Multiple Powers Allocation Strategy under Sensing/Transmission Frame Structure in Cognitive Radio Networks

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Abstract-In the traditional opportunity spectrum access cognitive radio system(.OSA), the state of the primary user is assumed to be constant during the entire frame duration, the secondary user accesses licensed band with proper power only when primary user is detected idle. In fact, the state of the primary user may changes at any time, on the premise of ensure that the quality of service of primary user and secondary user to make full use of the channel, we proposed a new power allocation strategy based on the four states and three power sunder sensing/transmission frame structure. In this strategy, secondary user is assigned to three different types of access power for the four kinds of states in the process of sensing, considering the effect of activity of the primary user to the system throughput, adopting frame structure that maximize the sensing period and the data transmission period at the same time, to avoid the sensing-throughput tradeoff, aiming to achieve the maximize throughput. The simulation results show that compared to the new power allocation strategy in this paper and conventional power allocation strategy, proposed strategy in this paper has obvious improvement for throughput. In addition, the proposed system throughput is associated with the target detection probability and the primary user's signal-to-noise ratio received at the secondary user.

Keywords-cognitive radio; power allocation; spectrum sensing; spectrum sharing; throughput maximization

I. INTRODUCTION

With the rapid development of wireless communication technology, the requirements of information transmission speed and quality unceasing enhancement, the spectrum scarcity has been significantly amplified out.Cognitive radio technology[1]-[3] is a effective method to improve spectrum utilization, allowing secondary users to access the licensed frequency band under the condition of protecting the quality of service (QOS) of the primary users,greatly improving the spectrum efficiency and system capacity. Now the main spectrum access technology are opportunity spectrum access(OSA)[4]-[5] and spectrum sharing(SS)[6]-[8].

At present, most of the research assume that the state of the primary user not changed in a frame, and to do two powers optimization for secondary user to access licensed band, namely that secondary user uses low power to access when primary user is detected busy; Otherwise using the high power. But the primary user stats may change in a frame, if secondary user uses a high power to access, may be causing that secondary user failed to make full use of licensed band when primary from busy to idle. In [9], discussing the problem of sensing-throughput tradeoff on the promise of fixing high target detection probability, in order to determine the optimal time of sensing to maximize the system throughput. The new frame structure proposed in [10] (named sensing/transmission frame structure in this paper),this frame structure overcome the problem of sensing-throughput tradeoff, the simulation results show that this method compared with the conventional frame structure to increase the system throughput.The influence of activity of primary user to cognitive network is studied in [11]-[12],primary user can random arrive and departure during the frame with the actual environment. Among them, in [12] discussing the influence of multiple primary users of random arrive and departure to throughput. The conventional power allocation strategy is proposed in [13],which based on sensing/transmission frame structure with four states and two powers, achievable throughput have a sharply higher than[9]-[11], but the power allocation strategy in [13] still can be further optimized in regard to the power of secondary user access the licensed band.

In this paper, we proposed a new power allocation strategy which based on the sensing/transmission frame structure with four states and three powers in spectrum sharing cognitive radio networks, to do three powers optimization for secondary user to access the licensed band, and take into account the primary user activity, to maximize the system throughput as the optimization goal. The channel capacity and system achievable throughput of new power allocation strategy are derived in theory, in addition we discuss the influence of multiple primary users of random arrive and departure to throughput. The conventional power allocation strategy is proposed in [13],which based on sensing/transmission frame structure with four states and two powers, throughput have a sharply higher than[9]-[11], but the power allocation strategy in [13] still can be further optimized in regard to the power of secondary user access the licensed band.

