p_{12}, \\ p_1, p_2, p_5, p_6, p_7, p_9, p_{10}, p_{11}, p_{12}, \\ p_3, p_4, p_5, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, \\ p_2, p_3, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12} \end{matrix} \right\},$$

The cardinality of ρ^*, denoted by$|\rho^*|$, is the number of points in ρ^* including repeated memberships.

Definition 6 Point multiplicity:

For each point$p_i \in \rho^*$, its multiplicity is the number of instances of p_i inρ^*. For example, in Figure 1 (a), p_1,p_2,p_3 and p_4 all have multiplicity 2; p_5, p_6, p_7 and p_8 all have multiplicity 3; p_9, p_{10}, $p_{11}$and p_{12} all have multiplicity 4. The Inclusion points ρ^{s_i}can be rewritten

as $\rho^{s_1} = \{p_1{}^2, p_4{}^2, p_5{}^3, p_6{}^3, p_8{}^3, p_9{}^4, p_{10}{}^4, p_{11}{}^4, p_{12}{}^4\}$.

$\rho^{s_2} = \{p_1{}^2, p_2{}^2, p_5{}^3, p_6{}^3, p_7{}^3, p_9{}^4, p_{10}{}^4, p_{11}{}^4, p_{12}{}^4\}$.

$\rho^{s_3} = \{p_3{}^2, p_4{}^2, p_5{}^3, p_7{}^3, p_8{}^3, p_9{}^4, p_{10}{}^4, p_{11}{}^4, p_{12}{}^4\}$.

$\rho^{s_4} = \{p_2{}^2, p_3{}^2, p_6{}^3, p_7{}^3, p_8{}^3, p_9{}^4, p_{10}{}^4, p_{11}{}^4, p_{12}{}^4\}$.

Definition 7 Multiplicity degree:

Let d be the number of occurrence of the point in the multiset ρ^{*d}, in the example above we have:

$$\rho^* = \{p_1{}^2, p_2{}^2, p_3{}^2, p_4{}^2, p_5{}^3, p_6{}^3, p_7{}^3, p_8{}^3, p_9{}^4, p_{10}{}^4, p_{11}{}^4, p_{12}{}^4\}$$
$$|p_9{}^4, p_{10}{}^4, p_{11}{}^4, p_{12}{}^4| = 4$$
$$|\{p_5{}^3, p_6{}^3, p_7{}^3, p_8{}^3\}| = 3$$
$$|\{p_1{}^2, p_2{}^2, p_3{}^2, p_4{}^2\}| = 2$$

The maximum coverage degree is $\max d = 4$.

Definition 8 Sub-region associated sensor $C_i^{k^{s_i}}$

Let s_i be the associated sensor of C_i^kif and only the head-point of the sub regions occur in inclusion points of s_i

$$\varphi\ C_i^{k^{s_i}} \subset \rho^{s_j}.$$

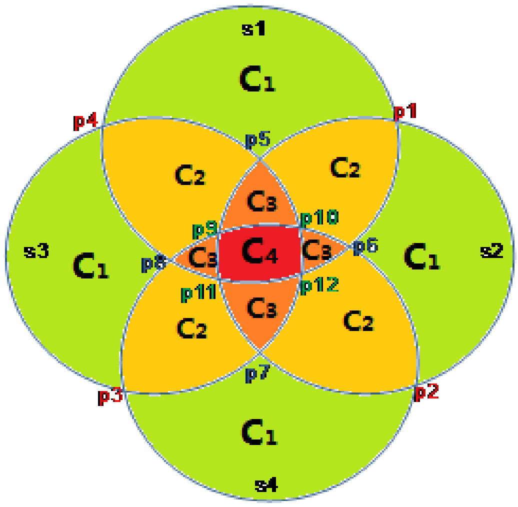

Figure 3. Shows the coverage degree of sub-regions.

III. GRAPH REPRESENTATION

Our proposed weighted undirected graph consists of a finite nonempty set V of vertices and a set E of 2-elements of V called edges. In the graph theory, the notation V(G) is the set of vertex and E(G) is a set of edges. Using the idea of graph theory, we can say that each vertex represents an intersection point in the system S, the linked circle of each two intersection points is represented with an edge weighted with the according linked circle (sensor). There will be a path from the source node to the base station as long as the graph is connected. In our work, we assume that connectivity and coverage of network are managed well.

The graph can be represented mathematically by:

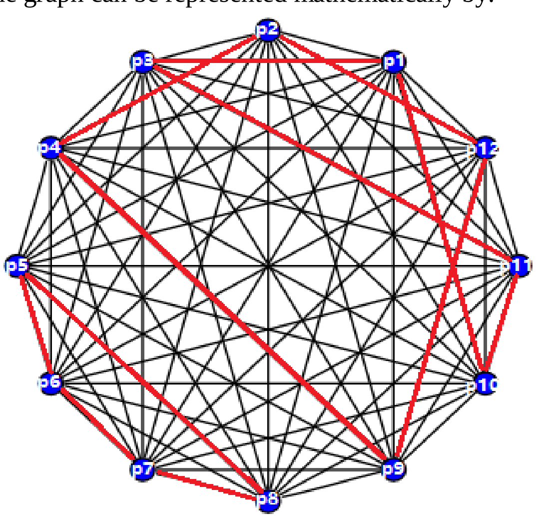

Figure 4. Graph model.

Our proposed undirected graph is the graph with black edges, the red edges are added to show the remaining of our graph to be complete K12 (complete graph with 12 vertices).

A. Graph Edges (ARCS)

$V(G) = \{p_1(s_1,s_2), p_2(s_2,s_4), p_3(s_3,s_4), p_4(s_1,s_3), p_5(s_2,s_3), p_6(s_1,s_4), p_7(s_2,s_3), p_8(s_1,s_4), p_9(s_2,s_4), p_{10}(s_3,s_4), p_{p11}(s_1,s_2), p_{12}(s_1,s_3)\}$.

