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DOA Estimation Using Log Penalty under Large Arrays

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Abstract. This paper proposes a new direction-of-arrival (DOA) estimation algorithm, which is suitable for the scenario that the number of sensors is large, and is comparable with the number of samples in magnitude. Instead of utilizing classical subspace technique, sparse-recovery-based approach with log penalty is exploited. In detailed implementation, we use DC (Difference of Convex function) decomposition to solve the non-convex optimization problem, and weighted L_1-norm penalty to provide the initial estimation, where the weights are constructed via the orthogonality between the noise subspace and signal subspace in large-scale random matrix theory framework. As a result, an improved DOA estimation performance is achieved. Simulation results validate the effectiveness of the proposed algorithm.

Introduction

Direction-of-arrival (DOA) estimation is an important issue in array signal processing fields, such as mobile communication systems, radar, sonar and acoustic tracking [1-2]. Existing DOA estimation algorithms mostly assume that the sensor number is fixed, while the snapshot number tends to infinity. In constrast, the sensor number is large, but the snapshot number is limited in actual detecting systems. In this situation, the consistency of classical statistical theory based DOA estimation algorithms, such as Capon [3], MUSIC [4] and ESPRIT [5], is no longer satisfied, and the robustness is hard to be guaranteed.

In order to provide improved DOA estimation performance under the scenario that both the number of sensors and the number of samples are large, and are comparable in magnitude. Xavier *et al* [6] first obtain some asymptotic results about MUSIC spatial spectrum function based on random matrix theory, and then apply them for DOA estimation. This algorithm (termed as G-MUSIC) outperforms the classical MUSIC algorithm in large array and finite sample-size situation.

Recently, a different framework, namely sparse signal recovery, has been introduced in signal processing field, and many algorithms based on this framework including FOCUSS [7], L_1-SVD [8], NSW-L_1 [9] and JLZA [10] have been presented for DOA estimation. These kinds of algorithms show salient advantages in comparison with subspace based algorithm, such as high resolution and suitable for low samples number.

By fully using sparse recovery and the asymptotic results obtained via random matrix theory, this paper proposes another DOA estimation algorithm under large arrays. Instead of utilizing general L_1-norm penalty, a more appropriate log penalty [11] is exploited. Through jointly using DC decomposition [12] and the orthogonality between the noise subspace and signal subspace, the non-convex optimization problem is solved

efficiently, and a good DOA estimation is also guaranteed. Finally, we verify the effectiveness of the proposed algorithm via several numerical simulations.

Problem Formulation

Assume K narrowband, uncorrelated source signals from distinct angles impinging on a uniform linear array (ULA) with M sensors. After sampling with a proper rate, the array output $\mathbf{y}$ can be expressed as

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad t = 1, 2, \ldots, N \tag{1}$$

where $\mathbf{A}=[\mathbf{a}(\theta_1),\ldots, \mathbf{a}(\theta_K)]$ denotes $M\times K$ array steering matrix, whose kth column is the steering vector of k source, given by

$$\mathbf{a}(\theta_k) = [1, e^{-j2\pi d\sin(\theta_k)/\lambda}, \ldots, e^{-j2\pi(M-1)d\sin(\theta_k)/\lambda}]^T \tag{2}$$

λ and d represent the carrier wavelength and the inter-sensor spacing, respectively. To avoid phase ambiguity problem, typically assuing $d\leq\lambda/2$. $\mathbf{s}(t)=[s_1(t), \ldots, s_K(t)]^T$, $\mathbf{n}(t)$ is the zero mean Gaussian white noise embedded in the array sensors. The superscript T denotes the transpose operation.

Based on Eq. 1, we can also obtain the array covariance matrix

$$\mathbf{R} = E\{\mathbf{y}(t)\mathbf{y}^H(t)\} = \mathbf{ASA} + \sigma^2\mathbf{I} \tag{3}$$

where $\mathbf{S}=[P,\ldots, P_K]$ denotes the signal covariance matrix, $\mathbf{I}$ is an $M\times M$ identity matrix. $E\{\cdot\}$ and H denote the expectation and the conjugate transpose, respectively.

