The Influence of Atmospheric Turbulence on Radio Vortex Wave

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Abstract-**Spiral space phase distribution of the vortex wave form specific orbital angular momentum. This paper studies the effect of atmospheric turbulence on the vortex wave phase through the numerical simulation. In order to analyze the influence of atmospheric turbulence on the vortex wave, we make comparison between various effects by different atmosphere structure constants, transmission distances and topological charges. When the refractive index structure constant is $8 \times 10^{-15} \mathrm{m}^{-2/3}$, the spiral phase change is about 37.5% ; When the refractive index structure constant increases up to a value of $8 \times 10^{-13} \mathrm{m}^{-2/3}$ accordingly, spiral phase increases to 50%; when transmission distance reaches up to 1000 m, phase change increases to 62.5%. At the same time, in different azimuth space, spiral phase change effect is also different. The research results show that the strength of the atmospheric turbulence, space orientation, and wave transmission distance will make the OAM phase distribution of the vortex wave changes, therefore, they are the important factors of affecting the characteristics of the vortex waves OAM.**

Keywords-orbital angular momentum; atmospheric turbulence; vortex wave; phase characteristic

I. INTRODUCTION

In order to improve the communication system capacity and spectrum efficiency, it's necessary to explore innovative technology. In recent years, orbital angular momentum (OAM) becomes a study hotspot and is causing more and more widely concerned in wireless communication. However, the performance of communication system in free-space is inevitably affected by space propagation environment[10]. In this paper we analyze the influence of atmospheric turbulence on radio vortex wave signal phase, give the simulation of transmission signal phase under different transmission distance and different atmospheric turbulence intensity, and make comparison of different influences between the two topological charges of $l=1$ and $l=4$.

II. THE MAIN FEATURES OF VORTEX WAVE ORBITAL ANGULAR MOMENTUM

We can add a twiddle factor associated with spatial phase angle on the normal electromagnetic wave to generate vortex electromagnetic wave, the vortex electromagnetic waves can be represented as follows[8]:

$$U(r,\theta) = A(r) \times e^{il\theta} \tag{1}$$

In which, $A(r)$ is the amplitude of normal electromagnetic wave, r is the radiation distance from the center of a beam, θ is space phase angle and l is the intrinsic value of orbital angular momentum.

At the transmitter, the plane electromagnetic wave revolve and form vortex electromagnetic wave; at the receiver, in order to gain the normal electromagnetic waves, the vortex electromagnetic wave should rotate according to the opposite direction. OAM beams with different topological charges of l values are mutually orthogonal, and they satisfy the following formula [3]:

$$(U_1(r,\theta), U_2(r,\theta)) = \int U_1(r,\theta) U_2^*(r,\theta) r dr d\theta$$
$$= \begin{cases} 0, \forall l_1 \neq l_2 \\ \int \left| U_1(r,\theta) \right|^2 r dr d\theta, \quad l_1 = l_2 \end{cases} \tag{2}$$

We can multiply OAM beams with different l values, and embed these different electromagnetic OAM beams on the same frequency, therefore, improve the system capacity and spectrum efficiency.

In this paper, we use Laguerre-Gaussian (LG) beam to produce the OAM beams. The expression for the cylindrical coordinate of LG vortex waves which transmit along Z axis about z distance is:

$$A(r,\phi,z) = R_{l,p}(r,z) \cdot e^{il\phi} \cdot \exp\{-i(2p+|l|+1) \cdot \tan^{-1} \frac{zc}{\pi f w_0^2}\} \tag{3}$$

where

$$R_{l,p}(r,z) = \frac{1}{w(z)} \cdot \left[\frac{2p!}{\pi(|l|+p)!} \right]^{\frac{1}{2}} \times$$
$$\left[\sqrt{2}r \middle/ w(z) \right] \cdot L_p^l \cdot \left(\frac{2r^2}{w^2(z)} \right) \cdot e^{-\frac{r^2}{w^2(z)}} \cdot e^{-\frac{i\pi f r^2}{Rc}} \tag{4}$$

$$w(z) = w_0 \cdot \sqrt{1 + \left(zc \middle/ \pi f w_0^2 \right)^2} \tag{5}$$

$$R = z \left[1 + \left(\pi f w_0^2 \middle/ z \cdot c \right)^2 \right] \tag{6}$$

In formula(3), l is the intrinsic value of OAM, p , the index of radiation, w_0 ,the waist radius, r is the radiation radius, $L_P^{|l|}$,the Laguerre polynomials, Φ, the radiation angle, c, the velocity of wave propagation, f , the frequency of the wave.