$|V(G)| = |\rho|$

In the figure3: $|V(G)| = |\rho| = 12$

E(G)={$(p_1,p_2),(p_1,p_4),(p_1,p_5),(p_1,p_6),(p_1,p_7),(p_1,p_8),(p_1,p_9),(p_1,p_{11}),(p_1,p_{12}),(p_2,p_3),(p_2,p_5),(p_2,p_6)(p_2,p_7)(p_2,p_8)(p_2,p_9)(p_2,p_{10})(p_2,p_{11}),(p_3,p_4)(p_3,p_5)(p_3,p_6)(p_3,p_7)(p_3,p_8)(p_3,p_9)(p_3,p_{10}),(p_3,p_{12}),(p_4,p_5),(p_4,p_6),(p_4,p_7),(p_4,p_8),(p_4,p_{10}),(p_4,p_{11}),(p_4,p_{12}),(p_5,p_7),(p_5,p_9),(p_5,p_{10}),(p_5,p_{11}),(p_5,p_{12}),(p_6,p_8),(p_6,p_9),(p_6,p_{10}),(p_6,p_{11}),(p_6,p_{12}),(p_7,p_9),(p_7,p_{10}),(p_7,p_{11}),(p_7,p_{12}),(p_8,p_9),(p_8,p_{10}),(p_8,p_{11}),(p_8,p_{12}),(p_9,p_{10}),(p_9,p_{11}),(p_{10},p_{12}),(p_{11},p_{12})$}.

$$|E(G)| = 54$$

The remaining edges to be complete graph K_{12}:

C(G)= {$(p_1,p_3),(p_1,p_{10}),(p_2,p_4),(p_2,p_{12}),(p_3,p_{11}),(p_4,p_9),(p_5,p_6),(p_5,p_8),(p_6,p_7),(p_7,p_8),(p_9,p_{12}),(p_{10},p_{11})$} ,$|C(G)| = 12$.

$K_{12} = E(G) \cup C(G)$.

Number of edges of $K_{12} = 54 + 12 = \frac{12(12-1)}{2} = 66.$

B. Arcs Weights Matrix

The element of the matrix A_{ij} is defined as a sensor (cycle) linker of the couple of points $(p_{i,}p_j)$. This element will represent a weight of the arc linking the two respective points

TABLE I. SHOWS THE ARC WEIGHT FOR EACH COUPLE (p_i, p_j)

	$p_1^{(s_1,s_2)}$	$p_2^{(s_2,s_4)}$	$p_3^{(s_3,s_4)}$	$p_4^{(s_1,s_3)}$	$p_5^{(s_2,s_3)}$	$p_6^{(s_1,s_4)}$	$p_7^{(s_2,s_3)}$	$p_8^{(s_1,s_4)}$	$p_9^{(s_2,s_4)}$	$p_{10}^{(s_3,s_4)}$	$p_{11}^{(s_1,s_2)}$	$p_{12}^{(s_1,s_3)}$
$p_1^{(s_1,s_2)}$		$\{s_2\}$	$\emptyset$	$\{s_1\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2\}$	$\emptyset$	$\{s_1,s_2\}$	$\{s_1\}$
$p_2^{(s_2,s_4)}$	$\{s_2\}$		$\{s_4\}$	$\emptyset$	$\{s_2\}$	$\{s_4\}$	$\{s_2\}$	$\{s_4\}$	$\{s_2,s_4\}$	$\{s_4\}$	$\{s_2\}$	$\emptyset$
$p_3^{(s_3,s_4)}$	$\emptyset$	$\{s_4\}$		$\{s_3\}$	$\{s_3\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_4\}$	$\{s_3,s_4\}$	$\emptyset$	$\{s_3\}$
$p_4^{(s_1,s_3)}$	$\{s_1\}$	$\emptyset$	$\{s_3\}$		$\{s_3\}$	$\{s_1\}$	$\{s_3\}$	$\{s_1\}$	$\emptyset$	$\{s_3\}$	$\{s_1\}$	$\{s_1\}$
$p_5^{(s_2,s_3)}$	$\{s_2\}$	$\{s_2\}$	$\{s_3\}$	$\{s_3\}$		$\emptyset$	$\{s_2,s_3\}$	$\emptyset$	$\{s_2\}$	$\{s_3\}$	$\{s_2\}$	$\{s_3\}$
$p_6^{(s_1,s_4)}$	$\{s_1\}$	$\{s_4\}$	$\{s_4\}$	$\{s_1\}$	$\emptyset$		$\emptyset$	$\{s_1,s_4\}$	$\{s_4\}$	$\{s_4\}$	$\{s_1\}$	$\{s_1\}$
$p_7^{(s_2,s_3)}$	$\{s_2\}$	$\{s_2\}$	$\{s_3\}$	$\{s_3\}$	$\{s_2,s_3\}$	$\emptyset$		$\emptyset$	$\{s_2\}$	$\{s_3\}$	$\{s_2\}$	$\{s_3\}$
$p_8^{(s_1,s_4)}$	$\{s_1\}$	$\{s_4\}$	$\{s_4\}$	$\{s_1\}$	$\emptyset$	$\{s_1,s_4\}$	$\emptyset$		$\{s_4\}$	$\{s_4\}$	$\{s_1\}$	$\{s_1\}$
$p_9^{(s_2,s_4)}$	$\{s_2\}$	$\{s_2,s_4\}$	$\{s_4\}$	$\emptyset$	$\{s_2\}$	$\{s_4\}$	$\{s_2\}$	$\{s_4\}$		$\{s_4\}$	$\{s_2\}$	$\emptyset$
$p_{10}^{(s_3,s_4)}$	$\emptyset$	$\{s_4\}$	$\{s_3,s_4\}$	$\{s_3\}$	$\{s_3\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_4\}$		$\emptyset$	$\{s_3\}$
$p_{11}^{(s_1,s_2)}$	$\{s_1,s_2\}$	$\{s_2\}$	$\emptyset$	$\{s_1\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2\}$	$\emptyset$	$\{s_1\}$	$\{s_1\}$
$p_{12}^{(s_1,s_3)}$	$\{s_1\}$	$\emptyset$	$\{s_3\}$	$\{s_1,s_3\}$	$\{s_3\}$	$\{s_1\}$	$\{s_3\}$	$\{s_1\}$	$\emptyset$	$\{s_3\}$	$\{s_1\}$	$\{s_1\}$

IV. NET ARCS METHOD (NA)

Definition Net Arcs:

Net arc is the arc that link two direct neighbors in the same cycle, such that there is no other arc crossing or dividing this arc.Two net arcs can intersect only in the end arc points.

