

Power Splitting and Power Allocation for Energy Harvesting Multi-antenna Relay Networks in the Presence of CSI Errors

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Abstract. Adaptive power splitting and power allocation can provide a substantial performance gain in energy-hungry multi-antenna relay wireless networks when perfect channel state information (CSI) is available at relay. In the case when only imperfect CSI is available, the performance may significantly degrade, and robust power-splitting and power-allocation schemes are developed to save this kind of degradation. In this paper, we investigate the energy-constrained multi-antenna relay system, in which one source communicates with one destination (S-D) with the aid of an energy constrained relay. We focus on *power splitting* (PS) protocol to enable simultaneous energy harvesting and information forwarding, and *power allocation* (PA) to support both the battery power and harvested energy. The optimization problem is established under partial CSI conditions. The objective is to maximize the transmission rate of S-D pair by jointly optimizing PS and PA. Through mathematical analysis, the PS and PA are obtained.

Introduction

Facing energy-constrained relay networks [1], simultaneous wireless information and power transfer (SWIPT) has been a popular research spot [2,3,4], in which the energy-hungry relay harvests energy of radio frequency signals from the transmitters and performs information forwarding by using the harvested energy and its own battery energy. Chen et al. in [3] study the distributed power splitting (PS) scheme in relay interference channels. Our previous work [4] considers PS and beamforming vectors optimization in multi-antenna cognitive relay networks.

However, these work focus on SWIPT under perfect CSI conditions. For practical communication systems, the perfect CSI at the transmitter side is often unavailable. Work [5] develops robust power allocation with error analysis for multicarrier amplify-and-forward relay systems. But, it does not consider the SWIPT and multi-antenna functions. In this paper, we investigate robust PS and PA for SWIPT-enabled multi-antenna relay networks with only imperfect CSI available.

Notations: CN means the complex Gaussian distribution, $E[x]$ stands for the expectation of a variable x , and $\mathbb{C}^{M \times N}$ is the space of $M \times N$ -dimensional complex matrices. Operators $(\bullet)^T$, $(\bullet)^H$ and $\|\bullet\|$ denote the transpose, conjugate transpose and Frobenius norm, respectively..

System Model and Problem Formulation

As shown in Fig. 1, this paper considers a multi-antenna relay system, in which one source transmits data to the destination with the aid of an energy-constrained relay. The

transmission is composed of two slots. In the first slot, source transmits to the relay, and the relay harvests energy from received signals through PS. The relay equips with a power splitter through which the received signal could be partitioned into two streams intended for EH and IF respectively. In the second slot, the relay allocates the harvested energy plus its own battery to forward signals to the destination.

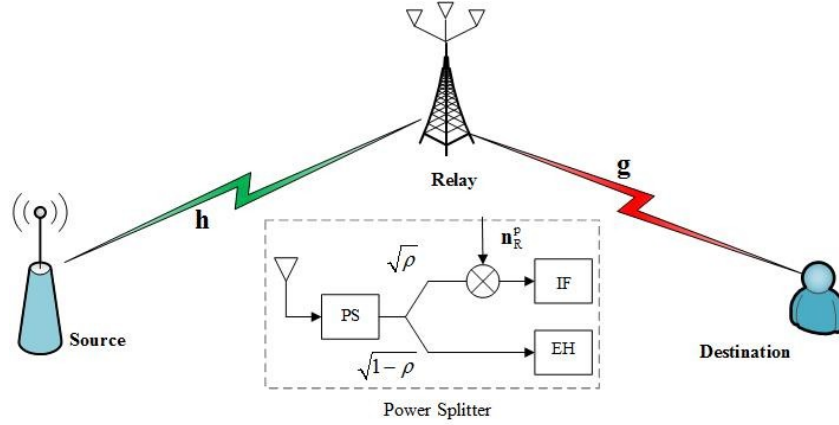


Figure 1. System model of the EH-enabled multi-antenna relay network

It is assumed that the relay is equipped with N antennas. The transmission channel from the source to the relay and from the relay to the destination are denoted as $\mathbf{h} \in \mathbb{C}^{N \times 1}$ and $\mathbf{g} \in \mathbb{C}^{N \times 1}$, respectively. Due to erroneous, outdated, or quantized feedback, the CSI obtained by the transmitter is typically subject to uncertainty. In this paper, we focus on the relay network, and thus assume that \mathbf{h} is perfectly known at the relay while \mathbf{g} is imperfectly known at the relay. That is, only the estimated channel $\hat{\mathbf{g}}$ is known at the relay, the real channel \mathbf{g} can be expressed as the superposition of the estimated channel $\hat{\mathbf{g}}$ and a random estimation error \mathbf{e} , i.e., $\mathbf{g} = \hat{\mathbf{g}} + \mathbf{e}$ with $\|\mathbf{e}\| \leq \varepsilon$.

