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From Fuzzy Models to Granular Fuzzy Models

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Abstract

In this study, we offer a general view at the area of fuzzy modeling and elaborate on a new direction of system modeling by introducing a concept of granular models. Those models constitute a generalization of existing fuzzy models and, in contrast to existing models, generate results in the form of information granules (such as intervals, fuzzy sets, rough sets and others). We present a rationale and some key motivating arguments behind the emergence of granular models and discuss their underlying design process. Central to the development of granular models are granular spaces, namely a granular space of parameters of the models and a granular input space. The development of the granular model is completed through an optimal allocation of information granularity, which optimizes criteria of coverage and specificity of granular information. The emergence of granular models of type-2 and type-*n*, in general, is discussed along with an elaboration on their formation. It is shown that achieving a sound coverage-specificity tradeoff (compromise) is of essential relevance in the realization of the granular models.

Keywords: fuzzy models, Granular Computing, information granules of higher type, granular spaces

1. Introduction

Fuzzy models and fuzzy modeling are one of the most visible applied manifestations of fuzzy sets. While there are a number of ways to define fuzzy models and the environment of fuzzy modeling, in general, the following somewhat generic definition could be offered. Fuzzy models can be regarded as models whose architecture dwells upon the constructs of fuzzy sets (fuzzy sets, fuzzy relations, fuzzy set operators), their functioning adheres to the fundamental ways of processing of fuzzy sets and their development is supported by the design methodology pertinent to fuzzy sets. Fuzzy models appeared quite early after the emergence of the discipline of fuzzy sets and have undergone evolution in parallel to the developments observed in fuzzy sets themselves. As of now, the area of fuzzy modeling is enormously diversified and capitalizes upon the advances in fuzzy sets as well as impacts methodologies and practices of system modeling. At the same time fuzzy modeling is impacted by the developments occurring in other areas including optimization.

Fuzzy models offered a new way of thinking about system modeling bringing interesting and innovative facets to the area of system modeling. The objective of this study is to position the area of fuzzy modeling in a general framework of system modeling. We also venture into the future directions of fuzzy models and a way in which further developments can be envisioned.

The material is structured into six sections. We start by casting the pursuits of fuzzy models, their design agenda in a certain historical context and identifying the key phases of the evolution of the overall area of fuzzy modeling. This helps us visualize the evolution of the area of fuzzy modeling. Section 3 provides some selected topics of Granular Computing central to fuzzy models. Granular fuzzy models are discussed in Section 4 whereas Section 5 is devoted to granular fuzzy rule – based models and Section 6 includes conclusions.

2. Fuzzy modeling and fuzzy models: a retrospective

Fuzzy models and fuzzy modeling have emerged soon after the inception of the concept of fuzzy sets in 1965. They have undergone a substantial transformation and what we are witnessing today barely resembles the architectures from the beginning of the 1970s. In the retrospect, some essential development phases can be delineated. There are no specific dates of transitions from one phase to another but we rather witness some periods of time over which the transition from one phase to another becomes increasingly visible. The evolution of fuzzy modeling and fuzzy models has been implied by several factors, both internal and external. The internal driving force is inherently associated with the progress occurring in fuzzy sets themselves with the area progressing at the conceptual, theoretical level, which in the sequel has triggered emergence of new architectures, algorithmic constructs, and ensuing applications. With regard to the external factors, one can point at the ongoing progress in the analysis and synthesis of intelligent systems, integration and holistic treatment of existing technologies giving rise to concepts of Computational Intelligence, autonomous agents, and web intelligence, to point at a few examples. In this interaction, one can easily witness its bi-directional and synergistic character. On the one hand, fuzzy sets are influenced by various external developments. On the other hand, they also become an integral and vital contributor to the external areas such as this was evidently visible in the case of Computational Intelligence.

The main periods of fuzzy modeling can be briefly characterized as follows:

Period of early studies and development of fuzzy models
This period has been initiated shortly after fuzzy sets have been introduced to the research community. The models were mostly of prescriptive character where the variables present in system description are described by means of fuzzy sets whereas the relationships among input, state, and output variables are treated as fuzzy relations. All in all, there has been a growing understanding of different roles of fuzzy sets, especially in the formation of fuzzy sets either to describe system variables or being used at the structural level of capturing the dependencies among inputs and outputs in a certain relational format, refer to Figure 1.