Algorithm:

Finding the net arcs (NA) algorithm for the system S is broken up into three steps for each sensor:

For each node $s_j \in S$ do

Initialization:

$\delta^+ := \emptyset; \delta^- := \emptyset;\ |\delta^+| = 0;\ |\delta^-| = 0;$

***Step 1**:* Arranging (ordering) the points in (y-Axis) in two groups δ^+ and δ^-.

For each $p_i \in \Delta(s_j)$ do

If $y_{p_i} > y_{s_j}$, then $\delta^+ := \delta^+ \cup \{p_i\}$; $|\delta^+| := |\delta^+| + 1$;

Otherwise $\delta^- := \delta^- \cup \{p_i\}$;$|\delta^-| := |\delta^-| + 1$;

End;

Step 2: Arranging the points in (X-Axis) in ascending order [9]

- Arranging the points belong to δ^+ in Array V[i] inascending order inthe (X-Axis). [9].

$x_{v[1]} < x_{v[2]} < x_{v[3]} < \cdots < x_{v[|\delta^+|]}$.

V=	p_f	p_l	p_j				p_m
	i=1						i= $|\delta^+|$

- Arranging the points belong toδ^-in an Array W[i] in ascending order inthe (X-Axis).[9]

Such that: $x_{w[1]} < x_{w[2]} < x_{w[3]} < \cdots < x_{w[|\delta^+|]}$.

W=	p_c	p_v	p_o				p_k
	i=1						i=$|\delta^+|$

$W[1] = p_c, .. W[|\delta^+|] = p_k$.

Step 3: Linking the points

Remark1:

When applying (NA) for each sensor the arc linker of the points is weighted with the value equal to the current sensor.

Remark2:

If the circle (sensor) contains only two points (Number of its intersection points equal to number of its inclusion points is equal to two) then we will link these two points with two net arcs with the same weight and the same end points but we will add use different indexes to distinguish between them.

Remark3:

If there is two different net arcs $(p_i, p_j)^{s_j}$with the same end points and the same weight means there are the only two points in the circle (sensor s_j) but with different index i to show that there are two and not one. $(p_i, p_j)^{s_j}{}_i, (p_i, p_j)^{s_j}{}_j$.

Remark4:

If there is sensor s_j that has only one inclusion point(intersection point) $\rho^{s_j} = \{p_i\}$we will link the point with itself by using a net arc weighed s_j(The value of the current sensor).

In each array, we link between two successive points

- Linking V array points elements;

$\forall i = \overline{1, |\delta^+| - 1}$ *there is an arc linking* $(v[i], v[i+1])$

- Linking W array elements;

$\forall i = \overline{1, |\delta^-| - 1}$ *there is an arc linking* $(w[i], w[i+1])$

- Linking between extremities elements of the vectors.

If v [1] =v [[$|\delta^+|$] and W [1] =w[$|\delta^+|$] do

Link v [1] and W [1] with two arcs with same weight but different index to distinguish between them.

Otherwise link (v [1], w [1]) and we link (v [$|\delta^+|$], w[$|\delta^+|$])

End;

End;

Example: let's apply the algorithm to the figure (6-a)

By applying step 1 to the figure (6-a)

We will get: $\delta^+ = \{p_4, p_1\}$, $\delta^- = \{p_6, p_8, p_{12}, p_{11}\}$

By applying step 2 we will get:

V:	p_4	p_1

W:	p_8	p_{11}	p_{12}	p_6

Step3:

We link (p_4, p_1) .

We link (p_8, p_{11}),(p_{11}, p_{12}),(p_{12}, p_6),

We link (p_4, p_8),(p_1, p_6).

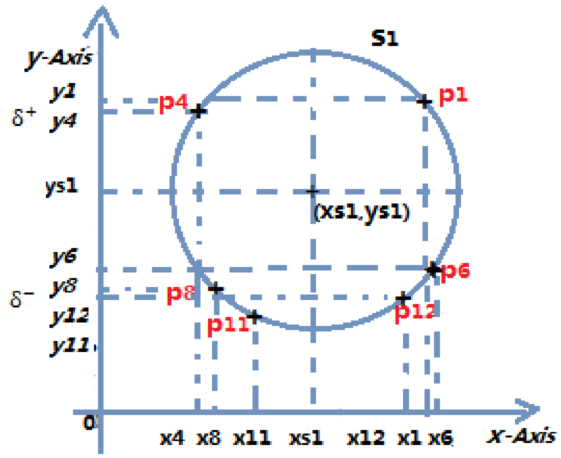

Figure 5. The net arcs for the circle (sensor S_1).

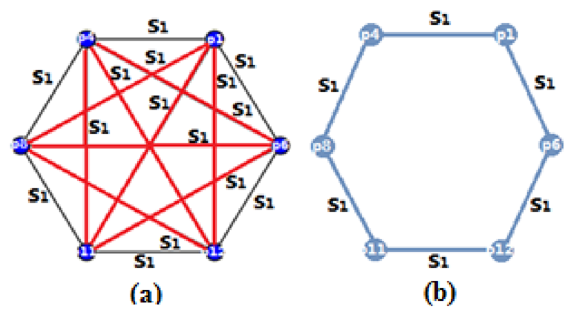

Figure 6. Shows the reduction of numbers of edges in sensor s_1 from the state (a) before using net arcs method to figure (6-b) after the use of our net arcs method.