II. SYSTEM MODEL AND DERIVATION

We consider the cognitive radio system presented in Fig.1.Let g and h denotes the channel from the secondary transmitter (SU-TX) to the secondary receiver (SU-RX) and the primary receiver (PU-RX), respectively. The channels g and h are assumed to be ergodic, stationary and known at the secondary users. Whereas the noise is assumed to be circularly symmetric complex Gaussian (CSCG) with mean zero and variance $\sigma^2_n$, namely $CN(0,\sigma^2_n)$.
\[ p_b = \frac{\lambda}{\lambda + \mu}, \quad \text{and idle with probability } P_e = 1 - P_b. \]

The transition probability is given by [11]

\[ p(T) = \begin{pmatrix} p_0(T) & p_1(T) \\ p_0(T) & p_1(T) \end{pmatrix} = \begin{pmatrix} \frac{\lambda + \mu}{\lambda + \mu} & \frac{\mu}{\lambda + \mu} \\ \frac{\mu}{\lambda + \mu} & \frac{\lambda}{\lambda + \mu} \end{pmatrix} \]

(2)

It is further assumed that the primary user state transition occurs at most once within each frame. The case of two transitions in one frame is also examined by simulation but is not analyzed. Based on the above assumptions, the quaternary hypothesis testing problem [13] given by (3)

**C. The Four States and Three Power Allocation Strategy Based on Sensing/Transmission Frame Structure**

In the conventional power allocation strategy, primary user is assumed that state remains unchanged in the frame, so when primary user is detected busy, secondary user use a low transmit power; otherwise, secondary user use a high transmit power. In fact, state of primary user may have changed in any time, if secondary user in a high power to access, may be causing a larger interference to primary user when primary user from busy to idle; if secondary user in a low power to access, may be causing that secondary user failed to make full use of licensed band when primary from busy to idle. Giving consideration to the above two cases, we proposed a new power allocation strategy.

\[
\begin{align*}
H_{00} &= \sum_{i=1}^d n_i^2, \\
H_{10} &= \sum_{i=1}^d (x_{pi} + n_i)^2 + \sum_{i=d+1}^j n_i^2, \\
H_{11} &= \sum_{i=1}^d (x_{pi} + n_i)^2, \\
H_{01} &= \sum_{i=1}^d n_i^2 + \sum_{i=d+1}^j (x_{pi} + n_i)^2.
\end{align*}
\]

(3)

In this paper, assuming that primary user state transition occurs at most once within adjacent two frame, and calculate throughput of the latter frame, secondary user access licensed band by the spectrum sharing model, as follow: H00 state: The primary user is busy for d samples and then stay idle during the rest of the frame, in this case, secondary user use a high transmit power p0; H01 state: The primary user is idle for a samples and then stay busy during the rest of the frame, in this case, secondary user use a low transmit power p1; H11 state: this state is similar to state H00, secondary user use a medium transmit power pm. Among them: p1 < pm < p0.

The unconditional probability of false alarm and the unconditional probability of detection can be found in [13] as follow.
\[ P_{H_{i}\epsilon} (\epsilon, J) = \frac{P_{H_{i}} (T) P_{\text{data}} (\epsilon, J)}{P_{H_{i}} (T) + P_{H_{i}} (T)} + \frac{P_{H_{i}} (T) P_{\text{data}} (\epsilon, J)}{P_{H_{i}} (T) + P_{H_{i}} (T)} \]

\[ P_{H_{i}\epsilon} (\epsilon, J) = \frac{P_{H_{i}} (T) P_{\text{data}} (\epsilon, J)}{P_{H_{i}} (T) + P_{H_{i}} (T)} + \frac{P_{H_{i}} (T) P_{\text{data}} (\epsilon, J)}{P_{H_{i}} (T) + P_{H_{i}} (T)} \] (4)

where \( P_{H_{i}} (T) \) is the probability of state \( H_{i} \), \( P_{H_{i}\epsilon} (\epsilon, J) \) and \( P_{H_{i}\epsilon} (\epsilon, J) \) respectively is the conditional probability of state \( H_{i} \) and \( H_{i} \) respectively, \( x, y \in \{0, 1\} \).

Once the spectrum sensing in (3) is completed, the secondary transmission stats according to the sensing information to decide transmit power, as follow:

H00 state: similar to state H00, the channel capacity is affected by the primary user traffic, and the instantaneous channel capacity of primary user remain idle in a later frame system can be derived as

\[ K_{H_{00}} = \log_2 (1 + \frac{g_{s}P_{m}}{N_0}) \].

(5)

The instantaneous channel capacity of primary user from idle to busy in a later frame system can be derived as

\[ K_{H_{i}(a)} = \log_2 (1 + \frac{J - \alpha}{J} \frac{g_{s}P_{m}}{g_{p}P_{s}}) \], \( \alpha \leq J \).

