An Iterative Algorithm for Linear-Phase Paraunitary FIR Filter Banks with Impulse Responses of Different Lengths

Li CHEN [a] and Hai-tao DENG [b]

Department of Electronic and Information Engineering, Shantou University,

Shantou 515063, Guangdong, P. R. China

[a]lchen@stu.edu.cn; [b]14htdeng@stu.edu.cn

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Abstract. In this paper, a new design algorithm is presented for paraunitary filter banks with extended filter length. Equal-length filter banks are designed where the filter length is not restricted to integral multiples of the number of channels. Filter banks with unequal length are also studied. In the new algorithm, the filter lengths are iteratively reduced through a cascade of lattice structures and delay chains. Design examples of 5 channels with different filter lengths and stop band attenuations are presented.

Introduction

In recent years, there have been increasing interests in the design of multirate subband filter banks [1]-[14]. In [2], biorthogonal filter banks were designed for image compressing by using genetic algorithm. A computationally efficient digital FIR filter bank with adjustable subband distribution is proposed in [3][4][5]. In [6], an eigenfilter-based method was proposed for the design of 4-band perfect reconstruction (PR) filter banks with linear phase and orthogonality. In [7], a class of 2-channel quadrature mirror filters were optimized. Paraunitary filter banks have been designed in the past and the filters are mainly of equal lengths and the filter lengths are restricted to integral multiples of the number of channels[8]. Design of filter banks with sharp transition band have been reported in [9][10]. Lattice structures of biorthogonal wavelet basis were optimized in image compression using the SPIHT algorithm in [11]. In [12], a low delay oversampled critical-band division filter bank was designed and applied to a two-state modeling speech enhancement system. The multi-channel filter banks are to be applied to wireless sensor networks for data acquisition [14]. In this paper, a new design algorithm is presented for paraunitary filter banks with generalized filter length. The algorithm for linear-phase filter banks is extended to general paraunitary filter banks. Equal-length filter banks are designed where the filter length is not restricted to integral multiples of the number (M) of channels. Filter banks with unequal length are also designed.

Filters with Equal Length

Fig. 1 is referred to as a paraunitary filter bank if $E^T(z^{-1})E(z)=I$ where $E(z)$ is the polyphase matrix of the analysis filters $H_i(z)$, $0 \le i \le M-1$. Paraunitary filter banks can be treated as an extension of non-overlapped transforms to lapped transforms [1]. A filter bank with the paraunitary property has several advantages: The polyphase matrix satisfies $E^{-1}(z)=E^T(z^{-1})$

and the synthesis filters $F_i(z)$, $0 \le i \le M-1$ can be obtained from $E(z)$ without matrix inversion. The polyphase matrix $E(z)$ of an FIR subband filter bank can be expressed as

$$E_K(z) = e(0) + e(1)z^{-1} + \cdots + e(K-1)z^{-(K-1)}, \tag{1}$$

where $E_K(z)$ can be factored as

$$E_K(z) = Q_K E'_K(z), \tag{2}$$

$$E'_K(z) = e'(0) + e'(1)z^{-1} + \cdots + e'(K-1)z^{-(K-1)}, \tag{3}$$

$$e'_l(0) = \begin{array}{c} 0 \\ \vdots \\ \sigma-1 \\ \sigma \\ \vdots \\ M-1 \end{array} \begin{pmatrix} * & * & \cdots & * \\ \vdots & \vdots & & \vdots \\ * & * & \cdots & * \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}. \tag{4}$$

Filters with Unequal Lengths

The polyphase matrix $E(z)$ of a filter bank with length $N_i = k_i M$ can also be expressed as in (1). Suppose among the M filters, there are ρ_K filters that have the longest length KM. Let $e(0)$ and $e(K-1)$ in (1) be partitioned as

$$e(0) = \begin{pmatrix} e_l(0) \\ e_s(0) \end{pmatrix}, e(K-1) = \begin{pmatrix} e_l(K-1) \\ 0 \end{pmatrix}, \tag{5}$$

where the subscripts l and s specify the polyphase components of the ρ_K filters with the longest length KM and the remaining $M-\rho_K$ short filters, respectively. The sub-matrix on the lower part of $e(K-1)$ is set to 0, as the filters have length shorter than the longest length KM .The paraunitary property $\tilde{E}(z)E(z) = I$ in time-domain shows that $e^T(K-1)e(0) = 0_M$. Equations (3)(4)(5) yield $e_l^T(K-1)e_l(0) = 0_{\rho_K}$, where ρ_K is the number of filters that have the longest length KM.