By applying the net arc method to figure of graph G we will get a new graph G^*that contains the following net arcs:

$E(G^*) = \{(p_1, p_2)^{s2}, (p_1, p_4)^{s1}, (p_1, p_5)^{s2}, (p_1, p_6)^{s1}, (p_2, p_3)^{s4}, (p_2, p_6)^{s4}, (p_2, p_7)^{s2}, (p_3, p_4)^{s3}, (p_3, p_8)^{s4}, (p_3, p_7)^{s3}, (p_4, p_5)^{s3}, (p_4, p_8)^{s1}, (p_5, p_9)^{s2}, (p_5, p_{10})^{s3}, (p_6, p_{10})^{s4}, (p_6, p_{12})^{s1}, (p_7, p_{11})^{s2}, (p_7, p_{12})^{s3}, (p_8, p_9)^{s4}, (p_8, p_{11})^{s1}, (p_9, p_{10})^{s4},$

$(p_9, p_{11})^{s2}, (p_{10}, p_{12})^{s3}, (p_{11}, p_{12})^{s1}\}$.
$|E(G^*)| = 24$.

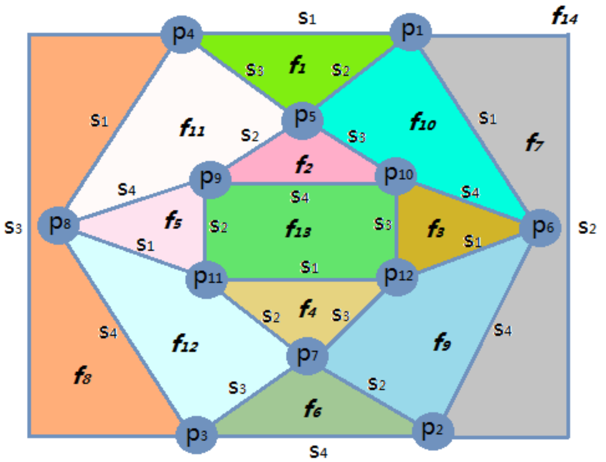

Figure 7. The graph G* of the system S (sub-regions of overlapped sensors), such that E(G*) is the set of net arcs for S.

$|V(G^*)| = 12, |E(G^*)| = 24$

Remark:

We note that Euler's formula for the planar graph is satisfied.

$v - e + f = 2.\ f = 2 - v + e = 2 - 12 + 24 = 14$

$f = 14$. (It can't prove yet that the graph is planar it will be proved in further research).

V. FINDING THE OVERLAPPED COVERED SUB-REGIONS (C^2, C^3) AND THE MAXIMUM COVERED REGIONS.

A. *Finding the Overlapped Covered Sub-regions (2-Edges)* C^2

Definition 1 Covered Sub-region (2-edges (arcs)) C^2: It's a sub-region bounded by two net arcs linking the same two distinct points, and covered by a number of sensors.

Case1: If in G^*there are two points linked with twonet arcs with different weights ,it means there is a covered sub-region (2.edges) C^2 between the respective points bounded by the respective net arcs ,(we assume that WSN system is more than 3 sensors), see figure (8).

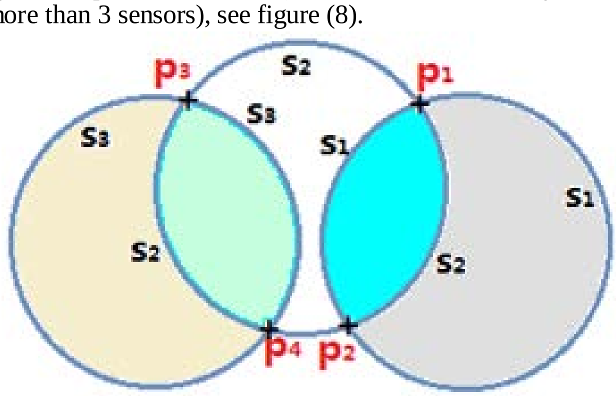

Figure 8. WSN of 3 sensors.

In figure (8) The two points (p_1, p_2) are linked by net arcs weighted withs_1 : $(p_1, p_2)^{s1}{}_1, (p_1, p_2)^{s1}{}_2$and one net arc $(p_1, p_2)^{s2}$ weighted with s_2.

The two points $(p_3, p_4)_1^{s3}, (p_3, p_4)_2^{s3}$ and one net arc $(p_3, p_4)^{s2}$. Link the two points (p_3, p_4).

$C_1^2 = \{(p_1, p_2)^{s1}{}_1, (p_1, p_2)^{s2}\}$=$p_1$<u>s1</u> p_2<u>s2</u> p_1

$C_2^2 = \{(p_1, p_2)^{s1}{}_2, (p_1, p_2)^{s2}\}$ = p_1<u>s1</u> p_2<u>s2</u> p_1

$C_3^2 = \{(p_3, p_4)_1^{s3}, (p_3, p_4)^{s2}\}$=$p_1$<u>s3</u> p_2<u>s2</u> p_1

$C_4^2 = \{(p_3, p_4)_2^{s3}, (p_3, p_4)^{s2}\}$=$p_1$<u>s3</u> p_2<u>s2</u> p_1

R_1^2 = p_1<u>s1</u> p_2<u>s1</u> p_1, R_2^2 = p_3<u>s3</u> p_4<u>s3</u> p_3.

The R_1^2 and R_2^2 are regions and contains sub-regions inside. Because the sub-region can't be bounded by edges with the same weight and index.

C_1^2, C_2^2, C_3^2 and $C_4^2$are (faces or sub-region) bounded by (2- edges), because each sub-region is bounded by two net arcs with different weighted.

In the above figure, each two net arcs with different weights satisfy the case 1 thus forms a sub-region bounded by these two net arcs.

