

3D 9^+ - Model Representing Topological Relation between 3D Bodies with Holes

Yunfei Shi^{1,2,*}, Shuang Di¹, Lingling Zhang¹ and Ting Mi²

¹Key Laboratory of Urban Land Resources Monitoring and Simulation Ministry of Land and Resources, Shenzhen, 518034, China

²College of Resource and Environment, Linyi University, Linyi, 276000, China

³Linyi Bureau of Land Resources, Linyi, 276000, China

*Corresponding author

Abstract—Three-dimensional topological relation is the basis of spatial analysis, spatial reasoning, and other applications in 3D GIS. Based on 9-intersection model, the proposed 3D 9-intersection uses 5 values {0, 1, 2, 3, 4} to replace two values {0, 1} of original model, which not only can distinguish whether two bodies intersection but can get the dimensions of the intersection results. Based on 3D 9-intersection model, the broad boundary of TRCR model was introduced to divide several sub-sections of bodies with holes into interior, broad boundary, and exterior. Moreover, two methods to divide broad boundary were proposed. In order to further distinguish the topological relations, 3D 9^+ -intersection model was proposed by referring the nested matrix of 9^+ -intersection model for representing broad boundary. Therefore, this model allows topological relationships between complex bodies to be implemented.

Keywords—3D body with holes; expression of 3D topological relation; 3D 9-intersection model; 3D 9^+ -intersection model

I. INTRODUCTION

The expression of 3D topological relation is the foundation of spatial analysis, spatial reasoning, and other fields in 3D GIS. With the application of 3D GIS in 3D city planning [1], geological disaster analysis [2], indoor navigation [3], mining engineering and underground excavation [4], shopping guide in supermarket, and other fields, the expression of 3D topological relation has become one of research hotspots in academic circle. The present researches are mainly concentrated in the expression of topological relation between simple 3D bodies, rather than complex bodies. However, spatial objects in actual world are always complex involving holes or separating sub-parts. Therefore, it is significantly practical to explore the expression of topological relation between complex bodies (with holes).

Holes are called as “handles” in topology, and the number of “handles” in surface called as Genus which is an invariant in topological transformation. The most present researches are mainly concentrated in 2D space when exploring the expression of topological relation of objects with cavity. Egenhofer et al. [6] started to study the expression of topological relation between regions with holes since 1997. By introducing the concepts of general area and hole, the topological relation between region A (with n holes) and B (with m holes) was described. In 2007, Egenhofer [8] expanded 8 face/face relations to 23 relations by refining holes area. Vasardani [15] proposed HFM model for expressing topological

relation between face with holes and those without hole. Climentini [7][13] proposed TRCR model by expanding the 9-intersection model. Taking the area between hole and boundary as broad boundary, this model can express the topological relation of complex region with one hole. However, this model cannot distinguish if these two objects are connected to (or separated from) internal boundary or external boundary. 9-intersection model is difficult to express object with hole, so Kurata [9] proposed 9^+ -intersection model. He divided spatial objects into several sub-parts and expressed topological relation between two complex objects with holes. Ouyang Jihong [5] expanded elements in 9-intersection matrix to five binary coding $mij = x_4 x_3 x_2 x_1 x_0$ ($1 \leq i, j \leq 3$), and judged the intersection between two regions by transforming binary coding into corresponding decimal number. Liu Bo [10] proposed 4-4ID model based on 4-intersection model. This model can only describe the topological relation between regions with single hole. Expanding the research object to three dimension. Zhang Jun et al. [11] treated the spatial body A with several holes as the difference between simple body A^* and several simple bodies a_i . This model is very simple when expressing the relation between simple bodies and single hole, but it cannot distinguish some topological relations. Moreover, a 9-intersection element has several explanations if the body has multiple holes.

