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Improvement of Dirichlet Observation Matrix based on Super Prime Numbers

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Abstract. The reconstruction quality of the signal is directly determined by the observation matrix in the sense of signal compression. The observation matrix has a strong randomness to ensure the quality of the signal reconstruction, but the hardware implementation of the random observation matrix is more difficult. The certainty observation matrix is easy to be realized, but the reconstruction quality is poor. In order to balance the randomness and the realization of the observation matrix, this paper proposes the use of super prime numbers to generate pseudo-random sequence, which can make up the deficiency of the non-randomness of the deterministic observation matrix. Combined with the structure characteristics of Dirichlet observation matrix to design an observation matrix. Compared to the Gaussian random observation matrix, Dirichlet observation matrix and based on super prime Dirichlet observation matrix, the quality of the reconstructed signal was improved significantly and easy to be realized with hardware.

Introduction

Compressed sensing has been widely concerned by academia and industry, and has been applied to various fields[1]. The theory pointed out, if the signal is compressible or in a transform domain is sparse, using a observation matrix to project the signals from high dimension space to low dimensional space, finally from a small of projection to reconstruct the original signal by solving an optimization problem[2-3]. Observation matrix is an important part of the compressed sensing technology, which directly determines the quality of the reconstructed signal[4]. In order to ensure not lost signal during the projection process, the observation matrix should satisfy the Restricted Isometry Property (RIP)[5]. At present, the extensive use of the observation matrix has a random observation matrix and a deterministic observation matrix[6-7]. The random observation matrix mainly has the Gauss random observation matrix, the Bernoulli random observation matrix and so on. This kind of observation matrix can reconstruct the original signal in theory, but it is difficult to implement in hardware and the cost is high[8-9]. The deterministic observation matrix has the Dirichlet observation matrix, the sparse binary observation matrix and so forth. This kind of matrix has the advantages of low computational complexity and easy hardware implementation, but its reconstruction precision is low. The contribution of this paper lies in introducing super primes method to generate pseudo-random numbers during constructing the Dirichlet observation matrix, in relying on the Dirichlet observation matrix deterministic, try to improve random of Dirichlet observation matrix, so as to optimize the signal reconstruction accuracy[10-11].

Observation Matrix of Constructing

The sparsity of the signal refers to the m dimensional signal has n ($n<m$) nonzero elements, the other elements are all zero, and the signal is sparse. The essence of compressed sensing is to find out the characteristics of sparse signal, or the simple representation of signal on a ξ basis. The length of signal is m, the base vector is ξ_i , the signal is transformed as follows:

$$\chi = \xi\theta \tag{1}$$

where χ is the signal expressed in the time domain, θ is the signal in the ξ domain representation.

Under the condition of signal compression, the compressed sensing is divided into two processes:

1) design the p×q dimensional observation matrix η that not related to the transformation to observe the signal and get the p dimensional observation vector.

2) reconstruct the signal from the p dimensional observation vector.

Because the signal is n sparse, the observation matrix satisfies the RIP, the n coefficients can be reconstructed from the p observations, in other words, for any n sparse signal χ and constant δn ∈ (0,1), observation matrix:

$$1-\delta_k \le \frac{\|\eta\xi\|_2^2}{\|\chi\|_2^2} \le 1+\delta_k \tag{2}$$

The process of Dirichlet observation matrix is as follows: firstly, generate the vector $v=(v_1, v_2, ...v_m)$, secondly, according to the Dirichlet function looping construct residual vector, then, the observation matrix is obtained by the normalization process, Dirichlet function as follows:

$$D(v) = \frac{\sin \pi m v}{m \sin \pi v} e^{-j\pi(m-1)v} \tag{3}$$

Under normal circumstances, the value of the vector v is ±1, and each element is independent of the two distribution. The Dirichlet observation matrix is obtained by the vector loop shift, which is easy to implement, but the reconstruction accuracy is not high.

The Construct of Dirichlet Observation Matrix Based on Super Prime Numbers

At present, the random of Dirichlet observation matrix is not very satisfactory, resulting in low quality of signal reconstruction. The super prime method produces a longer period and the same distribution of pseudo random sequence, which satisfies the requirement of the random of the observation matrix. In the prime, there is a kind of special prime, which is special: if E is a prime number, $F_i \in N^+$ ($N^+$is expressed as a positive integer), $0<F_i<E$, and exist the score F_i/E expressing pure recurring decimal $0.\zeta_1\zeta_2\ldots\zeta_n\zeta_1\zeta_2\ldots\zeta_n\ldots$, circular section number is $\tau=E-1$, F_i is called super prime.