Remark1:

If the two points are linked with three net arcs (figure8)

It means that two of them must have the same weight and the third one is different because we assume that the overlapping is partially and the system is minimum3sensors.

In figure (8), each two net arcs with different weight satisfies the case 1 form a sub-region bounded by these two net arcs.

Definition2:

Coverage sub-region degree:$C_i^{2^d}$(case 1) for $d > 3$. Let d be the number of occurrence of the sub-regionC_i^khead-points sequence in the Inclusion multiset of the system S, ρ^*.

Remark2:

This definition is always satisfied for Coverage degree more than 3. $d > 3$,But if it is less than 3 we can't apply it. Like figure (3).

$C_1^2 = \{(p_1, p_2)^{s1}{}_1, (p_1, p_2)^{s2}\}$= p_1<u>s1</u> p_2<u>s2</u> p_1

$C_2^2 = \{(p_1, p_2)^{s1}{}_2, (p_1, p_2)^{s2}\}$ = p_1<u>s1</u> p_2<u>s2</u> p_1

d=2 (by using the above definition) for both sub-regions but as shown in the graph only one of these two sub-regions is covered by 2 sensors, is the sub –region in blue color.

And the same for:

$C_3^2 = \{(p_3, p_4)_1^{s3}, (p_3, p_4)^{s2}\}$=$p_1$<u>s3</u> p_2<u>s2</u> p_1

$C_4^2 = \{(p_3, p_4)_2^{s3}, (p_3, p_4)^{s2}\}$=$p_1$<u>s3</u> p_2<u>s2</u> p_1

d=2 for both sub-regions but as shown in the graph only one of these two sub-regions are covered by 2 sensors, the sub –region in green color.

As we explained our coverage definition for (2-edges) sub-region is applicable only in case thatd>3.

But for d=2. The two neighbor'ssub-regions that share one edge, one of them satisfy the definition and one not, so we check by adding points in one sub-region and after finding this point belongs to how many sensors and conclude the coverage degree of that sub-region we can conclude the other, because the coverage degree founded by the above definition is the maximum degree that can be found between these two sub-regions. And by finding other we reduce one degree for the other or we add one. Then we will have at the last (d, d-1) for the coverage degree of these two neighbor's sub-regions.

Definition3:

For each node $p_i \in \rho$, there is $s_i \in S$, such that $p_i \in \Delta(s^i)$. then s_i is defined as the associated sensor for p_i defined as $p_i^{s^i}$.

Case 2: If in G^*there is two points linked only with two net arcs with the same weight (Two intersections points belong to same circle(sensor)) and the two points have one and only one common associated sensor means there is a sub-region bounded by these two net arcs and two head-points (the respective points), and covered by only the current sensor (The common associated sensor),see figure (9).(We assume that we have more than 3sensors).

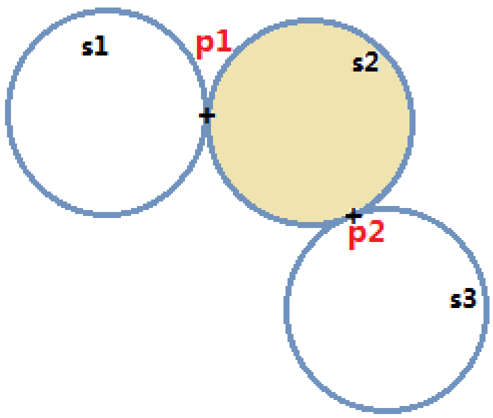

Figure 9. Figure (9): WSN of 3 sensors

$C^2 = \{(p_1, p_2)^{s2}, (p_1, p_2)^{s2}\}$
$\Delta(s^1) = p_1, \Delta(s^2) = \{p_1, p_2\}, \Delta(s^3) = p_3,$
$p_1 \in \Delta(s^1), p_1 \in \Delta(s^2)$
$p_2 \in \Delta(s^1), p_2 \in \Delta(s^2)$
$\{p_1, p_2\} \in \Delta(s^1).$

Mean there is a sub-region between these two points bounded by net arc weighted with the associated sensor value (Sub-region is the sensor itself)

The coverage degree is C=1.

B. Finding the Overlapped Covered Sub-regions Bounded by (3-Edges) C^3

Definition1Sub-region (3-edges) C^3:

It's a sub-region bounded by three net arcs linking the same three distinct points, such that the intersection points between each two net arcs is only one point, these points are the head-points.

Definition 2 Point Neighbor List:

For any point$p_i \in \rho$, its neighbor list is defined as a set of points $nl(p_i) = \{p_j | p_j \in \rho \text{ and } (p_i, p_j) \in E(G^*)\} \cup \{p_i\}$.we assume that each point its neighbor of itself in order to get the 3 -sequence of three points in the respective three points.

Definition 3 Let's R_i^3 be the 3-sequence:

It's a sequence composed of three points.

Algorithm of finding the sub-regions (3-Edges) C_i^3

Step 1: Finding all points' neighbor lists.

Step 2: Finding the all R^3(3-sequence) in the neighbor lists.

Step 3: Checking the all regions (3- sequences) foundedin (step 2) whether are sub-regions or not (sub-region can't be decomposed into sub-regions inside). See the figure

Step3-1: If in 3- sequenceR_i^3there is at least two points linked with two different net arcs mean that the region $R_i^3$is not a Sub-region $C_i^3$to find sub-regions apply method 1.

Step 3-2: Otherwise if in the 3-sequence there is no two net arcs linking two same points, mean that this region bounded by these three points and the respective three net arcs is Sub-region C_i^3.

Method 1: In each 3-sequence find the sensors that one of the three points doesn't belongs to it, then form regions of three points using the net arcs weighted with these sensors (belongs to these sensors) and eliminate the remaining net arcs for each two points linked with two net arcs. These formed regions are sub-region (3-edges) C_i^3.

Definition4: Coverage sub-region degree:$C_i^{3^d}$

Let d be the number of occurrence of the sub-regionC_i^khead-points sequence in the Inclusion multiset of the system S, ρ^*.