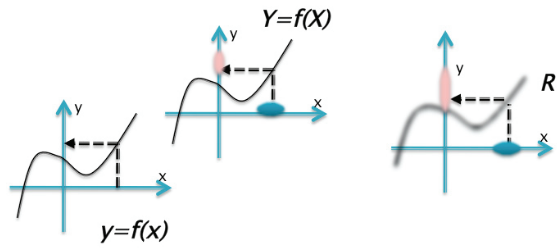


Figure 1. From numeric models to their fuzzy counterparts: a role of fuzzy sets played in describing variables and relationships in system modeling

The period has been predominantly of exploratory character. The ultimate objective was to understand the abilities offered by fuzzy sets as linguistically meaningful and operationally sound entities. These fuzzy models were often referred to as *linguistic* models with the term “fuzzy” being treated as a synonym of linguistic variables with the aid of which fuzzy models are structured. In virtue of the nature of the main direction present in the period, in most situations fuzzy models came in a tabular fashion merely forming a collection of existing (prescribed) relationships existing between fuzzy sets. The models are mostly of prescriptive character.

First structural and identification maturity period I
While in the previous period, fuzzy models augmented existing models by incorporating fuzzy sets in describing input, state, and output variables but being rooted in fuzzy relational equations and fuzzy regression were some of the existing constructs. The mechanisms of identification or parameters estimation techniques matured in this development phase. There has been a well-recognized need to engage data (typically numeric data) in the construction of fuzzy models. As a result, the models became more descriptive constructs adhering to the nature of experimental data.

Second structural and identification maturity period II
In this period, more optimization tools and topologies were involved in the construction of the fuzzy models. Advanced rule-based fuzzy models became more visible and dominant in system modeling. Mechanisms of calibration of fuzzy sets helped enhance the quality of fuzzy models when it comes to their abilities to approximate data. The level of flexibility was substantially elevated by engaging logic-based

constructs of fuzzy sets (including t-norms and t-co-norms).

Computational Intelligence – guided fuzzy modeling
Along with the emergence of Computational Intelligence, there is a visible impact of the methodologies and constructs dominant there on the development and evaluation of fuzzy models. This is particularly visible when dealing with the characterization of fuzzy models as universal approximators. A flurry of hybrid neuro-fuzzy architectures (say, fuzzy cognitive maps, fuzzy neural networks, etc.) and learning schemes associated with them became present. More complex topologies, multilevel architectures directly mimicking neural networks architectures while creatively adopting some terminology of fuzzy sets started to dominate in fuzzy modeling. The focus was on accuracy fuzzy models. Fuzzy models were constructed by guided by their numeric manifestation (defuzzification, decoding) so the numeric nonlinear characteristics play a central role and they are pivotal to the assessment of the quality of the fuzzy models. Interpretation (given that more advanced architectures are considered) became less visible on the agenda of fuzzy modeling.

Fuzzy modeling and fuzzy models with higher order and higher type fuzzy sets
The involvement of fuzzy sets of higher type, in particular type-2 fuzzy sets and interval –valued fuzzy sets have triggered a new direction in fuzzy modeling. A general motivation behind these models relates with the elevated generality of the concepts of fuzzy sets of higher type, which translates into a higher flexibility of type-2 fuzzy models. While this argument is valid, there are a number of ongoing challenges. This concerns an increase of complexity of the development schemes of such fuzzy models. A significantly larger number of their parameters (in comparison with the previously considered fuzzy models) require more elaborate estimation mechanisms. This has immediately resulted in essential optimization challenges (which owing to the engagement of more advance population-based optimization tools have been overcome to some extent but at expense of intensive computing). At the end, type-2 fuzzy models are assessed as numeric constructs with the chain of transformations: type reduction (from type 2- to type-1) followed by defuzzification (reduction from type-1 to type-0 information granules, viz. numbers) thus resulting in a numeric construct.

Ironically, in spite of all significant progress being observed, fuzzy models seem to start losing identity, which was more articulated and visible at the very early days of fuzzy sets. While one may argue otherwise, there is a visible identity crisis: at the end of the day fuzzy models have been predominantly perceived and evaluated as numeric constructs with the quality expressed at numeric level (through accuracy measures).

3. Perspectives of fuzzy modeling: future directions

Obviously, it is difficult, if impossible, to precisely project future directions of development of fuzzy models. Nevertheless one can envision several promising and useful routes.