Implementing the eigenvalue decomposition (EVD) on $\mathbf{R}$, we have

$$\mathbf{R} = [\mathbf{E}_s\ \mathbf{E}_n]\begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I}_{M-K} \end{bmatrix}[\mathbf{E}_s\ \mathbf{E}_n]^H \tag{4}$$

where $\mathbf{E}_s=[\mathbf{e}_1,\ldots,\mathbf{e}_K]$ and $\mathbf{E}_n=[\mathbf{e}_{K+1},\ldots,\mathbf{e}_M]$ denote the signal subspace matrix and noise subspace matrix, respectively. $\mathbf{\Lambda}_s$ is a $K\times K$ diagonal matrix containing the largest eigenvalues of $\mathbf{R}$. If the number of samples increases without bound ($N\rightarrow\infty$), whereas the number of sensors is set to be a fixed quantity ($M<\infty$), then the DOA can be estimated accurately by the following object function

$$f(\theta) = \min_{\theta} \mathbf{a}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{a}(\theta) \tag{5}$$

However, the number of samples is finite and its magnitude is comparable to the number of sensors in practical applications. In this situation, the consistency of Eq. (5) cannot be guaranteed according to the large-scale random matrix theory [13].

Proposed DOA Estimation Algorithm

Unlike subspace technique, the sparse-recovery-based approach is better suited for DOA estimation under small number of samples. We first divide the spatial domain to $G(\gg K)$ sampling grid and form the overcomplete basis matrix $\mathbf{\Phi}=[\bar{\mathbf{a}}(\theta_1),\ldots,\bar{\mathbf{a}}(\theta_G)]$. Consequently, Eq. (1) can be rewritten as

$$\mathbf{y}(t) = \mathbf{\Phi}\mathbf{x}(t) + \mathbf{n}(t), \quad t = 1, 2, \ldots, N \tag{6}$$

where $\mathbf{x}(t)$ is a K-sparse vector, whose ith element is nonzero and equal to $s_k(t)$ if source k comes from θ_k and zero otherwise. The matrix form of (6) is given by

$$\mathbf{Y} = \mathbf{\Phi X} + \mathbf{N} \tag{7}$$

In order to reduce the computational complexity, we take the singular value decomposition (SVD) on **Y**, which yields

$$\mathbf{Y}_{\mathrm{SV}} = \mathbf{\Phi X}_{\mathrm{SV}} + \mathbf{N}_{\mathrm{SV}} \tag{8}$$

where $\mathbf{Y} = \mathbf{ULV}^H$, $\mathbf{Y}_{\mathrm{SV}} = \mathbf{YVW}_K$, $\mathbf{N}_{\mathrm{SV}} = \mathbf{NVW}_K$, and $\mathbf{W}_K = [\mathbf{I}_K, \mathbf{0}]^T$, $\mathbf{I}_K$ and $\mathbf{0}$ denote $K\times K$ identify matrix and $K\times(M\text{-}K)$ zero matrix, respectively.

Furthermore, the DOAs can be achieved by solving the following L_0-norm optimization problem

$$\min \left\| \mathbf{x}^{(L_2)} \right\|_0 \quad s.\ t.\ \left\| \mathbf{Y}_{\mathrm{SV}} - \mathbf{\Phi X}_{\mathrm{SV}} \right\|_F^2 \le \eta \tag{9}$$

where $\mathbf{x}^{(L_2)} = [x_1^{(L_2)}, \mathrm{K}, x_G^{(L_2)}]^T$, and $x_i^{(L_2)}$ denote the L_2-norm of ith row of $\mathbf{X}_{\mathrm{SV}}$. η is a penalized parameter that controls the tradeoff between F-norm term and L_0-norm term.

It is well known that the L_0-norm penalty optimization problem is a NP-hard problem. Alternatively, we exploit log penalty to enforce sparsity, whose function is defined as

$$g(x) = \log\left(|x| + \varepsilon\right) - \log\left(\varepsilon\right) \tag{10}$$

where $\varepsilon > 0$ is a tuning parameter controlling the degree of approximation. The log penalty is continuous and very closely approximates the L_0 penalty, thus it can be predicted that the corresponding object function will lead to a good estimation result.