Applying the Algorithm for Figure (7)

Applying step 1

$nl(p_1) = \{P_1, P_2, P_4, P_5, P_6\}, nl(p_2) = \{P_1, P_2, P_3, P_6, P_7\}$
$nl(p_3) = \{P_2, P_3, P_4, P_7, P_8\}, nl(p_4) = \{P_1, P_3, P_4, P_5, P_8\}$
$nl(p_5) = \{P_1, P_4, P_5, P_9, P_{10}\}, nl(p_6) = \{P_1, P_2, P_6, P_{10}, P_{12}\}$
$nl(p_7) = \{P_2, P_3, P_7, P_{11}, P_{12}\}, nl(p_8) = \{P_3, P_4, P_8, P_9, P_{11}\}$
$nl(p_9) = \{P_5, P_8, P_9, P_{10}, P_{11}\},$
$nl(p_{10}) = \{P_5, P_6, P_9, P_{10}, P_{12}\}$
$nl(p_{11}) = \{P_8, P_9, P_{10}, P_{11}, P_{12}\},$
$nl(p_{12}) = \{P_6, P_7, P_{10}, P_{11}, P_{12}\}$

Applying step2: the set R^3 of all 3-sequences in the system$S$.

$R_1^3 = \{P_1, P_4, P_5\}, R_2^3 = \{P_5, P_9, P_{10}\}, R_3^3 = \{P_6, P_{10}, P_{12}\}$
$R_4^3 = \{P_7, P_{11}, P_{12}\}, R_5^3 = \{P_8, P_9, P_{11}\}, R_6^3 = \{P_2, P_3, P_7\}$
$R_7^3 = \{P_1, P_2, P_6\}, R_8^3 = \{P_3, P_4, P_8\}.$
$R^3 = \{R_1^3, R_2^3, R_3^3, R_4^3, R_5^3, R_6^3, R_7^3, R_8^3\}.$

Applying step 3 for the result of step 2

In the result of step 2 ; there is no 3- sequence satisfying the step(3-1),the all 3-sequences satisfy the step (3-2) which mean the whole regions founded in step 2 are Sub-regions (faces) $C^3{}_i$ with the coverage degree $C_1^{3^d}$ .Its already explained how to calculate the coverage degree of the sub-region(Section2)

$C_1^3 = f_1 = P_1 \underline{\ s_1\ } P_4 \underline{\ s_3\ } P_5 \underline{\ s_2\ } P_1$. d=1, $C_1^{3^1}$. Covered by 1 sensor.

$C_2^{3^3} = f_2 = P_5 \underline{\ s_2\ } P_9 \underline{\ s_3\ } P_{10} \underline{\ s_3\ } P_5$, covered by 3sensors.

$C_3^{3^3} = f_3 = P_6 \underline{\ s_3\ } P_{10} \underline{\ s_3\ } P_{12} \underline{\ s_1\ } P_6$, covered by 3sensors.

$C_4^{3^3} = f_4 = P_7 \underline{\ s_2\ } P_{11} \underline{\ s_1\ } P_{12} \underline{\ s_3\ } P_7$, covered by 3sensors.

$C_5^{3^3} = f_5 = P_8 \underline{\ s_3\ } P_9 \underline{\ s_2\ } P_{11} \underline{\ s_1\ } P_8$, covered by 3sensors.

$C_6^{3^1} = f_6 = P_2 \underline{\ s_3\ } P_3 \underline{\ s_3\ } P_7 \underline{\ s_2\ } P_2$. covered by 1 sensor.
$C_7^{3^1} = f_7 = P_1 \underline{\ s_2\ } P_2 \underline{\ s_3\ } P_6 \underline{\ s_1\ } P_1$. covered by 1 sensor.
$C_8^{3^1} = f_8 = P_3 \underline{\ s_3\ } P_4 \underline{s_1} P_8 \underline{\ s_3\ } P_3$. covered by 1 sensor.

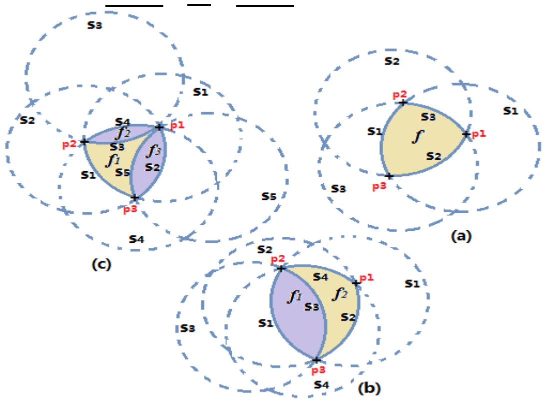

Figure 10. Shows some cases of forming regions by three points. R^3.

In Figure (10-a): There is no two net arc linking the same two points in the 3 -sequence$(p_1, p_2)^{s3}, (p_1, p_3)^{s2}, (p_2, p_3)^{s1}$. R^3={P_1, P_2, P_3}

Hence, the 3-sequence $R^3$is a sub-region (face) bounded with 3-edges C^3. without any sub-region inside.

In Figure (10-b): There is two net arcs linking the two points:

$p_2$and p_3, $(p_1, p_2)^{s4}, (p_2, p_3)^{s1}, (p_2, p_3)^{s3}, (p_1, p_3)^{s2}$.

R^3={P_1, P_2, P_3} it's a region bounded by three points but it has sub-regions inside. So $R^3$is not a sub-region.

In Figure (10-c): There is two net arcs linking the two points $p_1$and p_2, $(p_1, p_2)^{s3}, (p_1, p_2)^{s4}$ and two net arcs linking the two points$p_1$and p_3,$(p_1, p_3)^{s2}$,$(p_1, p_3)^{s5}$.

R^3={P_1, P_2, P_3} it's a region bounded by three points but it has sub-regions inside. So $R^3$is not a sub-region (face) $\boldsymbol{C^3}$.