(6)

\[ N_0 \] is additive white Gaussian noise (AWGN), \( g_{s} \) is channel gain of channel, \( g_{p} \) is the channel gain of primary user to secondary user. Therefore, the achievable throughput can be derived as

\[ R_{H_{i0}} = \log_2 (1 + \frac{g_{s}P_{s}}{N_0 + \frac{J - \alpha}{J} g_{p}P_{s}}) \] \( \alpha \leq J \).

(7)

Where \( \alpha_1 = P_{H_{i0}} (T), \alpha_2 = P_{H_{i0}} (T) P_{H_{i0}} (T) K_{H_{i}(a)} \) and \( \alpha_1 \) and \( \alpha_2 \) respectively corresponding to the probability of channel capacity \( K_{H_{i}} \) and \( K_{H_{i}(a)} \). H01 state: according to the correctness of detection of the primary user by secondary user, we divided it into two case. The instantaneous channel capacity of no false alarm occurring can be derived as

\[ K_{H_{i}} = \log_2 (1 + \frac{g_{s}P_{m}}{N_0}) \].

(8)

The instantaneous channel capacity of right detect by secondary user can be derived as

\[ K_{H_{i}} = \log_2 (1 + \frac{g_{s}P_{s}}{N_0 + g_{p}P_{s}}) \].

(9)

Therefore the achievable throughput under this state, as follow

\[ R_{H_{i}} = P_{H_{i0}} (T) \left[ 1 - P_{\text{fa}} (\epsilon, J) \right] K_{H_{i0}} + P_{H_{i0}} (T) P_{\text{fa}} (\epsilon, J) K_{H_{i}}, \]

\[ = \alpha_1 K_{H_{i0}} + \alpha_2 K_{H_{i}(a)} \].

(10)

Where \( \alpha_1 = P_{H_{i0}} (T) \left[ 1 - P_{\text{fa}} (\epsilon, J) \right], \alpha_2 = P_{H_{i0}} (T) P_{\text{fa}} (\epsilon, J), \alpha_1 \) and \( \alpha_2 \) respectively corresponding to the probability of channel capacity \( K_{H_{i0}} \) and \( K_{H_{i}(a)} \). H11 state: similar to state H00, the channel capacity is affected by the primary user traffic, and the instantaneous channel capacity of primary user remain busy in a later frame system can be derived as

\[ K_{H_{i1}} = \log_2 (1 + \frac{g_{s}P_{s}}{N_0 + \frac{J - \alpha}{J} g_{p}P_{s}}) \] \( \alpha \leq J \).

(11)

The instantaneous channel capacity of primary user from busy to idle in a later frame system can be derived as

\[ K_{H_{i1}(a)} = \log_2 (1 + \frac{g_{s}P_{m}}{N_0 + \frac{J - \alpha}{J} g_{p}P_{s}}) \] \( \alpha \leq J \).

(12)

Therefore the achievable throughput can be derived as

\[ R_{H_{i1}} = P_{H_{i1}} (T) K_{H_{i1}} + P_{H_{i1}} (T) P_{H_{i1}} (T) K_{H_{i1}(a)} \] \( \beta_1 \) and \( \beta_2 \) respectively corresponding to the probability of channel capacity \( K_{H_{i1}} \) and \( K_{H_{i1}(a)} \). H10 state: according to whether occurring false alarm for the primary user by secondary user, we divided it into two case. The instantaneous channel capacity of no false alarm occurring can be derived as

\[ K_{H_{i0}} = \log_2 (1 + \frac{g_{s}P_{m}}{N_0}) \].

(13)

The instantaneous channel capacity of false alarm occurring can be derived as

\[ K_{H_{i0}} = \log_2 (1 + \frac{g_{s}P_{s}}{N_0}) \].