III. ATMOSPHERE STRUCTURE CONSTANT C_n^2 OF MICROWAVE BAND

About C_n^2, ITU-R has presented its model, but this model is applicable for the band of light wave. For microwave band, we have to consider some new factors such as atmosphere relative humidity or water vapor content and other factors. In other words, an important factor influencing the microwave band C_n^2 is the humidity in the air, so the microwave band $C_n^2(T,R)$ is a function of temperature and humidity. As the elevation altering, temperature and humidity will change and therefore cause variation of C_n^2.

Based on the light band C_n^2 model presented by ITU-R, microwave band C_n^2 model may be given by the temperature of T, the humidity of U as follows[1]:

$$C_n^2 = 8.148 \times 10^{-56} v_{rms} h^{10} \cdot e^{\frac{-h}{1000}} + 2.7 \times 10^{-16} \cdot e^{\frac{-h}{1500}} \\ + C_0 \cdot e^{\frac{-h}{100}} + 6.4048 \times 10^{-12} \cdot h^{\frac{-11}{6}} N_w^2 \quad (7)$$

$$N_{wet} = \frac{22814 U \exp^{(17.5T/T + 240.97)}}{(T + 273)^2} \quad (8)$$

where, C_0 is $3.9 \times 10^{-12} \text{m}^{-2/3}$, N_{wet} is the humidity in ITU-R model. T and U vary with the elevation of h. According to the experimental results, we gain the expression of T changed with height [1]:

$$T = t_0 - 0.006047 \times h(^\cdot c) \quad (9)$$

where t_0 is the ground temperature and its value is $20.85^{\cdot c}$. The expression of U changed with height is[1]:

$$U = \begin{cases} u_0 - 0.0096h & h \le 5000 \\ 28.8 & 5000 < h \le 10000 \\ 28.8 - 0.00556(h - 10^4) & 10^4 < h < 1.5 \times 10^4 \end{cases} \quad (10)$$

where u_0 is the relative humidity of the ground and its value is 76.8%.

IV. ATMOSPHERIC TURBULENCE GIVEN BY KOLMOGOROV MODEL

Atmospheric refraction wave spectrum reflects the effect on signal caused by the atmospheric refractive. There are various forms of atmospheric refraction wave spectrum. In form of Kolmogorov model, it can be expressed as:

$$\phi_n(k_x, k_y) = 0.033 \times c_n^2 \times e^{\left[-\frac{k_x^2 + k_y^2}{k_l^2}\right]} \times \left[k_x^2 + k_y^2 + \frac{1}{L_0^2}\right]^{-\frac{11}{6}} \\ \times \left[1 + 1.802\sqrt{\frac{k_x^2 + k_y^2}{k_l^2}} - 0.254\left[\frac{k_x^2 + k_y^2}{k_l^2}\right]^{\frac{7}{6}}\right] \quad (11)$$

where k_X^2, k_Y^2 are frequency wave numbers on the X direction and Y direction respectively. L_0^2 is the outer scale of turbulence, l_0^2, the inner scale of turbulence, $k_l = 3.3/l_0$, V_{rms} is the RMS wind speed along the vertical path, its typical values is $21 m/s$.

Phase spectrum can be represented by atmospheric refraction wave spectrum as follows[6][7]:

$$\phi(k_x k_y) = 2\pi(k_x^2 + k_y^2) \cdot \Delta z \cdot \phi_n(k_x, k_y) \quad (12)$$

where Δz is the distance between adjacent phase screen on the wave propagation direction.

V. THE INFLUENCE OF ATMOSPHERIC TURBULENCE ON THE VORTEX WAVE PHASE

Radio waves propagating in the atmospheric space will be influenced by atmospheric turbulence[9], then the signal phase will change. So we can't accurately discern the signal at the receiving end. This paper sets the outer-scale of atmospheric turbulence as 20m, the inner-scale as 0.005m. According to the C_n^2 model of microwave band, we set C_n^2 as $8 \times 10^{-15} \text{m}^{-2/3}$, the waist radius W_0 as 3mm, the electromagnetic wave frequency as 30 GHz, the number of grid points N as 300, the grid distance $\triangle x$ as 0.0005m.We study the vortex waves which OAM intrinsic value is 1 and set the transmission distance as 400m. The simulations results are as follows:
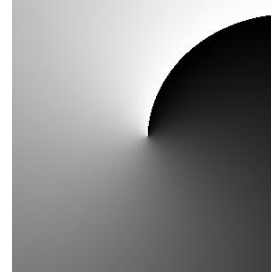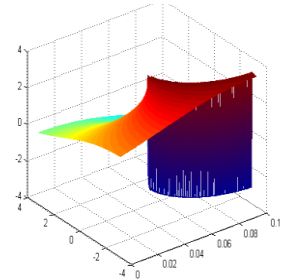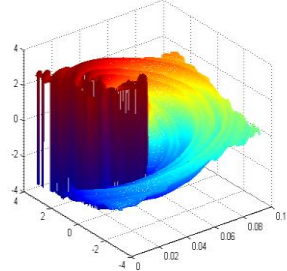