The expression of 3D topological relation between holes is a complex problem especially when holes number increases. This work combined TRCR model [7][13] with 9^+ -intersection model [9] to discuss the expression of topological relation between 3D holes. This work made following contributions: (1) 3D 9-intersection model was proposed with five values {0, 1, 2, 3, 4} replacing two values {0, 1} in 9-intersection model. This model can distinguish whether two bodies intersection but can get the dimensions of the intersection results express. (2) Body with holes has multiple boundaries (several internal boundaries and one external boundary), so it is difficult to distinguish interior, boundary, and exterior of 9-intersection model with too many topological sub-parts. To solve this problem, broad boundary of RTCR model was applied to turn the narrow boundary to broad boundary in 9-intersection model. (3) In order to solve the problem that some topological relations cannot be distinguished by broad boundary, 3D 9^+ -intersection model was proposed by further refining broad boundary with the nested matrix of 9^+ -intersection model.

II. 3D 9-INTERSECTION MODEL

Similar to 2D space, 3D space has 8 topological relations between two bodies, including disjoint, meet, overlap, covers, contains, coveredby, inside and equal. Among these relations, meet, covers, and coveredby are different from those in 2D space because the intersection can be empty, point, line segment, face, and body. In 9-intersection model, these five different conditions cannot be distinguished with 0 and 1. In order to further refining, matrix elements were reset to expand original value $\{0, 1\}$ to $\{0, 1, 2, 3, 4\}$, representing the intersection of empty, point, line, face, and body, respectively. The expression is as follows:

$$M = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap B^- \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^0 & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \quad (1)$$

where $R_{ij} \in \{0, 1, 2, 3, 4\}$.

The improved model can distinguish different intersections. In theory, there are 5^9 topological relations. Without practical significance, however, the most relations can be set with conditions. Compared with 9-intersection model, this model has stronger topological expressive ability. The meet between two bodies can be point, line segment, or face. Similarly, the matrix element can also be used to express covers and coveredby. When two bodies intersect, the intersection is also a body with 4 as the value of matrix element. Therefore, this model can distinguish 14 topological relations between two simple bodies: disjoint, point meet, line meet, face meet, overlap, point coveredby, line coveredby, face coveredby, inside, equal, point cover, line cover, face cover, and containing. For simple body A and B , these 14 topological relations correspond to 14 topological predicates: $D(A, B)$, $PM(A, B)$, $LM(A, B)$, $FM(A, B)$, $O(A, B)$, $PCB(A, B)$, $LCB(A, B)$, $FCB(A, B)$, $I(A, B)$, $E(A, B)$, $PC(A, B)$, $LC(A, B)$, $FC(A, B)$, and $C(A, B)$. This model is named as 3D-9 intersection model. According to FIGURE 1, the comparison of expression ability with 9-intersection model, 3D-9 intersection model has stronger expression ability.

| figure | 9-intersection model | 3D 9-intersection model | figure | 9-intersection model | 3D 9-intersection model |
|--------|---|---|--------|---|---|
| | $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \end{bmatrix}$ | | $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 3 & 4 \\ 3 & 3 & 3 \\ 4 & 3 & 4 \end{bmatrix}$ |
| | $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \end{bmatrix}$ | | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 3 \\ 4 & 3 & 4 \end{bmatrix}$ |
| | $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 2 & 3 \\ 4 & 3 & 4 \end{bmatrix}$ | | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ |
| | $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 3 & 3 \\ 4 & 3 & 4 \end{bmatrix}$ | | $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 3 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ |
| | $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 3 & 4 \end{bmatrix}$ | | | $\begin{bmatrix} 4 & 3 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ |
| | | $\begin{bmatrix} 4 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 3 & 4 \end{bmatrix}$ | | | $\begin{bmatrix} 4 & 3 & 4 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ |
| | | $\begin{bmatrix} 4 & 0 & 0 \\ 3 & 3 & 0 \\ 4 & 3 & 4 \end{bmatrix}$ | | $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 4 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ |

FIGURE 1. COMPARISON BETWEEN 3D 9-INTERSECTION MODEL AND 9-INTERSECTION MODEL

III. EXPRESSION OF TOPOLOGICAL RELATION FOR BODY WITH CAVITY

There are various bodies in 3D space, including non-manifold singular bodies. This work defined the body with holes as a hollow solid, allowing the body, multiple separated holes, divide 3D space into several parts: body A (closed set) itself, exterior of A (open set), and n ($n \geq 1$) interior A_1, A_2, \dots, A_n (closed set). Moreover, there are connected regular bounded closed sets inside of the body. FIGURE II shows that (a) is the body with holes and (b) is a concave simple body without hole. Because (c) connects internal to external, the hole can be directly treated as the exterior of body. Moreover, (d) is the singular body [12]. The body with holes A is a complex us in nature with n micro-bodies A_1, A_2, \dots, A_n . Here, $A_i = \partial A_i \cup A_i^h, i \in [1, n]$, and $\angle A_1, \angle A_2, \dots, \angle A_n$ are internal boundaries of A . $\angle A$ is the external boundary of A , and $A_1^h, A_2^h, \dots, A_n^h$ are holes of A (open set). A^{in} refers to the region between inner and outer boundaries, A^h to the general

hole: $A^h = \bigcup_{i=1}^n A_i^h$, and $\angle_{in} A$ to the general inner boundary:

$$\partial_{in} A = \bigcup_{i=1}^n \partial A_i$$

Structure with cavities is complex. If 9-intersection model is used to express body with multiple holes, the body cannot be simply divided into interior, boundary, and exterior any more. Therefore, every hole should be treated separately.

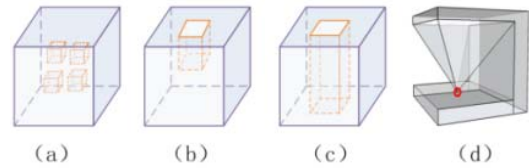


FIGURE II. CLASSIFICATION OF BODIES

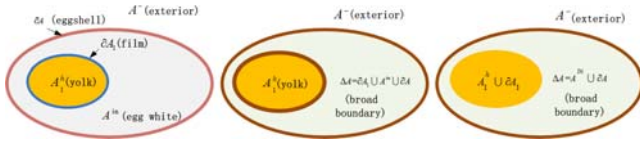


FIGURE III. TOPOLOGICAL EXPRESSION OF BODY WITH HOLE

Body A with n holes should be divided into $3+2n$ topological parts, $A^-, \angle A, A^{in}, \angle A_1, \angle A_2, \dots, \angle A_n, A_1^h, A_2^h, \dots, A_n^h$. However, there are several defects: (1) with several boundaries (several internal boundaries and one external boundary), it is difficult to determine the layer to be taken in boundary of 9-intersection model. (2) The body is a complex region, so it is difficult to determine which part should be taken in 9-intersection model, holes or the part between holes and external boundary. (3) There are too many topological sub-parts which is disadvantageous to the expression of topological relation. In order to solve these problems, these topological sub-parts are still classified into exterior, boundary, and interior. The division of exterior is unchanged. Based on broad boundary of TRCR model^[13], the boundary turns from narrow boundary in 9-intersection model to broad boundary of TRCR model ΔA by division. The broad boundary is composed of internal boundary, external boundary, and closed region between two boundaries. In addition, the interior is the union set of holes. This mode is called as “131 combination”, meaning exterior(1)+broad boundary (3-in-1) + interior (1).

A. “131 Combination” Pattern

A body with hole (taking single hole) as an example can be compared to egg. As shown in the left of FIGURE III, the body is composed of yolk (A_1^h), film ($\angle A_1$), egg white (A^{in}), egg shell ($\angle A$), and exterior A^- . FIGURE III shows three parts divided by this pattern in A .

(1) Interior $A^o = A^h = A_1^h$;

(2) Broad boundary ΔA contains internal boundary, external boundary, and closed area between two boundaries. In other words, $\angle A_1 \leq \Delta A \leq \angle A$ and $\Delta A = \angle A_1 \cup A^{in} \cup \angle A$;

(3) Exterior A^- is an open set in universal set Q . Therefore, $A^- = Q - A^o - \Delta A$.