(14)

Therefore the achievable throughput can be derived as

\[ R_{H_{i0}} = P_{H_{i0}} (T) \left[ 1 - P_{\text{fa}} (\epsilon, J) \right] K_{H_{i0}} + P_{H_{i0}} (T) P_{\text{fa}} (\epsilon, J) K_{H_{i0}} \] \( \beta_1 \) and \( \beta_2 \) respectively corresponding to the probability of channel capacity

\[ R_{H_{i0}} = P_{H_{i0}} (T) \left[ 1 - P_{\text{fa}} (\epsilon, J) \right] K_{H_{i0}} + P_{H_{i0}} (T) P_{\text{fa}} (\epsilon, J) K_{H_{i0}} \] \( \beta_1 \) and \( \beta_2 \) respectively corresponding to the probability of channel

\[ R_{H_{i0}} = P_{H_{i0}} (T) \left[ 1 - P_{\text{fa}} (\epsilon, J) \right] K_{H_{i0}} + P_{H_{i0}} (T) P_{\text{fa}} (\epsilon, J) K_{H_{i0}} \] \( \beta_1 \) and \( \beta_2 \) respectively corresponding to the probability of channel
capacity $K_{d_H}$ and $K_{H_{m_H}}$. Being able to get achievable throughput of cognitive system using with (5)-(16), as follow

$$R_N = R_{H_{m_H}} + R_{H_{m_H}} + R_{H_{m_H}} + R_{H_{m_H}},$$

(17) can be rewritten as:

$$\max_{\mu, \lambda} R_N = E_{r_{s-p}} \left\{ \alpha_i K_{H_{m_H}} + \alpha_i K_{H_{m_H}} (a)^i + \alpha_i K_{H_{m_H}} \right\} + \beta \left( \alpha_i K_{H_{m_H}} (d)^i + \beta \left( \alpha_i K_{H_{m_H}} + \beta \left( \alpha_i K_{H_{m_H}} \right) \right) \right),$$

(18) Subject to

$$E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_n + \alpha_i p_0 + \alpha_i p_m \right\} \leq \Gamma, \quad \text{(19)}$$

and

$$E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_m g_g + \alpha_i p_0 g_g + \alpha_i p_m g_g \right\} \leq \Gamma, \quad \text{(20)}$$

(19) and (20) respectively represents average transmit and interference power constraint, $p_0 \geq 0, p_1 \geq 0, p_m \geq 0$, gsp is channel gain of channel h.

The Lagrangian with respect the transmit powers $p_0, p_1$ and $p_m$ is given by

$$L(p_0, p_1, p_m, \lambda, \mu) = E_{r_{s-p}} \left\{ \alpha_i K_{H_{m_H}} + \alpha_i K_{H_{m_H}} (a)^i + \alpha_i K_{H_{m_H}} \right\} + \beta \left( \alpha_i K_{H_{m_H}} (d)^i + \beta \left( \alpha_i K_{H_{m_H}} + \beta \left( \alpha_i K_{H_{m_H}} \right) \right) \right) - \lambda \left[ E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_n + \alpha_i p_0 + \alpha_i p_m \right\} - p_n \right] - \mu \left[ E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_m g_g + \alpha_i p_0 g_g + \alpha_i p_m g_g \right\} - \Gamma \right].$$

Whereas the dual function can be obtained by

$$d(\lambda, \mu) = \sup_{p_0, p_1, p_m} L(p_0, p_1, p_m, \lambda, \mu).$$

(22) In order to calculate the dual function $d(\lambda, \mu)$, the supremum of the Lagrangian with respect to the transmit powers $p_0, p_1$ and $p_m$ needs to be obtained. We therefore apply the primal-dual-decomposition method[14], which facilitates the solution of the joint optimization problem by decomposing it into three convex single-variable optimization problems, one for each of the transmit powers $p_0, p_1$ and $p_m$, as follows:

**Subproblem 1:**

$$\max_{\lambda, \mu} \min_{p_n, p_m} e = f_1(p_n) = E_{r_{s-p}} \left\{ \alpha_i \log \left( 1 + \frac{\alpha_i p_0}{N_0} \right) \right\} + \beta_i \log \left( 1 + \frac{\alpha_i p_m}{N_0} \right) - \lambda E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_n \right\} - \mu E_{r_{s-p}} \left\{ \alpha_i p_0 \right\}.$$

(23)