In the first slot, the source transmits its data to the relay, and the received signal is

$$\mathbf{y}_R = \mathbf{h} \sqrt{p_1} x + \mathbf{n}_R^a. \quad (1)$$

where x , p_1 are the normalized transmitted data and transmit power at the source respectively, and $\mathbf{n}_R^a \sim \text{CN}(\mathbf{0}, s_{R,a}^2 \mathbf{I})$ is the noise vector at the relay from antennas. Then the relay adopts PS to split the received signals and harvest energy. Denote r as the PS factor at the relay. Assume $1-r$ portion of received signal is passed for EH, and the final harvested energy is

$$E_R = x(1-r)(p_1 \|\mathbf{h}\|^2 + s_{R,a}^2) = P_0 - r P_0. \quad (2)$$

where x is the energy conversion efficiency and $P_0 = x(p_1 \|\mathbf{h}\|^2 + s_{R,a}^2)$. The left r portion of signals is reserved for IF, which can be expressed shown as

$$\mathbf{y}_R^{\text{IF}} = \sqrt{r} \mathbf{y}_{R,k} + \mathbf{n}_R^p = \sqrt{r} \mathbf{h} \sqrt{p_1} x + \sqrt{r} \mathbf{n}_R^a + \mathbf{n}_R^p. \quad (3)$$

where $\mathbf{n}_R^p \sim \text{CN}(\mathbf{0}, \sigma_{R,p}^2 \mathbf{I})$ denotes the additional noise vector after PS, mainly incurred from necessary signal processing.

In the second slot, the relay forwards signal \mathbf{y}_R^{IF} with the forwarding matrix $\mathbf{W} \in \mathbb{C}^{N \times N}$. For brevity, the relay employs the match-filtered receiving first and then transmits the

received noisy signal with match-filtered precoding. That is, the forwarding matrix can be expressed as $\mathbf{W} = \mathbf{g} \times \mathbf{h}^H$ [6]. Meanwhile, since only the estimated channel vector $\hat{\mathbf{g}}$ is known at the relay, the forwarding matrix should be rewritten as

$$\mathbf{W} = \sqrt{p_2} \frac{\hat{\mathbf{g}}}{\|\hat{\mathbf{g}}\|} \times \frac{\mathbf{h}^H}{\sqrt{\beta} \|\mathbf{h}\|}. \quad (4)$$

where $\beta = \rho p_1 \|\mathbf{h}\|^2 + \rho \sigma_{R,a}^2 + \sigma_{R,p}^2$ is the power normalized factor, and p_2 is the really transmit power at the relay.

Therefore, the transmit signal is

$$\mathbf{s} = \sqrt{p_2} \frac{\hat{\mathbf{g}}}{\|\hat{\mathbf{g}}\|} \times \frac{\mathbf{h}^H}{\sqrt{\beta} \|\mathbf{h}\|} \mathbf{y}^{\text{IF}} = \sqrt{p_2} \frac{\hat{\mathbf{g}}}{\|\hat{\mathbf{g}}\|} \times \frac{1}{\sqrt{\beta}} (\|\mathbf{h}\| \sqrt{\rho p_1} x + \sqrt{\rho} \mathbf{n}_R^a + \mathbf{n}_R^p) \quad (5)$$