The above factorization process is applicable for filter banks with length $k_i M + \beta$. The difference between filter banks with length $k_i M + \beta$ and filter banks with length $k_i M$ lies in the initial matrix E_1. The polyphase matrix $E(z)$ can be expressed as

$$E(z) = e(0) + \cdots + e(K-1)z^{-K-1} + e(K)z^{-K}. \tag{6}$$

Because of the parameter β in $k_i M + \beta$, the highest order in the polyphase representation is K instead of $K-1$ in (1). Suppose that ρ_K filters have the longest length $KM + \beta$ and the remain $M-\rho_K$ filters have length shorter than $KM + \beta$. Let $e(0)$ and $e(K)$ be partitioned as

$$e(0) = \begin{pmatrix} e_{l,l}(0) & e_{l,r}(0) \\ e_{s,l}(0) & e_{s,r}(0) \end{pmatrix}, \tag{7}$$

$$e(K) = \begin{pmatrix} e_{l,l}(K) & 0 \\ 0 & 0 \end{pmatrix}, \tag{8}$$

where the left hand part consists of β columns; and the right hand part consists of the remaining $M-\beta$ columns. The right subscripts l and r represent the left and right hand part, respectively. The two sub-matrices at the bottom of $e(K)$ are set to 0, as the corresponding filters are shorter than $KM+\beta$. Since the maximum length of filters is $KM+\beta$, the two sub-matrices on the right hand side of $e(K)$ are set to zero matrices.

The paraunitary property $\tilde{E}(z)E(z) = I$ that $rank(e_l(0)) \leq \rho_K$, $K \geq 1$, i.e., the sub-matrix $e_l(0)$ formed by the polyphase components of the ρ_K filters with the longest length $KM+\beta$ are interdependent. The independent number of rows is less than ρ_K.

Given a linear-phase paraunitary filter bank, the polyphase matrix $E_K(z)$ can be gradually factored as the form $E_K(z) = Q_K\lambda(z^{-1})E_{K-1}(z)$ and the initial matrix E_1 (E_0 when β is 0). Conversely, recursively cascading the lattice structures $E_K(z) = Q_K\lambda(z^{-1})E_{K-1}(z)$ to the initial matrix E_1 yields the class of filter banks with lengths $N_i = k_iM+\beta$, $0 \leq k_i \leq K$, $0 \leq \beta \leq M-1$ (After each stage of the filter length changes $E_K(z) = Q_K^T\lambda(z^{-1})E_{K-1}(z)$, the polyphase matrix needs to be rearranged through row-wise permutations). A design example of 5-channel filter bank is shown in Fig. 2.

Conclusion

In this paper, an algorithm has been presented for designing a class of paraunitary filter banks with extended filter lengths. New properties in this class of filter banks were studied and the lattice factorizations were achieved through a succession of "length reduction" operations. The lattice structures and design examples were presented.

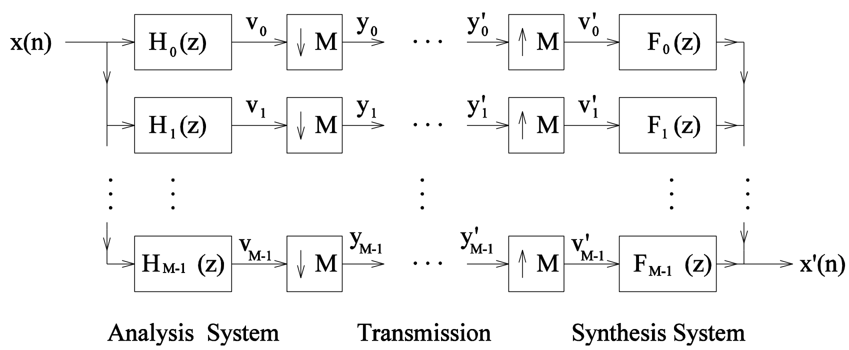

Figure 1. M-Channel Analysis/Synthesis Systems

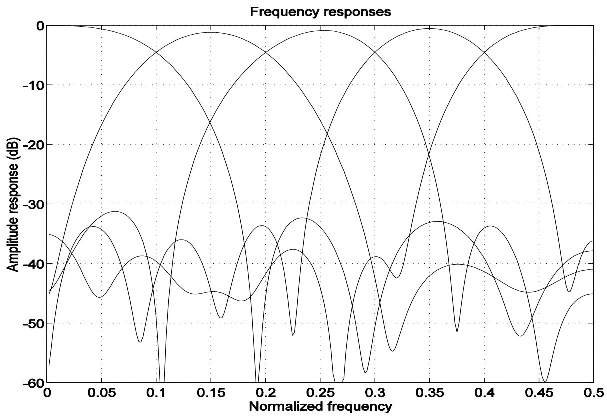

Figure 2. A 5-channel filter bank with filter lengths (18, 18, 18, 18, 18)

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