Adding-weight One- rank Local Predication Model of Busy Traffic based on Correlation

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Abstract. In order to improve accuracy of adding-weight one-rank local prediction, the paper proposed the correlation function to measure the correlation between different phase points, which is used to determine reference neighborhood of prediction center point, and made the values of expressing correlation to act on model by means of adding-weight, derived the identification algorithm of model parameters. Applied prediction model to prediction of busy traffic; the results show that the model effectively improves prediction accuracy of busy traffic, verifying the relevant function to measure the effectiveness of the phase correlation between different phase points.

Introduction

Forecasting of busy traffic, can undertake network security for network operation and provide decision-making support, it is an important basis for mobile operators to design network. Whether predicting result is accurate or not, it will affect communication quality. Busy traffic is a non-linear time series affected by multiple factors. The traditional forecast is season, ARMA model and regression, etc., but they mainly for smooth traffic forecast. It is difficult to satisfy the prediction of busy traffic. Some scholars put forward BP neural network [1], support vector machine [2], and echo state network [3], etc. these methods can deal with the non-linear relationship between traffic and its impact factors; they have more advantages than the traditional prediction.

Chaos is a new form of non-linear systems, its prediction has global and local [4], [5] using adding-weight one-rank local model to forecast short-term load, but model base on Euclidean distance to measure correlation between phase points, when embedding dimension is higher, Euclidean distance only reflect the distance of phase space, it can't fully reflect the correlation [6,7] of phase. Some scholars put forward to describe correlation of phase trajectory similar degree instead of Euclidean distance to measure correlation, which improves precision of short-term load. However the research of [8] and [9] shows that measured by phase Angle between the phase trajectories of similarity degree has better prediction effect than the correlation.

In fact, correlation between phase points, not only shorten space distance between phase points but also hope that the angle between phase trajectories of evolution direction is small to ensure the similarity of track changes. So, the paper considers the both influence on correlation, proposed discriminate correlation between phase points on the basis of the correlation function, deduced the new algorithm, finally model will be used in forecast of busy traffic to verify prediction effect.

Analysis of Adding-weight One-rank Local Model

Set observation data is $\{x_1, x_2, \dots, x_N\}$, N is the length of data, if the appropriate time delay τ and the best embedding dimension m is selected, it can turn into a new set of vector series.

$$X_i = [x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}]^T, i=1, 2, \dots, M, M=N-(m-1)\tau \quad (1)$$

The reconstruction of phase space is made of the observed data, τ and m . According to the law of Taken [6], the phase space can be recovery in the sense of topological equivalence dynamics

characteristic of time series, namely a trajectory under the high dimensional space.

If predicting point is x_{N+1} , the m th component of phase point X_{M+1} , X_M is predicting center. calculate the Euclidean distance with X_M and the rest of the phase point in phase space and choose q neighboring points which is close to X_{Mk} ($k = 1, 2, 3, \dots, q$) as its reference neighborhood, then adding-weighted one-rank local prediction model fitting k reference neighborhood points.

$$X_{M_k+1} = A + BX_{M_k} \quad (2)$$

$$\text{Which, } A = [a_1, a_2, \dots, a_m]^T, B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \dots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mm} \end{bmatrix}.$$

In type (2), the m th can be predicted:

$$x_{M_k+1}^m = a_m + \sum_{j=1}^m b_{mj} x_{M_k}^j \quad (3)$$

Set X_{Mk} normalized weight is P_k , the adding-weighted least square method as follows.

$$\sum_{k=1}^q P_k \left(x_{M_k+1}^m - a_m - \sum_{j=1}^m b_{mj} x_{M_k}^j \right)^2 = \min \quad (4)$$

Regard (4) as a function about $a_m, b_{mj} (j=1, 2, \dots, m)$, calculate the partial derivatives of a_m, b_{mj} , prediction model of fitting parameters a_m, b_{mj} , obtain prediction model of predicting point x_{N+1} :

$$\hat{x}_{N+1} = \hat{x}_{M+1}^m = a_m + b_{m1} x_M^1 + b_{m2} x_M^2 + \dots + b_{mm} x_M^m = a_m + \sum_{j=1}^m b_{mj} x_M^j \quad (5)$$

Predicting principle is shown in Fig. 1.