**Subproblem 2:**

$$\max_{\lambda, \mu} \min_{p_n, p_m} e = f_2(p_n) = E_{r_{s-p}} \left\{ \alpha_i \log \left( 1 + \frac{\alpha_i p_n}{N_0} \right) \right\} + \beta_i \log \left( 1 + \frac{\alpha_i p_m}{N_0} \right) - \lambda E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_n \right\} - \mu E_{r_{s-p}} \left\{ \alpha_i p_0 \right\}.$$

(24)

**Subproblem 3:**

$$\max_{\lambda, \mu} \min_{p_n, p_m} e = f_3(p_m) = E_{r_{s-p}} \left\{ \alpha_i \log \left( 1 + \frac{\alpha_i p_m}{N_0} \right) \right\} + \beta_i \log \left( 1 + \frac{\alpha_i p_m}{N_0} \right) - \lambda E_{r_{s-p}} \left\{ \left( \alpha_i + \alpha_i + \beta_i \right) p_n \right\} - \mu E_{r_{s-p}} \left\{ \alpha_i p_0 \right\}.$$

(25)

After forming their Lagrangian functions and applying the Karush-Kuhn-Tucker (KKT) conditions, the optimal powers $p_0, p_1$ and $p_m$ for given $\lambda, \mu$ are given by

$$p_0 = \left( \frac{\lambda + \mu}{2} \right)^{-1}, p_1 = \left( \frac{\lambda + \mu}{2} \right)^{-1}.$$
Algorithm: The four state and three power allocation strategy in the cognitive radio system.

1. Initialize $\lambda$, $\mu$.
2. Repeat:
   - calculate $p_0$, $p_i$ and $p_m$ using (26)–(29k);
   - update $\lambda$, $\mu$ using the ellipsoid method;
3. Until $\lambda$, $\mu$ converge.

### III. NUMERICAL RESULTS AND DISCUSSION

In this section, we present simulation results for the new power allocation strategy and compare it with the conventional power allocation strategy under sensing/transmission frame structure. The frame duration is set to $T=100$ms, the sampling frequency $f_s=6$MHZ, $g_{ss}$, $g_{sp}$ and $g_{ps}$ are exponentially distributed, and $E{g_{ss}} = E{g_{sp}} = E{g_{ps}} = 1$.

In Fig.4, we present the achievable throughput versus the additional channel power gain $g_{sp}$ of SU-TX to PU-TX for the new power allocation strategy and the conventional power allocation strategy in [13]. The target probability of detection is set to $P_d=0.99$, the primary user's signal-to-noise ratio (SNR) received at the secondary users is set to $\gamma_p=10$dB, and the mean channel holding time $\lambda$ and $\mu$ is set to $\lambda=\mu=10$. One can clearly see that the achievable throughput of the new power allocation strategy is higher compared to the conventional power allocation strategy. This is due to the increase in the quality of service of primary users, the new power allocation strategy allows secondary users to access licensed band for sending data when primary users are busy, thus improving the system achievable throughput.

Moreover, it can be seen from Fig.5 that with the increase of $g_{sp}$, the achievable throughput of the new power allocation strategy decreases, which is due to the increase in the channel gain $g_{sp}$, the interference of cognitive users to primary users increases, the primary users can withstand interference is limited, in order to protect the performance of the primary user, appropriate to reduce the sending power of cognitive users, thus caused the fall of system achievable throughput.

In Fig.5, the achievable throughput versus the target detection probability $P_d$ for the new power allocation strategy and the conventional power allocation strategy. $\gamma_p$ is set to $\gamma_p=-10$dB and the mean channel holding time $\lambda$ and $\mu$ is set to $\lambda=\mu=10$. One can clearly see that with the improvement of target detection probability, whether it is a new power allocation strategy or a traditional power allocation strategy, the system achievable throughput will present a downward trend. Moreover, the system of new power allocation strategy can achieve throughput significantly higher than the system of the conventional power allocation strategy. This is due to the increase in the probability of target detection, the new power allocation strategy based on three additional channel power gains $g_{sp}$ of SU-TX to PU-TX for the new power allocation strategy and the conventional power allocation strategy. The target probability of detection for the new power allocation strategy and the conventional power allocation strategy versus the target probability $\gamma_p$ of detection. Therefore, the secondary user is difficult to send the data access licensed channel, so cause the loss of system achievable throughput. Moreover, the system of new power allocation strategy can achieve throughput higher than the system of traditional power allocation strategy.