By using log penalty, the approximation of formulation (9) can be expressed as

$$\min \sum_{i=1}^{G} g\left(x_i^{(L_2)}\right) \quad s.\ t.\ \left\| \mathbf{Y}_{\mathrm{SV}} - \mathbf{\Phi X}_{\mathrm{SV}} \right\|_F^2 \le \eta \tag{11}$$

Note that formulation (11) is also nonconvex. To deal with this issue, we exploit DC decomposition strategy, whose key concept is to decompose a nonconvex function to two lower semi-continuous, proper convex functions. Then the optimization problem can be cast as the following form

$$\min \sum_{i=1}^{G} x_i^{(L_2)} - \sum_{i=1}^{G} \beta_i x_i^{(L_2)} \quad s.\ t.\ \left\| \mathbf{Y}_{\mathrm{SV}} - \mathbf{\Phi X}_{\mathrm{SV}} \right\|_F^2 \le \eta \tag{12}$$

where $\beta_i = h'(x_i^{(L_2)})$, $h\left(x_i^{(L_2)}\right) = x_i^{(L_2)} - \log(x_i^{(L_2)} + \varepsilon) - \log(\varepsilon)$.

Based on iterative procedure and replacing $h\left(x_i^{(L_2)}\right)$ by its minorization, we obtain

$$\beta_i = h'(x_i^{(L_2)}) = 1 - 1\Big/\left|x_i^{(L_2)} + \varepsilon\right| \tag{13}$$

Subsequently, the optimization problem for DOA estimation is formulated as

$$\min \sum_{i=1}^{G} \alpha_i x_i^{(L_2)} \quad s.\ t.\ \left\| \mathbf{Y}_{\mathrm{SV}} - \mathbf{\Phi X}_{\mathrm{SV}} \right\|_F^2 \le \eta \tag{14}$$

where $\alpha_i = 1/|x_i^{(L_2)} + \varepsilon|$. After the above process, the L_0-norm optimization problem reduces to a weighted L_1-norm problem, which is convex and can be solved easily via second-order cone programming (SOCP).

According to the DC decomposition theory, a good initial estimate should be provided to ensure the final DOA estimation accuracy, thus the weighted L_1-norm penalty using the orthogonality between the noise subspace and signal subspace is adopted. Since the number of sensors is large and comparable with the number of samples in magnitude, we present some theoretical results first in random matrix theory framework, which are shown as follows:

Thoerem1 [6]: Under the assumptions that the entries of noise are i.i.d process with zero mean, variance 1/2 and a finite moment of order higher than 8, as well as the correlation matrix **R** has uniformly bounded spectral radius for all *M*, the quantity $\mu^{(M,N)}(\theta)$ and the noise power σ^2 are consistently estimated by

$$\hat{\mu}^{(M,N)}(\theta) = \mathbf{a}^H(\theta)\left(\sum_{m=1}^{M}\phi(m)\hat{\mathbf{e}}_m\hat{\mathbf{e}}_m^H\right)\mathbf{a}(\theta) \tag{15}$$

$$\hat{\sigma}^2 = \frac{N}{M-K}\sum_{k=1}^{M-K}\left(\hat{\lambda}_k - \hat{\mu}_k\right) \tag{16}$$

with

$$\phi(m) = \begin{cases} 1+\sum_{k=M-K+1}^{M}\left(\dfrac{\hat{\lambda}_k}{\hat{\lambda}_m-\hat{\lambda}_k} - \dfrac{\hat{v}_k}{\hat{v}_m-\hat{v}_k}\right), & m \le M-K \\ -\sum_{k=1}^{M-K}\left(\dfrac{\hat{\lambda}_k}{\hat{\lambda}_m-\hat{\lambda}_k} - \dfrac{\hat{v}_k}{\hat{v}_m-\hat{v}_k}\right), & m > M-K \end{cases} \tag{17}$$

where $\hat{\lambda}_k$ is the *k*th eigenvalue of **R**, and $\hat{v}_1 \le \hat{v}_2 \le \mathrm{L} \le \hat{v}_M$ is the real-valued solutions to the following equation in $\hat{v}$

$$\sum_{k=1}^{M}\frac{\hat{\lambda}_k}{\hat{\lambda}_k - \hat{v}} = \frac{1}{N} \tag{18}$$

repeated according to the multiplicity of the *k*th sample eigenvalue.