The method of constructing pseudo random sequence by using super prime numbers is called super prime. A method of constructing a pseudo random sequence of super prime numbers is proposed in [8]. Its iterative model is as follows:

$$F_{i+1} = 10 \times F_i + C_{i+1} (\bmod E), C_{i+1} = C_i + 1 \tag{4}$$

where Ci is constant increment. In this paper, the incremental Ci improvements are as follows:

$$C_{i+1} = F_i + C_i (\bmod E) \tag{5}$$

The changing of incremental C_i makes the 0 elements of the periodic sequence generated by some super prime numbers (such as 7, 131, 179, 181 and so on) appear again in every ($E+1$) element, cycle from $E\times(E-1)$to $(E+1)\times(E-1)$. Due to the emergence of the 0 elements, we can choose the appropriate interval to get the pseudo random sequence containing more 0 elements, so that the observation matrix contains more 0 elements, reduce the amount of data, speed up the reconstruction speed. We needn't all the super prime numbers has the property, but can use the super prime structure sparse measurement matrix with this characteristic.

Simulation Experiment

In order to verify the performance of the signal reconstruction of the Dirichlet observation matrix based on the super prime numbers, and the experimental verification is carried out by using the power quality signal in this paper. By using compressed sensing OMP recovery algorithm, the performance of signal reconstruction is compared with that based on super prime numbers Dirichlet observation matrix, Gauss random observation matrix and Dirichlet observation matrix. The steps are as follows:

1) reads the power quality signal and selects the observation value.

2) generate sequences by the improved method, which is used to construct Dirichlet observation matrix.

3) the signal is transformed into frequency domain to get X1, and the observed value in frequency domain is obtained by using the observation matrix.

4) OMP algorithm is used to recover the signal to get the reconstructed signal s, and the residual error of the signal is obtained.

5) calculate the signal of the performance index.

When the observation value p=128, compare with Peak Signal to Noise Ratio (PSNR), Signal to Noise Ratio (SNR), Mean Square Error (MSE) and relative error of electric signal which reconstructed by different observation matrix. The results are shown in Table 1.

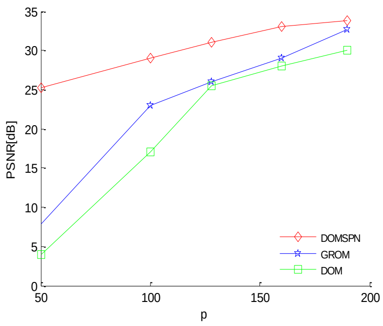
Table 1 The performance index of power quality reconstruction signal based on the observation matrix

	PSNR[dB]	SNR[dB]	MSE	relative error	running time[s]
DOMSPN	30.1346	16.2317	46.3622	0.0189	18.86
GROM	25.1717	12.2315	156.5456	0.0247	27.49
DOM	24.0311	11.8957	172.7665	0.0261	27.15

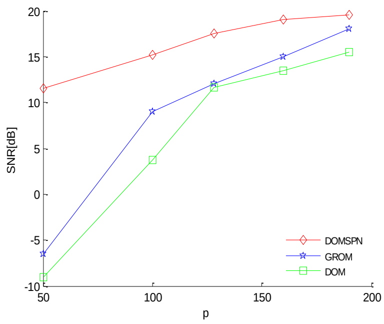
Notes: DOMSPN represents the Dirichlet observation matrix based on super prime numbers. GROM represents the Gauss random observation matrix. DOM represents the Dirichlet observation matrix.

Table 1 shows that the PSNR and SNR based on super primes Dirichlet observation matrix is high than that of Gaussian random observation matrix and Dirichlet observation matrix. However, its Mean Square Error, relative error and running time are lower than other observation matrices, which indicates that the improved Dirichlet observation matrix has better effect on signal reconstruction.

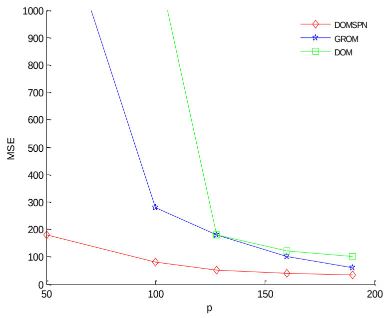
In order to better show based on super prime Dirichlet observation matrix statistical properties, the different sampling rate (*p=50, 100, 128, 160, 190*) is chosen to simulate the experiment, and the data obtained from the experiment are shown in Fig .1 (running time is the average value of running 20 times).



