

# Research and Application for The Numerical Analysis Method of PV Arrays Operating Characteristics

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**Abstract**—In this paper, based on the PV module performance parameters, five performance points of PV module are obtained by calculation. Based on five points, the curve of performance is generated by shape-preserving interpolation on the computer. The algorithm better describes the electrical characteristics of PV arrays performance curve, and can be used for the computer application software development of the photovoltaic power generation system.

**Keywords**—component; photovoltaic power station; performance characteristics; simulation; design of power station operation

## I. INTRODUCTION

With the decrease of the photovoltaic module costs and the adjustment of the feed-in tariff, photovoltaic power generation has played more and more important role in the energy structure in China. Large-scale photovoltaic power plants with capacities of tens megawatts and above are being built in the A-level solar energy resource area which is western region in China. As for the design of large-scale photovoltaic power plant, photovoltaic array unit is the basic design unit in the DC power generation. Therefore, designers have to know the operating performances of the array units under different running temperatures and radiation conditions as well as other factors to evaluate the electrical characteristics and economic benefits of the power plant. However, the available parameters are often basic parameters of the photovoltaic components under the reference conditions, which, caused data deficiencies when optimizing the power plant. Thus, how to use basic parameters to realize the running simulations of photovoltaic power generation system under different environmental conditions is currently an urgent issue in the fields of engineering design and computational simulation of photovoltaic power station.

Photovoltaic array unit, which is connected in series or in parallel by photovoltaic modules on a array base with specific structure features, has its electrical operation characteristics related to the connection modes and electrical characteristics of the modules, azimuth angles and inclination angles of array bases and shading conditions of peripheral environment. Among these influence factors, connection mode and electrical characteristic of the PV module are main factors affecting the array electrical performance while other factors have an effect on the radiation intensity on the surface of the PV array unit and then influence the electrical output characteristic through the first factor. In the current work, array unit I-V curve drawing algorithm used in the photovoltaic array unit design module of the large on-grid PV power station engineering

design and simulation analysis system was introduced in detail. The algorithm implements computer drawing of the IV performance curves of PV array units with certain radiation intensity and no shading by using basic parameters of PV module.

## II. PV ARRAY PERFORMANCE MODEL

The performance characteristic of PV array unit depends on the PV module characteristic and the series or parallel connections among them. Hence, PV module model should be the basis of the study of PV array units operating characteristics.[1] Generally, the curve of PV array performance can be protracted according to the PV module performance curve and the connection modes, because of which the PV module model and the determination of model parameters are keys to solve the above problem. Foreign researchers have carried out extensive researches to figure out how to set up PV module model using basic parameters provided by manufacturers and have already made some progress till now. The equivalent circuit model of the PV cell was described as single diode model and double diode model in literatures [1] and [2], respectively. In literature [3], five parameters were calculated approximately using five equations obtained from the equivalent circuit set in five different states combining with basic parameters provided by manufacturers. Based on the same single diode model but different from literature [3], the author of literature [4] proposed a method to calculate the model parameters through dynamic iteration to achieve approximation of maximum power point of model. The Sandia National Laboratory proposed a PV array model in literature [5]. This model was different from the above two model because it was a mathematical model based on empirical datas. In this paper, a computer drawing method of PV array unit operating performance curve was carried out based on the PV module models and conceptions and methods in the above three literatures.

### A. The Equivalent Circuit and the Shape Features of Performance Curve of Ideal PV Cells

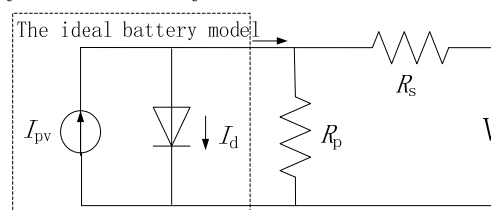


FIGURE I. THE PV EQUIVALENT CIRCUIT

The ideal PV cell was illustrated in FIGURE I according to the semiconductive theory, thus the PV equivalent circuit can be described as formula (1):

$$I = I_{pv,cell} - \underbrace{I_{0,cell} \left[ \exp\left(\frac{qV}{akT}\right) - 1 \right]}_{I_d} \quad (1)$$

Where  $I_{pv}$  is photo-generated current,  $I_d$  is dark current,  $I_0$  is diode saturation current,  $q$  is electron charge ( $1.6 \times 10^{-19}C$ ),  $k$  is the Boltzmann constant,  $T$  is absolute temperature and  $a$  is the curve constant of P-N junction. It can be known from formula (1) that the IV curve of the ideal PV cell can be illustrated as FIGURE II.