Furthermore, the initial weighted matrix is constructed by

$$\mathbf{W}_{in} = diag\left(\hat{\mu}^{(M,N)}(\theta)\right) \tag{19}$$

and the initial DOA is provided by the following optimization problem

$$\min\left\|\mathbf{W}_{in}\mathbf{x}^{(L_2)}\right\|_1 \quad s.\ t.\ \left\|\mathbf{Y}_{\mathrm{SV}} - \mathbf{\Phi}\mathbf{X}_{\mathrm{SV}}\right\|_F^2 \le \eta \tag{20}$$

The penalized parameter is selected via discrepancy principle as shown in [8], except that the noise power is estimated by Eq. (16).

Let $\hat{\mathbf{x}}$ denote the final estimation result of formulation (14), then the DOAs are obtained by finding the indexes of *K* larger coefficients.

Simulations

In this section, we validate the effectiveness of the proposed algorithm via numerical simulations. MUSIC [4], G-MUSIC [6] and Cramé–Rao lower bound (CRLB) are selected as the compared algorithms. Two closely spaced sources from DOAs of 35°

and 37° are considered. The number of sensors is fixed at 20. The root mean square error (RMSE) of DOA estimations is obtained by 200 independent Monte Carlo trails.

In the first experiment, the number of samples is set to be 50, and the SNR varies from 0dB to 12 dB in steps of 2dB. The simulation result is shown in Figure 1, from which we can easily observe that the proposed algorithm outperforms the MUSIC and G-MUSIC algorithms in the whole SNR region, and follows CRLB well.

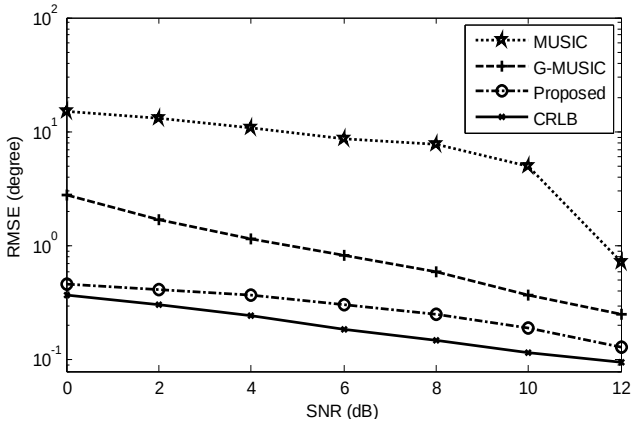

Figure 1. RMSE of DOA estimation versus SNR with 50 samples

In the second experiment, we evaluate the performance of DOA estimations against the number of samples. Unlike the first experiment, we fixed the SNR to be 10dB, and vary the number of samples from 30 to 70 in steps of 10. The simulation result is shown in Figure 2. The conclusion is similar with the first experiment, the proposed algorithm outperforms the compared algorithms, and follows CRLB well. These two simulations fully verify the superiority of the proposed algorithm.

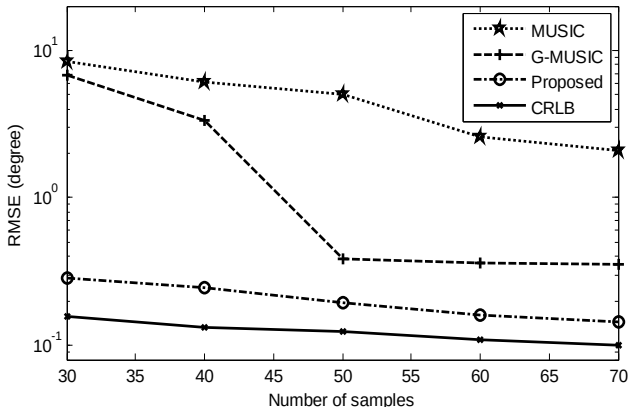

Figure 2. RMSE of DOA estimation versus the number of samples with 10dB SNR

Conclusions

This paper introduces a new algorithm for DOA estimation under large arrays. First, we exploit log penalty function to approximate the L_0 function, and successively utilize DC decomposition to solve the nonconvex optimization problem. Next, we make use of the asymptotic results obtained in large-scale random matrix framework, to construct the initial weighted matrix and further achieve a good DOA estimation. Simulations demonstrate that the proposed algorithm performs better than the compared algorithms in the small-sample-size regime, where the number of sensors and the number of samples are comparable in magnitude.

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