Figure 1(a)

Figure 1(b)

Figure 1(c)

Figure 1(d)

Figure1(a) is original signal phase; Figure1(b)is three dimensional graph of the original signal ; Figure1(c)is the variation of the signal phase which propagate through atmospheric turbulence; Figure1(d) is three dimensional graph of the signal which propagates through atmospheric turbulence

In the phase grayscale, the darker place represents the bigger phase value. By comparing Figure 1(a) and (c), we can clearly see the spiral phase changes obviously with a margin of about 37.5%. In the three dimensional graph, height represents the value of signal phase, we can also see the change of signal phase. Then, we change the value of

C_n^2, when C_n^2 is 8×10^{-13}m$^{-2/3}$, the simulation results are shown in Figure 2 (a) and 2(b).

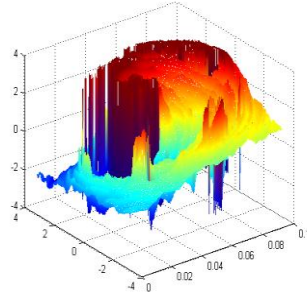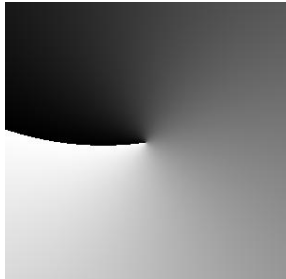Figure 2(a)   Figure 2(b)

Figure2(a) is the variation of the OAM phase when C_n^2 is 8×10^{-13}m$^{-2/3}$;Figure 2(b)is three-dimensional graph of the signal which propagates through turbulence with C_n^2 of 8×10^{-13}m$^{-2/3}$

By comparing Figure 1 (c) with Figure 2 (a), it can be seen that the signal change is more intense. When C_n^2 expands 100 times, phase fluctuation amplitude expand to 50％. From the corresponding three-dimensional graph we can see that the surface is very uneven, so the spiral phase in different spatial orientation is affected differently.

On the basis of Figure 2, we change the transmission distance. We set the transmission distance as 1000m, the simulations results are as follows: Figure 3 (a) is the original signal phase grayscale,(b) is the corresponding three-dimensional graph, (c) is the grayscale of the signal which propagates through atmospheric turbulence,(d) is the corresponding three-dimensional graph.
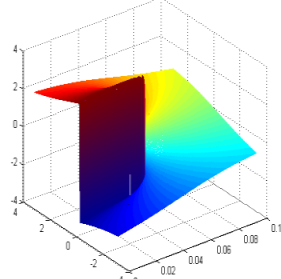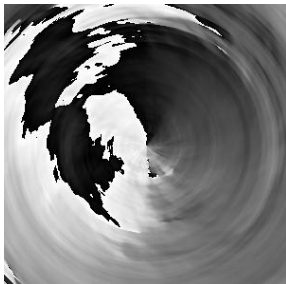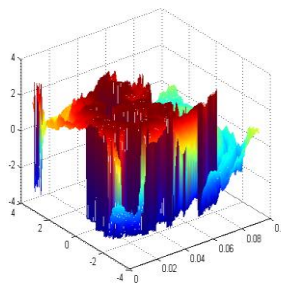

Figure3(a)   Figure3(b)

Figure3(c)   Figure3(d)

Figure3(a)is original signal phase grayscale; Figure3(b)is three dimensional graph of the original signal; Figure3(c)is the variation of the OAM phase when transmission distance is 1000m; Figure3(d)is three dimensional graph of the signal which propagates through a distance of 1000m

By comparing Figure 3(a) with 3(c), we can see the spiral phase change about 62.5%. Compared with the situation that the transmission distance is 400m, the phase fluctuation is more serious. So the change of the transmission distance will cause the phase change.