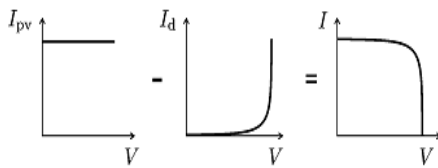


FIGURE II. THE DECOMPOSITION CURVE

It can be seen from FIGURE II that the performance curve of ideal PV cells has obvious shape features.

#### B. Establishment of the PV Module Model

Scholars at home and abroad have investigated the mathematical model of PV module with mainly two modeling approaches which are physical equivalent circuit model and empirical analytical model.

The single diode model which belongs to physical equivalent circuit model was illustrated in Fig.1. Some scholars have proposed other complicated models according to different needs of applications. As for the physical model, its equivalent circuit formula is a transcendental equation due to the diode characteristic of the battery. Formula (2) is the equivalent circuit current-voltage equation of the PV module single diode model.

$$I = I_{pv} - I_0 \left[ \exp\left(\frac{V + R_s I}{V_t a}\right) - 1 \right] - \frac{V + R_s I}{R_p} \quad (2)$$

Where  $I_{pv}$  is the photo-generated current,  $I_0$  is diode saturation current,  $V_t = N_s * k * T / q$  is the thermal voltage composed of  $N_s$  number of batteries connected in series. As is known, batteries in parallel can increase the current and in series can increase the voltage. Therefore, if the module is composed of  $N_p$  number of paralleled strings, then  $I_{pv} = I_{pv,cell} * N_p$ ,  $I_0 = I_{0,cell} * N_p$ . Where  $R_s$  is the equivalent series resistance,  $R_p$  is the parallel resistance.

According to this equation, the I-V performance curve can be obtained as shown in Fig.3. There are three feature points on this curve: short circuit current point, MPP point, open circuit voltage point. It can be observed from FIGURE III that the PV module has the mixed characteristic of current source and voltage source. When the PV module operated in the voltage source area, the curve shape was influenced by the series resistance significantly while operated in the current source area, the curve shape was affected dramatically by parallel resistance. On this basis, parameters ( $I_{sc}$ ,  $V_{mp}$ ,  $V_{oc}$ ,  $R_s$ ,  $R_p$  and others) are essential when drawing I-V curves. In order to draw PV module I-V curve based on the physical model, two problems are eager to be solved: (1) how to determine other coefficients of equations besides limited coefficients provided by the equipment manual; (2) how to solve transcendental equation. Fortunately, some solutions are available to address these problems in literatures [3] and [4], but inevitably with shortcomings of complications.

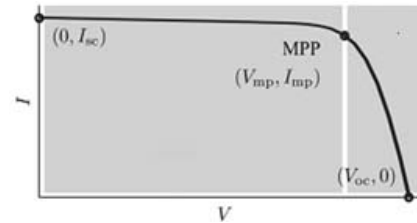


FIGURE III. THE CURVE OF PV MODULE

Based on empirical analysis, the American Sandia National Laboratory [2] proposed a PV module performance model in which each equation was derived through the analysis of a single battery. In addition, various factors related to the PV module performance was taken into account in order to ensure the accuracy and applicability of the model. For the purpose of industrial application, the model parameter data base was provided by the Sandia National Laboratory. Including the model parameters of the PV module manufactured by major manufactures all over the world, the data base was kept up to date for the use of model applications. The mathematical descriptions of the model are as follows:

$$I_{sc} = I_{sco} * f_1(AM_a) * \{ (E_b * f_2(AOI) + f_d * E_{diff}) / E_o \} * \{ 1 + \alpha_{Isc} * (T_c - T_o) \} \quad (3)$$

$$I_{mp} = I_{mpo} * \{ C_0 * E_e + C_1 * E_e^2 \} * \{ 1 + \alpha_{Imp} * (T_c - T_o) \} \quad (4)$$