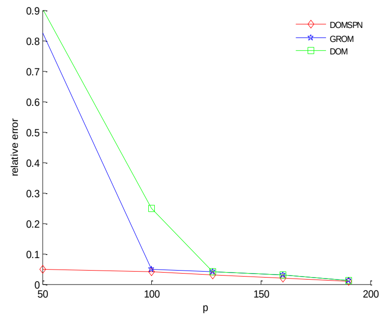
The curves of PSNR changing with *p*



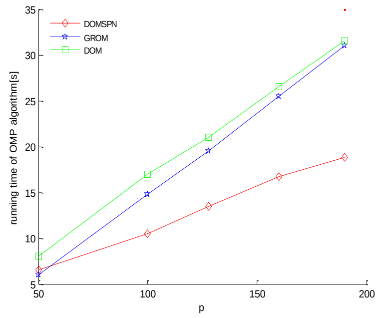
The curves of SNR changing with *p*



The curves of MSE changing with p



The curves of relative error changing with p



The curves of running changing with p

Figure 1. The performance indexes of different matrices under different p values

The results show that when the p value is smaller, in terms of the signal reconstruction accuracy, based on super primes Dirichlet observation matrix of PSNR and SNR high than that of Gaussian random observation matrix and Dirichlet observation matrix, but MSE and relative error is lower than that of Gauss observation matrix and Dirichlet observation matrix; when the value of p is increased, the reconstruction effect of Gauss random observation matrix and Dirichlet observation matrix is obviously improved, but the Dirichlet observation matrix based on super prime numbers also has certain advantages. In terms of the running time of OMP algorithm, the running time of the Dirichlet observation matrix based on super prime numbers is shorter than that of Gauss random observation matrix and Dirichlet observation matrix.

Conclusions

In the paper, based on the super prime numbers generated pseudo-random sequence to construct Dirichlet observation matrix to sparse sampling and reconstruction, which not only retains the certainty of Dirichlet matrix, but also increases the randomness of the matrix, so as to improve the reconstruction accuracy of the signal. Simulation results show that the proposed method has a good reconstruction effect, easy to hardware implementation, and strong practicability.

References

[1] Shree Krishna Sharma, Eva Lagunas, Symeon Chatzinotas, Bjorn Ottersten. Application of Compressive Sensing in Cognitive Radio Communications: A Survey. IEEE Communications Surveys & Tutorials, Vol.18 (2016) No.7, p.1838-1860.

[2] Christopher A. Metzler, Arian Maleki, Richard G. Baraniuk. From Denoising to Compressed Sensing. IEEE Transactions on Information Theory, Vol.62 (2016) No.9, p.5117-5144.

[3] Zhetao Li, Jingxiong Xie, Gengming Zhu, Xin Peng, Yanrong Xie, Youngjune Choi. Block-Based Projection Matrix Design for Compressed Sensing. Chinese Journal of Electronics, Vol.25 (2016) No.3, p.551-555.

[4] Pradip Sasmal, R. Ramu Naidu, Challa S. Sastry, Phanindra Jampana. Composition of Binary Compressed Sensing Matrices. IEEE Signal Processing Letters, Vol.23 (2016) No.8, p.1096-1100.

[5] Jian Wang. Support Recovery With Orthogonal Matching Pursuit in the Presence of Noise. IEEE Transactions on Signal Processing, Vol.63 (2015) No.21, p.993-1001.

[6] Marc Senneret, Yannick Malevergne, Patrice Abry,Gerald Perrin, Laurent Jaffres. Covariance Versus Precision Matrix Estimation for Efficient Asset Allocation. IEEE Journal of Selected Topics in Signal Processing, Vol.10 (2016) No.6, p.982-993.

[7] Hadi Zayyani, Mehdi Korki, Farrokh Marvasti. Dictionary Learning for Blind One Bit Compressed Sensing. IEEE Signal Processing Letters, Vol.23 (2016) No.2, p.187-191.

[8] J. Choi, F. Bien, W. Kim, J. Park. High-resolution offset-frequency PLL using properties of co-prime numbers. Electronics Letters, Vol.48 (2012) No.24, p.1-4.

[9] Mohammad Hossein Rohban, Venkatesh Saligrama, Delaram Motamed Vaziri. Minimax Optimal Sparse Signal Recovery With Poisson Statistics. IEEE Transactions on Signal Processing, Vol.64 (2016) No.13, p.3495-3508.

[10] Wache M, Murray D C. Application of Synchrophasor Measurements for Distribution Networks. IEEE Transactions on Power Delivery, Vol.57 (2015) No.2, p.1-4.

[11] Mallat S G, Zhang Zhifeng. Matching Pursuits with Time-frequency Dictionaries. IEEE Transactions on Signal Processing, Vol.41 (2016) No.21, p.3397-3415.