To find the sub-regions for Figure(9-b) and Figure(9-c) We apply Method1.

In figure (10-b):$\{p_2, p_3\} \in \rho^{s_1} \cup \rho^{s_2} \cup \rho^{s_3} \cup \rho^{s_4}$.
$\{p_1\} \in \rho^{s_1} \cup \rho^{s_2} \cup \rho^{s_4}$ but $\{p_1\} \notin \rho^{s_3}$.

The sensor s_3,has divided the overlapped region(3edges) composed by these three points. The sub-region of these three points must contain the net arc linking these two points and belongs to the sensor s_3, $(p_2, p_3)^{s3}$.and eliminate the remaining net arc that links the same two points.$(p_2, p_3)^{s1}$

Thus, we will get the Sub-region:
$\boldsymbol{C^3} = P_1 \underline{\ s_4\ } P_2 \underline{\ s_3\ } P_3 \underline{\ s_2\ } P_1$.

By applying the method 1 for figure (9-c) we will get:
$\boldsymbol{C^3} = P_1 \underline{\ s_3\ } P_2 \underline{\ s_1\ } P_3 \underline{\ s_5\ } P_1$.

C. Finding the Maximum Covered Regions

To find the maximum covered regions we will partition the graph to groups that contain maximum number of overlapped sensors.

Definition 1: Grouping technique:

Partitioning the sensors in groups will ensure theconnectivity of the network, the group is defined as the overlapped sensors, to find the groups we will use the sensor neighbor list idea.

The group $G^j = \{s_l, .., s_k\}$ composed of m sensors is defined as a repeated sequence of these m sensors in the all sensors neighbor list of these m sensors. j is used only to distinct between the groups.

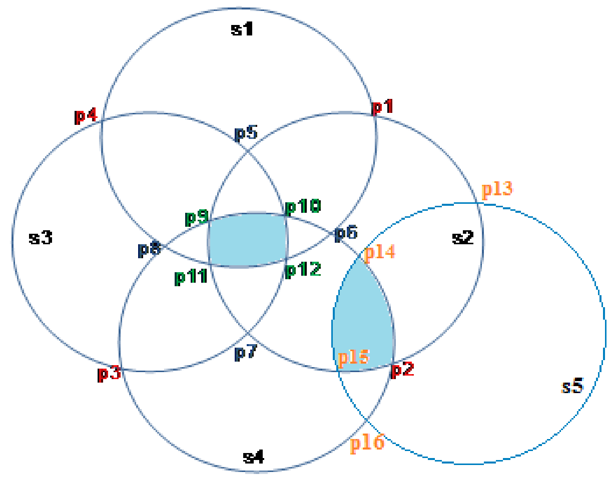

Figure 11. WSN with 6 sensors

$nl(s_1) = \{s_1, s_2, s_3, s_4\}$, $nl(s_2) = \{s_1, s_2, s_3, s_4, s_5\}$,
$nl(s_3) = \{s_1, s_2, s_3, s_4\}$, $nl(s_4) = \{s_1, s_2, s_3, s_4, s_5\}$,
$nl(s_5) = \{, s_2, s_4, s_5\}$

we have the sequence $\{s_1, s_2, s_3, s_4\}$ repeated in all $= \{nl(s_1), nl(s_2), nl(s_3), nl(s_4)\}$ which mean that there is a group formed with these sensors.
$G^1 = \{s_1, s_2, s_3, s_4\}$.

We have the sequence $\{s_2, s_4, s_5\}$ repeated in all $= \{nl(s_2), nl(s_4), nl(s_5)\}$ which mean that there is a group formed with these sensors.
$G^2 = \{s_2, s_4, s_5\}$.

Definition 2 Group points:

For each group of nodes$G^j = \{s_0, s_1, s_2 \dots\}$, the group points is a set, denoted by $\rho(G^j)$, of the intersection points of all elements in G^j.

For example, in Figure (11),$G^1 = \{s_1, s_2, s_3, s_4\}$,and the points of$G^1$are the set $\rho_{G^1} = \{p_1, p_2, p_3, p_4, p_5, .., p_{12}\}$.
$G^2 = \{s_2, s_4, s_5\}$, and the points of$G^2$are the set
$\rho_{G^2} = \{p_2, p_9, p_{13}, p_{14}, p_{15}, p_{16}\}$.

Definition3Maximum covered region:

The maximum covered region of a group G of m overlapped sensors is the region covered by the m respective sensors.

Forming the maximum covered regions:

For each group G after finding its inclusion points, the points with the maximum multiplicity degree will be the head- points of the maximum covered region in G bounded by these head-points and the edges(net arcs) linking between them.

Remark:

The multiplicity degree of maximum covered regions points doesn't change by using the grouping technique, but the multiplicity degree of others points which are not maximum covered regions points can change after using the grouping. Each maximum covered region formed by group has one maximum covered region with degree equal to number of sensors forming this group. This node in the maximum covered regions in the groups keep the same coverage degree in the system.

For each ρ_{G^i} we have already found the multiplicity degree of its points after using the grouping some points may not have the same multiplicity degree but the maximum covered region points will keep the same multiplicity degree. we will find the set of points with maximum multiplicity degree in each group points this set will be the head-points of maximum coverage degree region, bounded of the net arcs linking these head-points.

Ex in figure (11):

The multiplicity degree of system points before the repartition of the system to groups is:

$$\rho^* = \begin{Bmatrix} {p_1}^2, {p_2}^3, {p_3}^2, {p_4}^2, {p_5}^3, {p_6}^3, {p_7}^3, {p_8}^3, {p_9}^4, {p_{10}}^4, {p_{11}}^4, {p_{12}}^4, \\ {p_{13}}^2, {p_{14}}^3, {p_{15}}^2, {p_{16}}^2 \end{Bmatrix}$$

The multiplicity degree inclusion points of $G_1 = \{s_1, s_2, s_3, s_4\}$

$$\rho^{*G_1} = \{{p_1}^2, {p_2}^2, {p_3}^2, {p_4}^2, {p_5}^3, {p_6}^3, {p_7}^3, {p_8}^3, {p_9}^4, {p_{10}}^4, {p_{11}}^4, p_{12}\}$$

Maximum coverage degree= 4

$Points\ of\ maximum\ covered\ area\{{p_9}^4, {p_{10}}^4, {p_{11}}^4, {p_{12}}^4\}$

We can see that these points keep the same multiplicity degree in ρ^*, because they are head-points of the maximum covered region.