$$V_{oc} = V_{oco} + N_s * \delta(T_c) * \ln(E_e) + \beta_{Voc}(E_e) * (T_c - T_o) \quad (5)$$

$$V_{mp} = V_{mpo} + C_2 * N_s * \delta(T_c) * \ln(E_e) + C_3 * N_s$$

$$\begin{aligned} & * \{\delta(T_c) * \ln(E_e)\}^2 + \beta_{Vmp}(E_e) * (T_c - T_0) \\ & (6) \end{aligned}$$

$$P_{mp} = I_{mp} * V_{mp} \quad (7)$$

$$FF = P_{mp} / (I_{sc} * V_{oc}) \quad (8)$$

$$E_e = I_{sc} / [I_{sc0} * \{1 + \alpha_{Isc} * (T_c - T_0)\}] \quad (9)$$

$$\delta(T_c) = n * k * (T_c + 273.15) / q \quad (10)$$

Three key points on the I-V curve can be determined through equations (3), (4), (5) and (6). Two attach points ( $V_x, I_x$ ) and ( $V_{xx}, I_{xx}$ ) are defined to ensure the inclination of the curve.

$$V_x = 0.5 * V_{oc} \quad (11)$$

$$\begin{aligned} I_x &= I_{xo} * \{C_4 * E_e + C_5 * E_e^2\} \\ & * \{1 + (\alpha_{Isc}) * (T_c - T_0)\} \\ & (12) \end{aligned}$$

$$V_{xx} = 0.5 * (V_{oc} + V_{mp}) \quad (13)$$

$$\begin{aligned} I_{xx} &= I_{xx0} * \{C_6 * E_e + C_7 * E_e^2\} \\ & * \{1 + (\alpha_{Im p}) * (T_c - T_0)\} \\ & (14) \end{aligned}$$

The physical definitions of the model parameters are as follows:

$I_{sc}$  is short circuit current;  $I_{mp}$  is the current of maximum power point;  $I_x$  is the current in the circuit when the module voltage being half of the sum of the open circuit voltage and the voltage of maximum power point;  $V_{oc}$  is the open circuit voltage;  $V_{mp}$  is the voltage of maximum power point;  $P_{mp}$  is the power of maximum power point;  $FF$  is the filling factor;  $N_s$  is the number of series batteries;  $N_p$  is the number of the number of batteries in parallel connection;  $k$  is the Stefan-Boltzmann constant;  $q$  is the number of elementary charge;  $T_c$  is the module temperature;  $T_0$  is the reference temperature;  $E_0$  is the reference irradiance;

$\delta(T_c)$  is the battery thermal voltage under the temperature of  $T_c$ ;  $E_e$  is the effective irradiance and  $C_0, C_1, C_2, C_4, C_5, C_7$  are module voltage and temperature related experience coefficients provided by Sandia data base.

Generally, five key points on the I-V curve can be obtained from formula (3), (4), (5), (6), (11), (12), (13), (14) combined with Sandia parameter data base, as shown in FIGURE IV.

The accuracy of the Sandia model has been verified through tests and debugging over years. Compared with physical equivalent circuit model, this model is more suitable for the design practice of PV power system. However, the Sandia model just presented the calculations of five characteristic points instead of how to plot the I-V curve. In this paper, The Sandia model was chosen as the mathematical model of the PV array and then combined with the shape feature of the I-V curve to discuss how to plot the I-V curve on the computer and the numerical algorithm.

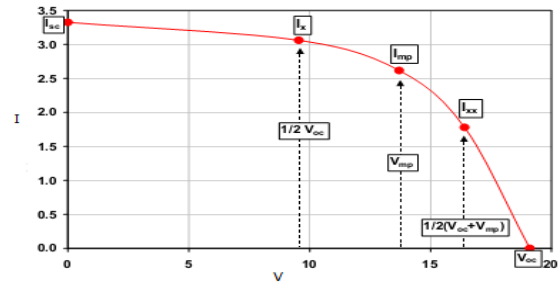


FIGURE IV. THE CURVE OF SANDIA MODEL

### C. Pv array Unit Curve Features Based on the Module Characteristics[1]

Since the PV module model was discussed in 1.2, in order to obtain the satisfactory power output in the photovoltaic engineering application, the PV modules should be connected in series or in parallel to form a PV array.