Figure 1. Prediction principle of adding-weight one-rank local

Adding-weighted One-rank Local Prediction Model Based on Relevance

Correlation Functions of Measured Phase Point. According to the reconstructed phase space, set Euclidean distance between point $X_i (i \neq M)$ and predicting center point X_M : Set d_{min} as the minimum value of d_i , and used y_i in describing phase point X_i to forecast the influence degree of center of X_M :

$$d_i = \left[\sum_{j=1}^m (X_M^j - X_i^j)^2 \right]^{\frac{1}{2}} \quad (6)$$

$$y_i = \frac{\exp\{-(d_i - d_{min})\}}{\sum_{i=1}^{M-1} \exp\{-(d_i - d_{min})\}} \quad (7)$$

Which, $d_i \in [0, \infty)$, $y_i \in (0, 1)$. If $d_i \rightarrow 0$, X_i and X_M is closer, then $y_i \rightarrow 1$, the greater influence of X_i to predict points.

Set $X_i (i \neq M)$ and Angle cosine of X_M is as follows.

$$\cos(X_M, X_i) = \frac{X_M \cdot X_i}{|X_M| \times |X_i|} = \frac{\sum_{j=1}^m X_M^j \times X_i^j}{\sqrt{\sum_{j=1}^m (X_M^j)^2} \times \sqrt{\sum_{j=1}^m (X_i^j)^2}} \quad (8)$$

Using z_i to show the shape similarity in trajectory with phase point X_i and X_M :

$$z_i = \frac{\cos(X_M, X_i) + 1}{2} \quad (9)$$

Which $\cos(X_M, X_i) \in [-1, 1]$, $z_i \in [0, 1]$. If $\cos(X_M, X_i) \rightarrow 1$, then $z_i \rightarrow 1$, the phase trajectory is similar, the higher the precision is.

Set a measure of the correlation of X_i and X_M correlation function for $c_i, c_i = y_i z_i$, and $c_i \in [0, 1]$, c_i considered influence of Euclidean distance and Angle cosine on the correlation between X_M and X_i , if $c_i \rightarrow 1$, then the stronger impact on prediction results in correlation.

Parameter Identification Adding-weight One-rank Local Forecast Model. Selected the largest phase points $X_{M_k} (k=1, 2 \dots q)$ of the q value of c_i as a neighborhood reference. Set c_{max} for maximum value of c_i , c_k is correlation of the k adjacent points, and the k reference neighborhood point's weight is as follows.

$$P_k = \frac{\exp(c_k - c_{max})}{\sum_{k=1}^q \exp(c_k - c_{max})} \quad (10)$$

Type (10) into type (4), calculate the partial derivatives of $a, b_{mj} (j=1, 2, 3, m)$ respectively, then obtain expansion equations of a, b_{mj} as follows.

$$\begin{aligned} \sum_{k=1}^q (P_k x_{M_k+1}^m) &= a_m + b_{m1} \sum_{k=1}^q (P_k x_{M_k}^1) + b_{m2} \sum_{k=1}^q (P_k x_{M_k}^2) + \dots + b_{mm} \sum_{k=1}^q (P_k x_{M_k}^m) \\ \sum_{k=1}^q (P_k x_{M_k+1}^1 x_{M_k}^1) &= a_m \sum_{k=1}^q (P_k x_{M_k}^1) + b_{m1} \sum_{k=1}^q [P_k (x_{M_k}^1)^2] + b_{m2} \sum_{k=1}^q (P_k x_{M_k}^1 x_{M_k}^2) + \dots + b_{mm} \sum_{k=1}^q (P_k x_{M_k}^1 x_{M_k}^m) \\ &\vdots \\ \sum_{k=1}^q (P_k x_{M_k+1}^m x_{M_k}^m) &= a_m \sum_{k=1}^q (P_k x_{M_k}^m) + b_{m1} \sum_{k=1}^q [P_k x_{M_k}^1 x_{M_k}^m] + b_{m2} \sum_{k=1}^q (P_k x_{M_k}^2 x_{M_k}^m) + \dots + b_{mm} \sum_{k=1}^q [P_k (x_{M_k}^m)^2] \end{aligned} \quad (11)$$