This mode can be expressed by following 3×3 matrix^[13]:

$$M = \begin{pmatrix} A^o \cap B^o & A^o \cap \Delta B & A^o \cap B^- \\ \Delta A \cap B^o & \Delta A \cap \Delta B & \Delta A \cap B^- \\ A^- \cap B^o & A^- \cap \Delta B & A^- \cap B^- \end{pmatrix} \quad (2)$$

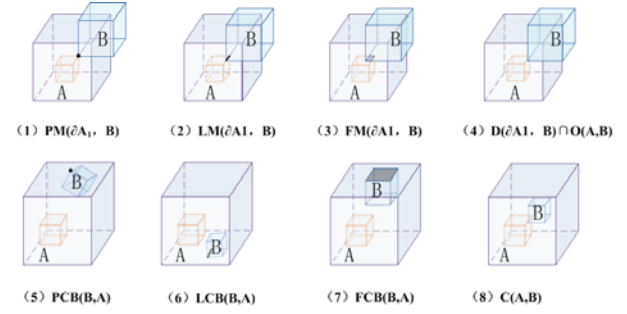


FIGURE IV. EXPRESSION EXAMPLES OF TOPOLOGICAL RELATIONS FOR BODY WITH CAVITIES

This model can express the topological relation of body with hole, but cannot distinguish different relations. In FIGURE IV, (1)—(4) present different topological relations. However, the expressions correspond to the same

matrix $R = \begin{bmatrix} 0 & 0 & 4 \\ 4 & 3 & 4 \\ 4 & 3 & 4 \end{bmatrix}$ when using this model. Obviously, there is deficiency.

B. “221 Combination” Pattern

In order to solve above problems, topological sub-parts are re-divided. As shown in the right of FIGURE III, the hole and internal boundary are treated as a whole, the area between internal and external boundaries as a whole, and the exterior as a separated part. The description is as follows:

(1) The interior of body $A^o = A_1$ is a closed set, i.e. $A_1 = \partial A_1 \cup A_1^h$. A_1^h correspond to yolk and $\angle A_1$ to the film out of the yolk. Yolk and film are combined into one.

(2) Boundary ΔA contains the part between internal and external boundaries and external boundary, excluding internal boundary. In other words, $\angle A_1 < \Delta A \leq \angle A$ and $\Delta A = A^{in} \cup \angle A$. The boundary corresponds to the egg white and shell in the egg, which are combined into one.

(3) Exterior A^- is an open set in complete set Q . Therefore, $A^- = Q - A^o - \Delta A$.

After re-division, (1)—(4) in FIGURE IV correspond to following matrixes:

$$R_{(1)} = \begin{bmatrix} 0 & 1 & 4 \\ 4 & 3 & 4 \\ 4 & 3 & 4 \end{bmatrix}, R_{(2)} = \begin{bmatrix} 0 & 2 & 4 \\ 4 & 3 & 4 \\ 4 & 3 & 4 \end{bmatrix},$$

$$R_{(3)} = \begin{bmatrix} 0 & 3 & 4 \\ 4 & 3 & 4 \\ 4 & 3 & 4 \end{bmatrix}, R_{(4)} = \begin{bmatrix} 0 & 0 & 4 \\ 4 & 3 & 4 \\ 4 & 3 & 4 \end{bmatrix} \quad (3)$$

The expression ability enhances.

However, there are still some problems in modified model. For example, in (5)—(8) of FIGURE IV, A is a body with single hole and B is simple body. Four conditions in figure are

obviously different. The first three conditions show that B is covered by A , while the forth condition shows that B is contained by A . These conditions cannot be distinguished when being expressed with this model because the same

matrix $R = \begin{bmatrix} 0 & 0 & 4 \\ 4 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ is obtained. In order to solve this problem, three topological parts are further subdivided into sub-topologies, “131 separate” and “121 separate” by introducing 9⁺-intersection model [9].

C. 3.3 “131 Separate” Pattern

Based on “131 combination”, broad boundary is subdivided into three sub-parts which are expressed with nested matrix. The expression is

$$M = \begin{bmatrix} A^0 \cap B^0 & A^0 \cap \Delta B & A^0 \cap B^- \\ \Delta A \cap B^0 & \Delta A \cap \Delta B & \Delta A \cap B^- \\ A^- \cap B^0 & A^- \cap \Delta B & A^- \cap B^- \end{bmatrix}$$

$$\Delta A = \begin{bmatrix} \partial_{in} A \\ A^{in} \\ \partial A \end{bmatrix}, \Delta B = \begin{bmatrix} \partial_{in} B & B^{in} & \partial B \end{bmatrix} \quad (4)$$