The voltage of the PV array is the sum of those of PV modules in series and the current keeps constant. However, when PV modules are connected in parallel, the PV array current is the sum of those of PV modules with the voltage remain the same. The above two IV curves are shown in FIGURE V and FIGURE VI respectively. When high power output is needed, a mixture way of in series and in parallel is adopted by the PV array. At this time, its IV curve is the sum of PV modules IV curves, as illustrated in FIGURE VII.

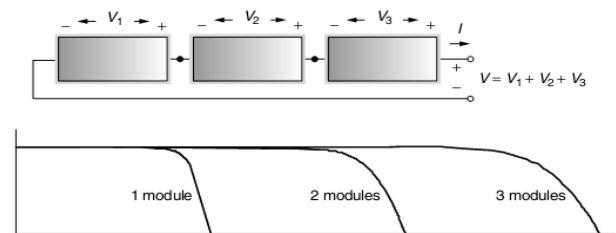


FIGURE V. THE CURVE FOR MODULES IN SERIES

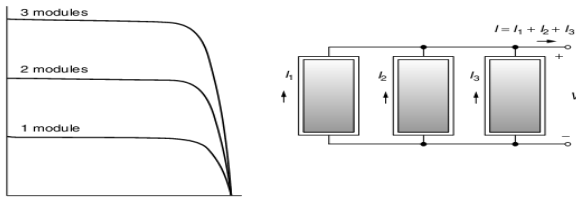


FIGURE VI. THE CURVE FOR MODULES IN PARALLEL

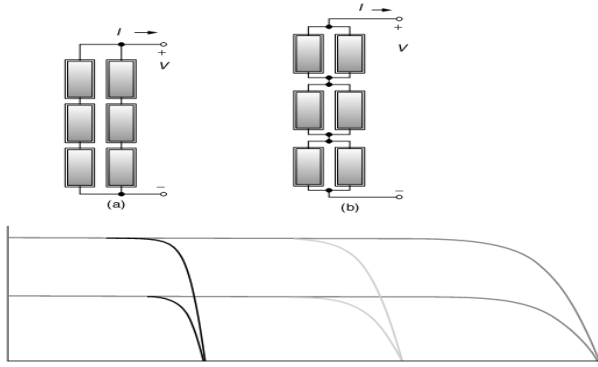


FIGURE VII. THE CURVE OF MODULES IN SERIES AND PARALLEL

Therefore, the drawing of PV array unit IV curve can be completed through the method of curve superposition after drawing PV module IV curves.

### III. PRINCIPLES OF CURVE GENERATION ALGORITHM

The performance curve is of great importance to the design of PV power station because it can deduce the design performance of the system by reflecting PV array unit electrical characteristics under different operating temperatures and radiation conditions. But how to draw a performance curve? As introduced in 1.2, five key points and shape features of IV curve can be obtained using the Sandia model, thus two requirements should be satisfied: (1) five key points must be on the curve; (2) the curve shape should reflect the mixed characteristics of voltage source and current source of PV module.

According to Hermit interpolation, there exist  $h_k = x_{k+1} - x_k$  with  $h_k$  representing the length of  $k$ th subinterval. The derivative of the interpolation function on  $x_k$  is  $d_k = P'(x_k)$ , thus in the range of  $x_k \leq x \leq x_{k+1}$ , there is an interpolation polynomial

$$P(x) = \frac{3hs^2 - 2s^3}{h^3} y_{k+1} + \frac{h^3 - 3hs^2 + 2s^3}{h^3} y_k + \frac{s^2(s-h)}{h^2} d_{k+1} + \frac{s(s-h)^2}{h^2} d_k \quad (15)$$

Where  $h = h_k$ ,  $s = x - x_k$ .