Now, we alter l values. When l is 4, C_n^2 is 8×10^{-13}m$^{-2/3}$ and transmission distance is 400m, the simulation results are as Figure 4(a) to (d).
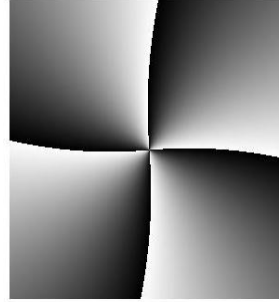

Figure 4(a)   Figure 4(b)
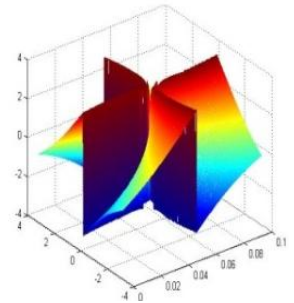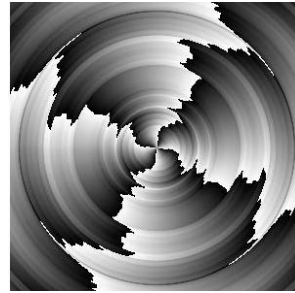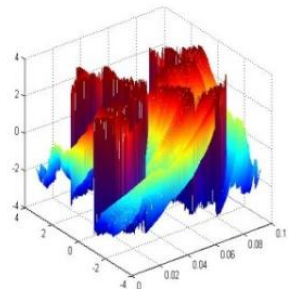

Figure 4(c)   Figure 4(d)

Figure4(a)is original signal phase for l=4; Figure4(b)is three dimensional graph of the original signal for l=4; Figure4(c)is the variation of the OAM phase when C_n^2 is 8×10^{-13}m$^{-2/3}$ for l=4; Figure4(d) is three dimensional graph of the signal which propagates through atmospheric turbulence for l=4.

With the same l values and transmission distance, we change C_n^2, the simulation results are as Figure 5(a) and 5(b).
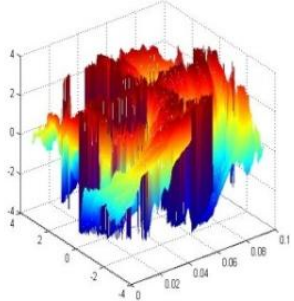

Figure 5(a)   Figure 5(b)

Figure5(a) is the variation of the signal phase which propagate through atmospheric turbulence; Figure5(b) is three dimensional graph of the signal which propagates through turbulence with C_n^2 of 8×10^{-13}m$^{-2/3}$ for l=4

Then on the basis of Figure 5, we change transmission distance, the simulation results are as Figure 6.

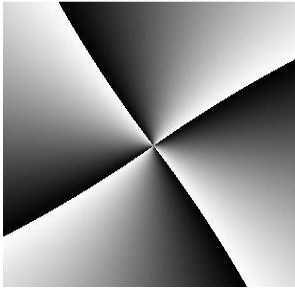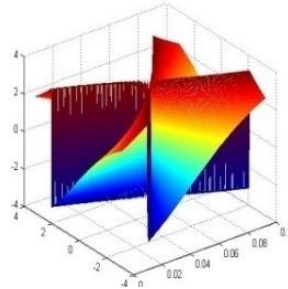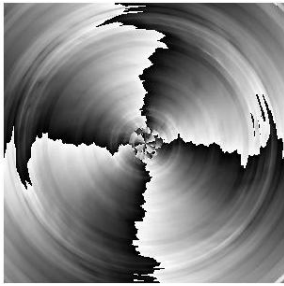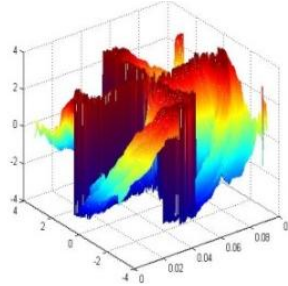Figure 6(a)

Figure 6(b)

Figure 6(c)

Figure 6(d)

Figure6(a) is original signal phase grayscale for *l*=4; Figure6(b) is three dimensional graph of the original signal for l=4; Figure6(c) is the variation of the OAM phase when transmission distance is 1000m for *l*=4; Figure6(d) is three dimensional graph of the signal which propagates through a distance of 1000m for *l*=4;

From the above simulation results, we can draw a conclusion that C_n^2 and transmission distance are factors which will cause the signal phase to change.

VI.   CONCLUSION

In this paper, we study the influence of atmospheric turbulence on the phase of vortex by simulation. We analyze the variations of vortex wave by changing the atmospheric refractive index structure constant and transmission distance. The results show that when atmosphere structure constant C_n^2 increases, the signal change will increases. Similarly, when the transmission distance increases, the fluctuation of vortex phase will increases.   It is important to know that the phase change of vortex wave passing through the atmospheric turbulence for discerning the correct signal at the receiving end. However, this paper focuses on studying the phase of the signal. Therefore, the future research will be comprehensive consideration of more parameters of the signal and how to reduce the influence of atmospheric turbulence on OAM communication system performance.

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