But ${p_2}^2$ has been changed in this group,then it is not a head-point of the group G_1.

The multiplicity degree inclusion points of $G_2 = \{s_2, s_4, s_5\}$

$\rho^{*G_2} = \{{p_2}^3, {p_9}^2, {p_{13}}^2, {p_{14}}^3, {p_{15}}^3, {p_{16}}^2\}$.

Maximum coverage degree=3,

$Points\ of\ maximum\ covered\ area\{{p_2}^3, {p_{14}}^3, {p_{15}}^3\}$

But the multiplicity degree of p_2 has been changed to ${p_9}^2$ this group instead of ${p_9}^4$ in ρ^* . Then it is not a head-point of the group G_2.

We can see that these points keep the same multiplicity degree in ρ^*. If they are head-points of the maximum covered region.

Then we will get the maximum covered regions by linking the head-points of each region by Net arcs.

$\{{p_9}^4, {p_{10}}^4, {p_{11}}^4, {p_{12}}^4\}$and $\{{p_2}^3, {p_{14}}^3, {p_{15}}^3\}$

${C_1^4}^4 = P_9 \underline{\ s_4\ } P_{10} \underline{\ s_3\ } P_{12} \underline{\ s_1\ } P_{11} \underline{\ s_2\ } P_9$.

${C_1^3}^3 = P_2 \underline{\ s_4\ } P_{14} \underline{\ s_5\ } P_{15} \underline{\ s_2\ } P_2$.

Definition4 Head- points;

Let φC_i^k be the set of head points. For each sub-region C_i^k formed by a k net arcs, (k=2,3) each two net arcs forming this sub-region are adjacent in (1,2) points respectively with k, these points are called Head-points of the sub-region C_i^k.

Definition 5 Sub-region associated group:

Let ${C_i^k}^{G^j}$, be the associated group of the sub-region (k-edges) if and only if the sub-regionC_i^k is included in the group G^j.

$$C_i^k \subset G^j.$$

Finding the associated group of each sub-region:

Case 1 If there is a sub-regionC_i^ksuch that $\varphi C_i^k \subset \rho(G^j)$ then the group G^j is called the associated group of the respective sub-region C_i^k.defined as ${C_i^k}^{G^j}$.

Ex In figure (11):

$\varphi C_1^3 = \{P_1, P_4, P_5\}, \{P_1, P_4, P_5\} \subset \{p_1, p_2, p_3, p_4, p_5., p_{12}\}$,

$\varphi C_1^3 \subset \rho_{G^1}. Then\ the\ sub - region\ C_1^3 \subset G^1$.

Case 2 For the sub-regions that don't satisfy the case (1) we already know the covering sensors of this sub-regions with the coverage degree see (def). The associated group of these sub-regions is the associated group of the sensors covering these sub-regions.

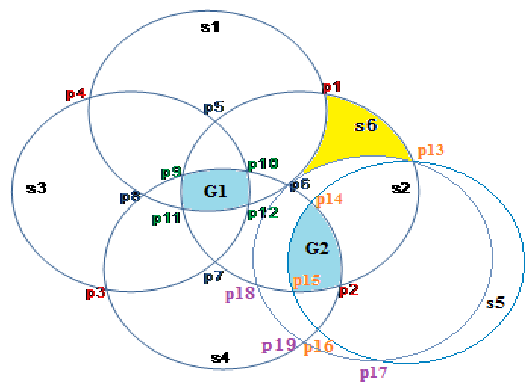

Figure 12. WSN with 7 sensors.

Ex in figure (12) The sub-region:

$\varphi C^3 = \{p_1, p_6, p_{13}\}, \varphi C^3 \not\subset \rho_{G^1}$ and $\varphi C^3 \not\subset \rho_{G^2}$. But we have: $\varphi C^3 \subset \rho^{s_6}$, means that the region is covered by sensor s_6.

We know through the grouping strategy, the sensors that form the groups. So, we have the associated group of each sensor.

By knowing the associated group of the sensor, we will conclude the associated group of each sub-region included in this sensor, which will be the same associated group of this sensor.

In this example: the associated groups of $s_6$are:

$G^1$and G^2. Hence, the associated groups of the sub-region Formed by $\{p_1, p_6, p_{13}\}$are: $\{G^1, G^2\}$.

Network Connectivity:

To ensure the connectivity of our founded sub-regions in the system S. We assume:

1. Each sub-regionis included in at least one sensor, we assume that each region inside our network is covered by at least onesensor, otherwise we can't have any data for elements inside this region. [3,6]
2. Each sensor belongs to at least one group. There is always connectivity between groups. [6]
3. Each sub-region is included in at least one sensor (The network is fully covered by assumption) and that sensor belongs at least to one group, which

mean that each sub-region is included at least in one group.

VI. CONCLUSION

Our proposed approach achieves the goal of finding the all coveredfaces(sub-regions) bounded by two or three (Edges(arcs)) and the maximum covered regions as well. In order to keep the network connected we explained that these founded sub-regions are always connected as long as the network lifetime is not over. Our Further research will aim to the following goals:

1. To prove that the implemented new method Net Arcs (NA) leads always to the planar graph, then we can use the properties of this graph to find the all overlapped covered sub-regions.
2. To find the all covered sub-regions in WSN.
3. To design more energy efficient routing protocol based in the overlapped covered sub-regions in WSN.

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