It can be inferred from the Sandia model, only  $d_k$  and  $d_{k+1}$  in formula (15) are unknown. Hence, how to calculate  $d_k$  and  $d_{k+1}$  is the key to ensure the curve shape. In literatures [6],[7], it is discussed how to meet the requirements of the curve shape characteristics by piecewise interpolation algorithm and the method can be summarized as follows:

$$\delta_k = \frac{y_{k+1} - y_k}{h_k}$$

Assuming  $\delta_k$ , then there exists:

If  $\delta_k$  and  $\delta_{k-1}$  are opposite in sign or one of them is zero, then the value of interpolation polynomial reached local minimum or maximum at  $x_k$ , thus  $d_k = 0$ ;

(1) If  $\delta_k$  and  $\delta_{k-1}$  are same in sign and  $h_k = h_{k-1}$ , then  $\frac{1}{d_k} = \frac{1}{2} \left( \frac{1}{\delta_{k-1}} + \frac{1}{\delta_k} \right)$ ;

(2) If  $\delta_k$  and  $\delta_{k-1}$  are same in sign but  $h_k \neq h_{k-1}$ , then  $\frac{w_1 + w_2}{d_k} = \frac{w_1}{\delta_{k-1}} + \frac{w_2}{\delta_k}$ ,

where  $w_1 = 2h_k + h_{k-1}$ ,  $w_2 = h_k + 2h_{k-1}$ .

It can be concluded that the curve of the PV array performance can be drawn well through a computer program based on the above method.

### IV. THE SOFTWARE IMPLEMENTATION OF THE ALGORITHM

According to part 2 and the Sandia model, the PV module performance curve can be divided into four sections which are presented as follows:

$$I_1(v) = I_x * H_{11}(v) + I_{sc} * H_{12}(v) + dI_x * H_{13}(v) + dI_{sc} * H_{14}(v) \quad (16)$$

$$I_2(v) = I_{mp} * H_{21}(v) + I_x * H_{22}(v) + dI_{mp} * H_{23}(v) + dI_x * H_{24}(v) \quad (17)$$

$$I_3(v) = I_{xx} * H_{31}(v) + I_{mp} * H_{32}(v) + dI_{xx} * H_{33}(v) + dI_{mp} * H_{34}(v) \quad (18)$$

$$I_4(v) = 0 * H_{41}(v) + I_{xx} * H_{42}(v) + 0 * H_{43}(v) + dI_{xx} * H_{44}(v) \quad (19)$$

Where,

$$H_{11}(v) = \frac{3 * (V_x - 0) * (v - 0) - 2 * (v - 0)^3}{(V_x - 0)^3},$$

$$H_{12}(v) = \frac{(V_x - 0)^3 - 3 * (V_x - 0) * (v - 0)^2 + 2 * (v - 0)^3}{(V_x - 0)^3},$$

$$H_{13}(v) = \frac{(v-0)^2 * (v-V_x)}{(V_x-0)^2},$$

$$H_{14}(v) = \frac{(v-0) * (v-V_x)^2}{(V_x-0)^2};$$

$$H_{21}(v) = \frac{3 * (V_{mp} - V_x) * (v - V_x) - 2 * (v - V_x)^3}{(V_{mp} - V_x)^3},$$

$$H_{22}(v) = \frac{(V_{mp} - V_x)^3 - 3 * (V_{mp} - V_x) * (v - V_x)^2 + 2 * (v - V_x)^3}{(V_{mp} - V_x)^3},$$

$$H_{23}(v) = \frac{(v - V_x)^2 * (v - V_{mp})}{(V_{mp} - V_x)^2},$$

$$H_{24}(v) = \frac{(v - V_x) * (v - V_{mp})^2}{(V_{mp} - V_x)^2};$$

$$H_{31}(v) = \frac{3 * (V_{xx} - V_{mp}) * (v - V_{mp}) - 2 * (v - V_{mp})^3}{(V_{xx} - V_{mp})^3},$$

$$H_{32}(v) = \frac{(V_{xx} - V_{mp})^3 - 3 * (V_{xx} - V_{mp}) * (v - V_{mp})^2 + 2 * (v - V_{mp})^3}{(V_{xx} - V_{mp})^3},$$

$$H_{33}(v) = \frac{(v - V_{mp})^2 * (v - V_{xx})}{(V_{xx} - V_{mp})^2},$$

$$H_{34}(v) = \frac{(v - V_{mp}) * (v - V_{xx})^2}{(V_{xx} - V_{mp})^2},$$

$$H_{41}(v) = \frac{3 * (V_{oc} - V_{xx}) * (v - V_{xx}) - 2 * (v - V_{xx})^3}{(V_{oc} - V_{xx})^3},$$