Matrix form of type (11) is as follows.

$$F = GH \quad (12)$$

$$\text{Which, } F = \begin{bmatrix} \sum_{k=1}^q (P_k x_{M_k+1}^m) \\ \sum_{k=1}^q (P_k x_{M_k+1}^1 x_{M_k}^1) \\ \vdots \\ \sum_{k=1}^q (P_k x_{M_k+1}^m x_{M_k}^m) \end{bmatrix}, G = \begin{bmatrix} 1 & \sum_{k=1}^q (P_k x_{M_k}^1) & \dots & b_{mm} \sum_{k=1}^q (P_k x_{M_k}^m) \\ \sum_{k=1}^q (P_k x_{M_k}^1) & \sum_{k=1}^q [P_k (x_{M_k}^1)^2] & \dots & \sum_{k=1}^q (P_k x_{M_k}^1 x_{M_k}^m) \\ \vdots & \vdots & \dots & \vdots \\ \sum_{k=1}^q (P_k x_{M_k}^m) & \sum_{k=1}^q [P_k x_{M_k}^1 x_{M_k}^m] & \dots & \sum_{k=1}^q [P_k (x_{M_k}^m)^2] \end{bmatrix}, H = \begin{bmatrix} a_m \\ b_{m1} \\ \vdots \\ b_{mm} \end{bmatrix}.$$

Obtain the parameter vector of predicting model.

$$H = G^{-1} F \quad (13)$$

$a_m, b_{mj} (j=1, 2, 3 \dots m)$ into predicting formula(5) can obtain predicting value.

The Analysis and Forecasting of Busy Traffic History Data

Through data collection of daily busy traffic, obtained 350 busy traffic data, the data points $\{x_1, x_2, \dots, x_{300}\}$ as a sample of forecasting model, $\{x_{301}, x_{302}, \dots, x_{350}\}$ as test samples, the proposed approach is adopted to improve the step prediction research.

Analyzing of Chaos Characteristic. Select the autocorrelation function to calculate $\tau=2$, though Cao's method to find $m=16$, and according to type (1), the 270 of $16d$ phase sequence, the phase space to reconstruct the busy traffic expressed as follows.

$$\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_{270} \end{pmatrix}^T = \begin{pmatrix} x_1 & x_3 & \dots & x_{31} \\ x_2 & x_4 & \dots & x_{32} \\ \vdots & \vdots & \dots & \vdots \\ x_{270} & x_{272} & \dots & x_{300} \end{pmatrix}^T \quad (14)$$

Using trajectory algorithm to calculate $\tau_{max} = 0.046601 > 0$, which shows busy traffic has chaos characteristics. $1/\tau = 21.459$ shows busy traffic is predictable, maximum forecast time is 21 days.

Busy Traffic Prediction. Step projections for the latter 50 points. Prediction steps are as follows. Step 1, determine the center of prediction and forecasting phase points (Tab.1).

Table 1 Prediction center points and prediction phase points

Prediction point	Prediction center point	Prediction phase point
301	$X_{270}=[x_{270},x_{272},\dots,x_{300}]^T$	$X_{271}=[x_{271},x_{273},\dots,x_{301}]^T$
302	$X_{271}=[x_{271},x_{273},\dots,x_{301}]^T$	$X_{272}=[x_{272},x_{274},\dots,x_{302}]^T$
$\vdots$	$\vdots$	$\vdots$

In Table 1, the m th point of prediction phase points is the prediction value of prediction point.