![Figure 4](image1.png)

**Figure 4.** Achievable throughput of the new power allocation strategy and the conventional power allocation strategy versus the additional channel power gain.

![Figure 5](image2.png)

**Figure 5.** Achievable throughput of the new power allocation strategy and the conventional power allocation strategy versus the target probability of detection.
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\[ B_0 = a' + b' + c' + d' - \alpha_i + \alpha_2 + \beta_1 + \beta_2, \]
\[ B_i = a'b' + a'c' + a'd' + b'c' + b'd' + c'd' - \frac{1}{M} \ln 2 \left[ a_i(b_i' + c_i') \right] + a_i'(a'_i + c'_i + d'_i) + \beta_i(a'_i + b'_i + d'_i) + \beta_i'(a'_i + b'_i + c'_i), \]
\[ B_i = a'b'c'd' - \frac{1}{M} \ln 2 \left[ (a'b'c' + a_i(c'_i + d'_i) + \beta_i(a'_i + b'_i + c'_i + b'_i') \right], \]
\[ \Delta_i = 2B_i^2 - 9B_iB_i + 27B_i^2 + 27B_i^2 - 72B_iB_i, \]
\[ \alpha' = \frac{N_0}{g_{ss}}, \quad b' = \frac{\frac{J - 2\mu - J + \mu_{sp}P_s}{g_{ss}}}{g_{ss}}, \quad c' = \frac{N_0 + \mu_{sp}P_s}{g_{ss}}. \]

IV. CONCLUSIONS

In this paper, we proposed a new power allocation strategy in the spectrum sharing access model, in a more realistic scenario, considering that the primary user arrives or leaves at any time, we distribute three different types of access power for four kinds of states of sensing, aiming to maximize the system throughput. Simulation results show that compared with the conventional power allocation strategy, the new power allocation strategy has the obvious enhancement and improvement in achievable throughput. Moreover the target probability of detection and the primary user's signal-to-noise ratio received at the secondary user have an impact on achievable throughput, but the overall performance of the new power allocation strategy is still superior to the conventional power allocation strategy in [13].

APPENDIX

The parameters in the formula (26) as shown:

\[ A_0 = \frac{\alpha_i + \beta_i}{\lambda(\alpha_i + \beta_i) + \mu(a_i + \mu g_{ss})} \ln 2 - \frac{2N_0 + \mu_{sp}P_s}{g_{ss}}, \]
\[ A_0 = A_i - \frac{g_{ss}}{4} \left[ \frac{N_0(N_0 + \mu_{sp}P_s)}{g_{ss}} - \frac{\alpha_iN_0 + \beta_i(N_0 + \mu_{sp}P_s)}{\lambda(\alpha_i + \beta_i) + \mu a_i + \mu g_{ss}} \right] \ln 2, \]
\[ A_i = \frac{\alpha_i + \beta_i}{\lambda(\alpha_i + \beta_i) + \mu(a_i + \mu g_{ss})} \ln 2 - \frac{2N_0 + \mu_{sp}P_s}{g_{ss}}, \]
\[ A_i = A_i - \frac{g_{ss}}{4} \left[ \frac{N_0(N_0 + \mu_{sp}P_s)}{g_{ss}} - \frac{\alpha_iN_0 + \beta_i(N_0 + \mu_{sp}P_s)}{\lambda(\alpha_i + \beta_i) + \mu a_i + \mu g_{ss}} \right] \ln 2, \]
\[ A_i = \frac{\sqrt{2}A_i}{3\Delta_i + \sqrt{-4\Delta_i^2 + \Delta_i^2}}, \]
\[ \Delta_i = B_i^2 - 3B_iB_i + 12B_i, \]

where

\[ M = \lambda(\alpha_i + \alpha_2 + \beta_1 + \beta_2) + \mu(\alpha_2 + \beta_2) g_{sp}, \]
\[ \alpha' = \frac{N_0}{g_{ss}}, \quad b' = \frac{\frac{J - \mu_{sp}P_s}{g_{ss}}}{g_{ss}}, \quad c' = \frac{N_0 + \mu_{sp}P_s}{g_{ss}}. \]

REFERENCE