$$H_{42}(v) = \frac{(V_{oc} - V_{xx})^3 - 3 * (V_{oc} - V_{xx}) * (v - V_{xx})^2 + 2 * (v - V_{xx})^3}{(V_{oc} - V_{xx})^3},$$

$$H_{43}(v) = \frac{(v - V_{xx})^2 * (v - V_{oc})}{(V_{oc} - V_{xx})^2},$$

$$H_{44}(v) = \frac{(v - V_{xx}) * (v - V_{oc})^2}{(V_{oc} - V_{xx})^2}.$$

The values of  $dI_x$ ,  $dI_{sc}$ ,  $dI_{mp}$ ,  $dI_{xx}$  can be calculated through derivative method in part 2.

The computer implementation process of the algorithm is shown in FIGURE VIII. Firstly, the

database of Sandia parameters is read to calculate five characteristic points. Secondly, the derivative approximation are calculated using the above results. Thirdly, the polynomial is calculated by formulas (16), (17), (18) and (19) combined with the above results. Finally, draw the curve by computer graphics method using the calculated polynomials.

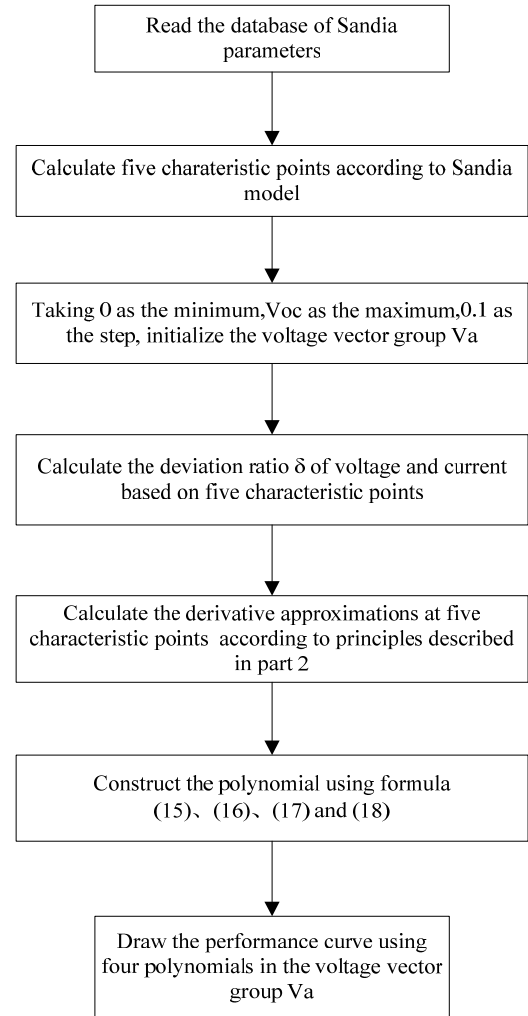


FIGURE VIII. THE PROGRAM FLOW

## V. ENGINEERING APPLICATION CASES OF THE ALGORITHM

The large PV power station project design and simulation analysis system is a set of large scale engineering design software covering the whole process of PV power station design. Its functions used to the engineering requirements of resources assessment, power station design, economic evaluation, operation simulation, construction and other stages in the design of power station. The algorithm introduced in this paper was applied to the software submodule of PV array unit design and a brief introduction to its design and application is stated in the following sections.

The PV array unit design module provided the project planners an interactive environment in which designers can select PV modules, choose connection and arrangement of



modules, determine tilt angle and elevation angle and analyze the force situation of PV array support structure. As a result, the calculation structure can be given and presented in a graphical manner according to the design of project planners.

As illustrated in FIGURE IX, user first select modules and inverters through the interface and then complete the string matching calculations under STC condition, after which the photovoltaic plane (consist of several PV array units) scale with single high power inverter can be determined.