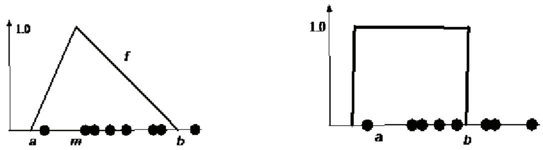
It is apparent that there are no ideal models. Numeric data are not ideally (without any error) captured by any model, no matter how complex such model could be. As usual, recalling the Ockahm's razor principle, we strive to build simple models and establish a crucial balance between accuracy and simplicity requirements. In spite of the diversity of the architectures of the models, especially those emerging in the realm of Computational Intelligence, the apparent challenges remain. An interesting, innovative, and promising direction is to pursue conceptualizing and building models that are formed at the higher level of abstraction and in this way become capable of coping with the system to be modeled. These models are constructed in terms of information granules and in the sequel are referred to as *granular* models. Information granules are formalized in various settings as sets (intervals), fuzzy sets, rough sets, etc. Depending upon the nature of the model, we can talk about granular neural networks, granular regression models, etc.

3.1. Information granules of higher type

By information granules of *higher type* (2^{nd} type and n^{th} type, in general) we mean granules in the description of whose we use information granules rather than numeric entities. For instance, in case of type-2 fuzzy sets we are concerned with information granules- fuzzy sets whose membership functions are granular. As a result, we can talk about interval-valued fuzzy sets, fuzzy fuzzy sets (or fuzzy^2 sets, for brief), probabilistic sets and alike. The grades of belongingness are then regarded as intervals in [0,1], fuzzy sets with support in [0, 1], probability functions truncated to [0,1], etc. In case of type -2 intervals we have intervals whose bounds are not numbers but information granules and as such can be expressed in the form of intervals themselves, fuzzy sets, rough sets or probability density functions. Information granules *of higher order* are those whose description is realized over a universe of discourse whose elements are information granules. In some sense rough sets could be sought as information granules of order-2. Information granules have been encountered in numerous studies reported in the literature; in particular stemming from the area of fuzzy clustering [3][13] in which fuzzy clusters of type-2 have been investigated [4] or they are used to better characterize a structure in the data and could be based upon the existing clusters [10].

3.2. The principle of justifiable granularity

The principle of justifiable granularity [9][11] delivers a comprehensive conceptual and algorithmic setting to develop an information granule. The principle is general as it shows a way of forming information granule without being restricted to certain formalism in which information granularity is expressed and a way experimental evidence using which this information granule comes from. Let us denote one-dimensional numeric data of interest (for which an information granule is to be formed) by $\boldsymbol{Z} = \{z_1, z_2, \ldots, z_N\}$. Denote the largest and the smallest element in $\boldsymbol{Z}$ by $z_{\min}$ and z_{max}, respectively. On a basis of $\boldsymbol{Z}$ we form an information granule A so that it attempts to satisfy two intuitively requirements of coverage and specificity. The first one implies that the information granule is justifiable, viz. it embraces (covers) as many elements of $\boldsymbol{Z}$ as possible. The second one is to assure that the constructed information granule exhibits a well-defined semantics by being specific enough. The specificity sp(A) [14] is related with the size of information granule with the following requirements satisfied: sp($\{z\}$)=1, if $A \subset B$ then sp(A) $\leq$ sp(B) where A and B are two information granules defined in the same space. When constructing a fuzzy set, say a one with a triangular membership function, we start with a numeric representative of $\boldsymbol{Z}$, typically represented as a mean or a modal value (denoted here by m) and then separately determine the lower bound (a) and the upper bound (b). In case of an interval A, we start with a modal value and then determine the lower and upper bound, Figure 2.



(a) (b)

Figure 2. Design of information granules with the use of the principle of justifiable granularity: (a) triangular membership function, (b) interval (characteristic function). The design is realized by moving around the bounds a and b so that a certain optimization criterion is maximized

The construction of the bounds is realized in the same manner for the lower and upper bound so in what follows we describe only a way of optimizing the upper bound (b).

The coverage criterion is expressed as follows

$$cov(A) = \sum_{z_k: z_k \in [m,b]} f(z_k)$$

where f is a decreasing linear portion of the membership function. For an interval form of A the coverage is a count of the number of data included in [m, b],

$$cov(A) = \mathbf{card}\{z_k \mid z_k \in [m,b]\}$$

The above coverage requirement states that we reward the inclusion of z_i in A. The specificity $sp(A)$ is one of those specified in the previous section. As we intend to maximize coverage and specificity and these two criteria are in conflict, an optimal value of b is the one, which maximizes the product of the two requirements

$$Q(b) = cov(A)*sp(A)^{\xi}$$

Furthermore the optimization performance index is augmented by an additional parameter ξ used in the determination of the specificity criterion, sp(A)$^{\xi}$ and assuming non-negative values. It helps control an impact of the specificity in the formation of the information granule. The higher the value of ξ, the more essential is its impact on A. If ξ is set to zero, then the only criterion of interest is the coverage. Higher values of ξ underline the importance of specificity; as a result A is more specific. The result of optimization comes in the form b_{opt} = arg max$_b$ $Q(b)$. The optimization of the lower bound of the fuzzy set (a) is carried out in an analogous way as above, a_{opt} = arg Max $Q(a)$.