Therefore,

$$M = \begin{bmatrix} A^0 \cap B^0 & [A^0 \cap \partial_{in} B & A^0 \cap B^{in} & A^0 \cap \partial B] & A^0 \cap B^- \\ \partial_{in} A \cap B^0 & [\partial_{in} A \cap \partial_{in} B & \partial_{in} A \cap B^{in} & \partial_{in} A \cap \partial B] & \partial_{in} A \cap B^- \\ A^{in} \cap B^0 & [A^{in} \cap \partial_{in} B & A^{in} \cap B^{in} & A^{in} \cap \partial B] & A^{in} \cap B^- \\ \partial A \cap B^0 & [\partial A \cap \partial_{in} B & \partial A \cap B^{in} & \partial A \cap \partial B] & \partial A \cap B^- \\ A^- \cap B^0 & [A^- \cap \partial_{in} B & A^- \cap B^{in} & A^- \cap \partial B] & A^- \cap B^- \end{bmatrix} \quad (5)$$

If above formula is used to express the topological relation in FIGURE IV, A is body with a hole, $A^0 = A_1^h$ and $\angle_{in} A = \angle A_1$.

There is no hole in B , so B^0 and $\angle B_1$ do not exist. The matrixes in (5)–(8) of FIGURE IV are as follows:

$$R_{(5)} = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}, R_{(6)} = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix},$$

$$R_{(7)} = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}, R_{(8)} = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 4 \end{bmatrix} \quad (6)$$

Obviously, the representation capacity of the pattern has been improved. This pattern supports the topological representation among bodies with multiple holes. We assume that there are n holes for A , and no holes for B , then the topological relations between A and B can be represented as follows:

$$M = \begin{bmatrix} \begin{bmatrix} A_1^h \cap B^{in} \\ A_2^h \cap B^{in} \\ \vdots \\ A_n^h \cap B^{in} \end{bmatrix} & \begin{bmatrix} A_1^h \cap \partial B \\ A_2^h \cap \partial B \\ \vdots \\ A_n^h \cap \partial B \end{bmatrix} & \begin{bmatrix} A_1^h \cap B^- \\ A_2^h \cap B^- \\ \vdots \\ A_n^h \cap B^- \end{bmatrix} \\ \begin{bmatrix} \partial A_1 \cap B^{in} \\ \partial A_2 \cap B^{in} \\ \vdots \\ \partial A_n \cap B^{in} \end{bmatrix} & \begin{bmatrix} \partial A_1 \cap \partial B \\ \partial A_2 \cap \partial B \\ \vdots \\ \partial A_n \cap \partial B \end{bmatrix} & \begin{bmatrix} \partial A_1 \cap B^- \\ \partial A_2 \cap B^- \\ \vdots \\ \partial A_n \cap B^- \end{bmatrix} \\ \begin{bmatrix} A^{in} \cap B^{in} \\ \partial A \cap B^{in} \end{bmatrix} & \begin{bmatrix} A^{in} \cap \partial B \\ \partial A \cap \partial B \end{bmatrix} & \begin{bmatrix} A^{in} \cap B^- \\ \partial A \cap B^- \end{bmatrix} \\ \begin{bmatrix} A^- \cap B^{in} \end{bmatrix} & \begin{bmatrix} A^- \cap \partial B \end{bmatrix} & \begin{bmatrix} A^- \cap B^- \end{bmatrix} \end{bmatrix} \quad (7)$$

The internal boundaries in this mode are not continuous, so the interior of body and boundary are not connected. Therefore, both interior and boundary should be divided into n topological sub-parts. When analyzing topological relation, interior and narrow boundary should be discussed for n times. The analysis is too tedious to express topological relation.