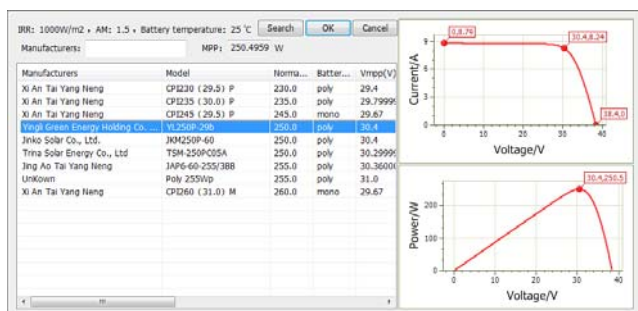


FIGURE IX. THE SOFTWARE INTERFACE 1

After the determination of the PV phalanx scale, user can design the PV array unit with the environment conditions of PV power plant. FIGURE X is the designer surface, the left of which is the input data including PV module parameters input by other submodules while the right of which is the user design operation panel. On the middle and lower section, lies the PV array unit design view which can be used by user to design and choose connections and arrangements of PV modules. The design result is displayed as an electrical characteristic curve in the top half of the design view and is then saved as a type of PV array unit which can be used in the design of PV power station phalanx.

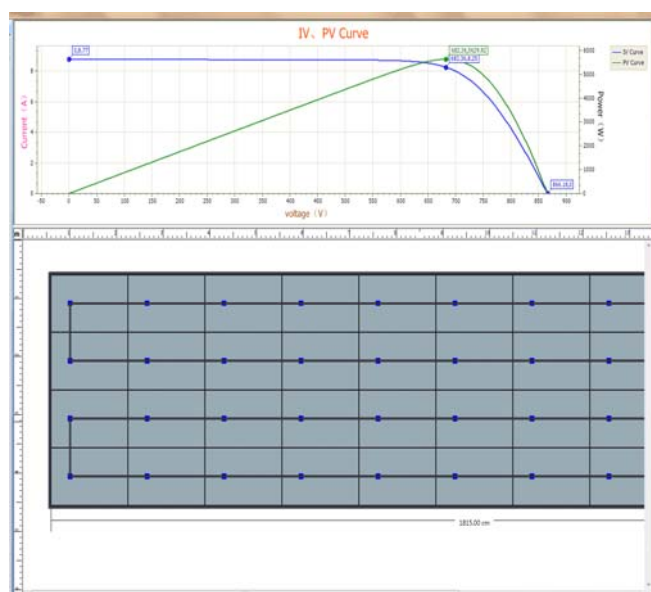


FIGURE X THE SOFTWARE INTERFACE 2

## VI. CONCLUSIONS

In this paper, the computer drawing algorithm ensuring the shape features of PV array unit IV curve was introduced as well as its engineering application cases using Sandia PV module performance model. Based on this algorithm, simulation and analysis of photovoltaic power generation systems operating characteristics can be implemented. At present, this algorithm was used in the computer application software development of the photovoltaic power generation system optimization design and operating simulation analysis, which achieved good application effects.

## ACKNOWLEDGEMENT

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## REFERENCES

- [1] Gilbert M.Masters. Renewable and Efficient Electric Power Systems[M]. Wiley, 2004, 460-472.
- [2] Antonio Luque,Steven Hegedus. Handbook of photovoltaic Science and Engineering 2nd [M].Wiley, 2011.120-123
- [3] WidalyS De Soto. Improvement and Validation of a Model for Photovoltaic [D]. Solar Energy Laboratory University of Wisconsin-Madison, 2004.
- [4] Marcelo Gradella Villalva,Jonas Rafael Gazoli, and Ernesto Ruppert Filho. Comprehensive Approach to Modeling and Simulation of Photovoltaic Arrays [J], IEEE TRANSACTIONS ON POWER ELECTRONICS,VOL.24,NO.5,MAY 2009.
- [5] King,D.L.,W.E.Boyson and J .A.Kratochvil. Photovoltaic Array Performance Model [C].Sandia National Laboratories,Albuquerque,NM,August,2004
- [6] F.N.Fritsch and R.E.Carlson.Monotone Piecewise Cubic Interpolation[J],SIAM J. Numerical Analysis 17,1980 238-246.
- [7] David Kahaner,Cleve Moler and Stephen Nash, Numerical Methods and Software[M],Prentice Hall,199FIGURE X The Software interface 2