Accordingly, the received signal at the destination is

$$y_D = \mathbf{g}^H \mathbf{s} + n_D = \sqrt{p_2} \frac{\mathbf{g}^H \hat{\mathbf{g}}}{\|\hat{\mathbf{g}}\|} \frac{1}{\sqrt{\beta}} (\|\mathbf{h}\| \sqrt{\rho p_1} x + \sqrt{\rho} \mathbf{n}_R^a + \mathbf{n}_R^p) + n_D. \quad (6)$$

where $n_D \sim \text{CN}(0, \sigma_D^2)$. The corresponding receiving rate is $R = 0.5 \log_2(1 + \gamma)$ with

$$\gamma = \frac{p_2 |\mathbf{g}^H \hat{\mathbf{g}}|^2 \times \rho p_1 \|\mathbf{h}\|^2}{p_2 |\mathbf{g}^H \hat{\mathbf{g}}|^2 \times (\rho \sigma_{R,a}^2 + \sigma_{R,p}^2) + \beta \|\hat{\mathbf{g}}\|^2 \sigma_D^2}. \quad (7)$$

which can be further written as

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}, \quad \gamma_1 = \frac{\rho p_1 \|\mathbf{h}\|^2}{\rho \sigma_{R,a}^2 + \sigma_{R,p}^2}, \quad \gamma_2 = \frac{p_2 |\mathbf{g}^H \hat{\mathbf{g}}|^2}{\|\hat{\mathbf{g}}\|^2 \sigma_D^2} = \frac{p_2 |(\hat{\mathbf{g}} + \mathbf{e})^H \hat{\mathbf{g}}|^2}{\|\hat{\mathbf{g}}\|^2 \sigma_D^2}. \quad (8)$$

where γ_1, γ_2 can be deemed as the transmission SNR from the source to the relay and the transmission SNR from the relay to the destination respectively. Note that since \mathbf{e} is a random estimation error, γ_2 (and accordingly γ) is also not a determined value, but a random value.

Our objective is to maximize the worst-case achieving rate at the destination by jointly optimizing the power splitting factor r and the power allocation p_2 subject to the energy neutral constraint at the relay. Hence the problem can be formulated as

$$\begin{aligned} \text{OP}_1 \quad & \max_{\rho, p_2} \min_{\|\mathbf{e}\| \leq \varepsilon} \gamma \\ \text{s.t.} \quad & p_2 \leq P_R + E_R \end{aligned} \quad (9)$$

where P_R is the available battery power, and E_R is harvest energy as shown in (2).

Algorithm Design

According to (8), it is easy to see that minimize γ for $\|\mathbf{e}\| \leq \varepsilon$ is equivalent to minimize γ_2 . Furthermore, γ_2 is minimized when $\mathbf{e} = -\varepsilon \hat{\mathbf{g}} / \|\hat{\mathbf{g}}\|$, and the resultant γ_2 can be written as $\gamma_2 = \alpha p_2 \|\hat{\mathbf{g}}\|^2 / \sigma_D^2$ with $\alpha = (1 - \varepsilon)^2 / \|\hat{\mathbf{g}}\|^2$. As a result, problem OP₁ can be simplified as

$$\begin{aligned} \text{OP}_2 \quad & \max_{\rho, p_2} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \\ \text{s.t.} \quad & p_2 + \rho P_0 \leq P_R + P_0 \\ & \gamma_1 = \frac{\rho p_1 \|\mathbf{h}\|^2}{\rho \sigma_{R,a}^2 + \sigma_{R,p}^2}, \gamma_2 = \frac{\alpha p_2 \|\hat{\mathbf{g}}\|^2}{\sigma_D^2} \end{aligned} \quad (10)$$

Substitute $p_2 = P_R + P_0 - P_0 \rho$ and the objective function can be simply expressed as a function about ρ , i.e.,

$$f(\rho) = \frac{\rho(P_R + P_0 - P_0 \rho)}{\rho(1 + a_1)c_1 + b_1c_1 + (P_R + P_0 - P_0 \rho)}. \quad (11)$$

with $a_1 = \sigma_{R,a}^2 / p_1 \|\mathbf{h}\|^2$, $b_1 = \sigma_{R,p}^2 / p_1 \|\mathbf{h}\|^2$, $c_1 = \sigma_D^2 / \alpha \|\hat{\mathbf{g}}\|^2$. Then by taking the first-order derivative of $f(\rho)$ and setting $f'(\rho) = 0$, we can easily obtain the optimal ρ^* as well as the resultant optimal p_2^* .

Conclusion

The paper briefly analyzes the power splitting and power allocation strategy for a multi-antenna relay system with imperfect channel state information at the relay, which is meaningful to the practical application of the integration of RF energy harvesting and information relaying.

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