Step 2, according to type (6) ~ (8) calculate Euclidean distance and Angle cosine of X_i and X_M , y_i and z_i . Finally, calculate c_i , select c_i as the largest of the q value of the phase point as the reference neighborhood X_{Mk} ; determine the prediction center reference neighborhood;

Step 3, type (10) to calculate the k reference neighborhood point weights of P_k , calculate the forecast model parameter by type (13), and plug type (5) to get the predicted point prediction.

Step 4, repeat the above process, until the last data points is predicted.

Prediction value of and Angle cosine and observed data are shown in Tab.2.

Table 2 Observed data comparison with prediction value of this paper and angle cosin

data point	observed data	correlation function	angle cosin	data point	observed data	correlation function	angle cosin
301	2.374	2.3485	2.2152	326	3.201	3.1155	3.4084
302	2.763	2.6556	2.9289	327	2.293	2.1712	2.1351
303	3.393	3.3094	3.2629	328	3.449	3.3878	3.3291
304	2.971	2.8341	2.7929	329	3.149	3.237	2.9562
305	2.511	2.488	2.6152	330	4.799	4.7024	4.9558
306	3.376	3.5276	3.1967	331	4.159	4.0904	4.2652
307	3.932	4.0216	4.1394	332	3.899	3.8315	4.0128
308	3.216	3.1486	3.3178	333	3.003	2.8992	2.8799
309	3.808	3.7158	3.7072	334	3.5	3.3919	3.3052
310	3.402	3.2938	3.5668	335	3.496	3.3997	3.3256
311	2.722	2.6323	2.989	336	3.141	3.0409	2.9481
312	4.618	4.7783	4.4253	337	2.485	2.5815	2.2986
313	4.094	4.2303	3.9204	338	2.431	2.3801	2.3075
314	2.644	2.6736	2.4723	339	3.289	3.3599	3.1316
315	2.746	2.8835	2.9369	340	3.103	3.201	2.9476
316	3.968	4.0655	3.7985	341	4.362	4.4026	4.2273
317	2.831	2.6955	2.6827	342	2.915	3.0063	3.0685
318	3.9	3.9887	3.6981	343	4.05	4.0958	4.1657
319	3.681	3.7301	3.4839	344	2.586	2.6915	2.4144
320	3.965	3.85	3.8326	345	2.829	2.7317	2.9547
321	4.361	4.4223	4.5274	346	3.064	2.9964	3.2716
322	3.066	2.9418	2.9172	347	3.112	2.9845	3.3538
323	2.46	2.3593	2.5851	348	5.012	4.9504	4.8366
324	2.015	1.9035	2.1694	349	2.333	2.4497	2.5056
325	3.3	3.2201	3.4256	350	3.435	3.3339	3.2216

Prediction Evaluation. The absolute error of prediction is the difference of observed data and predicted value ($P=1,2,\dots,50$), The paper adopts RMSE[10] to detect the effect of prediction.

$$RMSE = \sqrt{\frac{1}{P} \sum_{p=1}^P (x_{N+p} - \hat{x}_{N+p})^2} \quad (15)$$

The comparison of RMSE is shown as Table 3.

It can be seen by Table 3 that RMSE of correlation is less than Angle cosine, which shows that compared Angle cosine to measure phase point with correlation, the reference neighborhood and correlation of the prediction center correlation is stronger, obtained a good prediction effect.

Table 3 High and low settings of predictor variables

error	correlation function	angle cosine
RMSE	0.00967	0.1666

Conclusions

The paper reconstructed phase space of busy traffic and calculated $\max=0.046601$ shows that busy traffic has both of chaos characteristics and predictability. Its prediction model is adding-weight one-rank local model;

In order to overcome the shortage of the phase correlation that adding-weighted one-rank local prediction model using Euclidean distance or Angle cosine measure separately, the article considers both of influence on phase correlation putting forward correlation function, a new adding-weighted one-rank local prediction algorithm is deduced. And forecast busy traffic. The result shows that the root mean square prediction error is 0.0967, which shows that the correlation function is more suitable to describe the correlation between the phase points. The method is effective to improve the prediction precision of the busy traffic, and provides an effective way for the prediction of non-linear time series.

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