D. 3.4 “221 Separate” Pattern

Based on “221 combination”, topological part is subdivided. Interior A^0 is divided into A^h and internal boundary $\angle_{in} A$. Moreover, broad boundary ΔA is divided into interior A^{in} and boundary $\angle A$. The expressions are as follows:

$$M = \begin{bmatrix} A^0 \cap B^0 & A^0 \cap \Delta B & A^0 \cap B^- \\ \Delta A \cap B^0 & \Delta A \cap \Delta B & \Delta A \cap B^- \\ A^- \cap B^0 & A^- \cap \Delta B & A^- \cap B^- \end{bmatrix}$$

$$A^0 = \begin{bmatrix} A^h \\ \angle_{in} A \end{bmatrix}, \Delta A = \begin{bmatrix} A^{in} \\ \angle A \end{bmatrix}, B^0 = [B^h \quad \partial_{in} B], \Delta B = [B^{in} \quad \partial B] \quad (8)$$

Therefore,

$$M = \begin{bmatrix} \begin{bmatrix} A^h \cap B^h & A^h \cap \partial_{in} B \\ \angle_{in} A \cap B^h & \angle_{in} A \cap \partial_{in} B \end{bmatrix} & \begin{bmatrix} A^h \cap B^{in} & A^h \cap \partial B \\ \angle_{in} A \cap B^{in} & \angle_{in} A \cap \partial B \end{bmatrix} & \begin{bmatrix} A^h \cap B^- \\ \angle_{in} A \cap B^- \end{bmatrix} \\ \begin{bmatrix} A^{in} \cap B^h & A^{in} \cap \partial_{in} B \\ \angle A \cap B^h & \angle A \cap \partial_{in} B \end{bmatrix} & \begin{bmatrix} A^{in} \cap B^{in} & A^{in} \cap \partial B \\ \angle A \cap B^{in} & \angle A \cap \partial B \end{bmatrix} & \begin{bmatrix} A^{in} \cap B^- \\ \angle A \cap B^- \end{bmatrix} \\ \begin{bmatrix} A^- \cap B^h & A^- \cap \partial_{in} B \end{bmatrix} & \begin{bmatrix} A^- \cap B^{in} & A^- \cap \partial B \end{bmatrix} & \begin{bmatrix} A^- \cap B^- \end{bmatrix} \end{bmatrix} \quad (9)$$

If above expressions are used to express the topological relations in FIGURE IV, following matrixes are corresponding to (5)–(8):

$$\begin{aligned}
 R_{(5)} &= \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}, & R_{(6)} &= \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}, \\
 R_{(7)} &= \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}, & R_{(8)} &= \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}
 \end{aligned} \quad (10)$$

Above formula has the same number of matrix elements in "131 separate" pattern. However, in this formula, broad boundary is continuous, so the analysis is simpler. Moreover, the boundary between two holes can be treated as a simple body, so the reasoning of topological relation can be much simpler. This model can be called as 3D 9⁺-intersection model which can be transformed into 9-intersection model in following process. If an element in internal nested matrix is not 0, replace it with 1. Otherwise, replace it with 0. For example, above-mentioned $R_{(5)}$ matrix can be transformed to $R_{(5)} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. This transformation can simplify the operation

in some specific applications (including spatial reasoning). With the transformed matrix, the relationship between A and B can be judged. If the specific intersection should be checked, the nested matrix element should be further checked. This model also supports the expression of topological relation between bodies with multiple holes.

IV. CONCLUSIONS

Based on 9-intersection model, 3D 9-intersection model replaces matrix elements in 9-intersection model with dimensions to consider different intersection dimensions of 3D spatial objects. Therefore, element value can express both emptiness and dimension of intersection. This model is more suitable for the expression of topological relation between 3D spatial bodies by enhancing the differentiation and expression of topological relations. For body with cavity, 3D 9⁺-intersection model is proposed. Combining advantages of TRCR model and 9⁺-intersection model, this model solves the problem that TRCR model cannot distinguish if two objects are connected in internal boundary or external boundary (disjoint). Compared with 9⁺-intersection model, this model is simpler in expression form and analysis process. Moreover, introducing the expression with dimension from 3D 9-intersection model, this model has strong ability in expressing the topological relation of 3D body with cavity. Therefore, this model has theoretical value in expression, analysis, etc. of topological relations between objects in spatial reasoning, analysis, and other fields. However, there are some defects. For example, the more cavities of body object, the more matrix elements and the more complex the analysis will be. In the future, this model should be improved and modified to simplify the expression of topological relation.

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