4. An emergence of granular models: structural developments

Granular Computing [14][15][16][17] with its coherent framework of representing and processing information granules gives rise to the emergence of granular models. To introduce a concept of granular fuzzy models along with their design strategies, it could be instructive to start with a notion of granular spaces, namely spaces composed of information granules. In the context of fuzzy models we distinguish between granular parameter spaces and granular input spaces. The first category of spaces associates with a construction of granular fuzzy models whereas the second one help establish functioning of fuzzy models in the presence of granular data. There is an option in which both granular parameter and granular input spaces are involved.

4.1. Embedding fuzzy models in granular parameter spaces

The concept of the granular models form a generalization of numeric models no matter what their architecture and a way of their construction are. In this sense, the conceptualization offered here are of general nature. They also hold for any formalism of information granules. A numeric model M_0 constructed on a basis of a collection of training data ($\boldsymbol{x}_k$, target$_k$), $\boldsymbol{x}_k \in \boldsymbol{R}^n$ and $target_k \in \boldsymbol{R}$ comes with a collection of its parameters $\boldsymbol{a}_{\text{opt}}$ where $\boldsymbol{a} \in \boldsymbol{R}^p$. Quite commonly, the estimation of the parameters is realized by minimizing a certain performance index Q (say, a sum of squared error between $target_k$ and $M_0(\boldsymbol{x}_k)$), namely $\boldsymbol{a}_{\text{opt}}= \arg \text{Min}_{\boldsymbol{a}}$ Q($\boldsymbol{a}$). To compensate for inevitable errors of the model (as the values of the index Q are never equal identically to zero), we make the parameters of the model information granules, resulting in a vector of information granules $\boldsymbol{A} = [A_1 \;\; A_2\ldots \; A_p]$ built around original numeric values of the parameters $\boldsymbol{a}$. In other words, the fuzzy model is embedded in the *granular* parameter space. The elements of the vector $\boldsymbol{a}$ are generalized, the model becomes granular and subsequently the results produced by them are information granules. Formally speaking, we have

- granulation of parameters of the model $\boldsymbol{A} = G(\boldsymbol{a})$ where G stands for the mechanisms of forming information granules, viz. building an information granule around the numeric parameter;
- result of the granular model for any $\boldsymbol{x}$ producing the corresponding information granule Y, $Y= M_1(\boldsymbol{x}, \boldsymbol{A}) = G(M_0(\boldsymbol{x}))= M_0(\boldsymbol{x}, G(\boldsymbol{a}))$.

Information granulation is regarded as an essential design asset. By making the results of the model granular (and more abstract in this manner), we realize a better alignment of $G(M_0)$ with the data. Intuitively, we envision that the output of the granular model "covers" the corresponding target. Formally, let cov($target$, Y) denote a certain coverage predicate (either Boolean or multivalued) quantifying an extent to which target is included (covered) in Y.

The design asset is supplied in the form of a certain allowable level of information granularity ε which is a certain non-negative parameter being provided in advance. We allocate (distribute) the design asset across the parameters of the model so that the coverage measure is maximized while the overall level of information granularity serves as a constraint to be satisfied when allocating information granularity across the model, namely $\sum_{i=1}^{p} \varepsilon_i = \varepsilon$. The constraint-based optimization problem reads as follows

$$\max_{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p} \sum_{k=1}^{N} \text{cov}(target_k \in Y_k)$$

subject to

$$\sum_{i=1}^{p} \varepsilon_i = \varepsilon \text{ and } \varepsilon_i \geq 0$$

The monotonicity property of the coverage measure is obvious: the higher the values of ε, the higher the resulting coverage. Hence the coverage is a non-decreasing function of ε.

The underlying idea of the allocation of information granularity can be succinctly displayed in Figure 3.

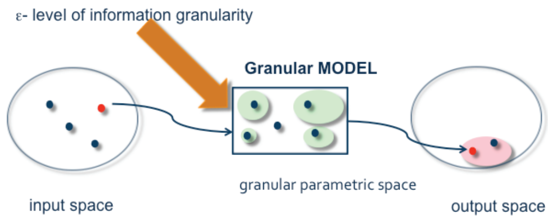
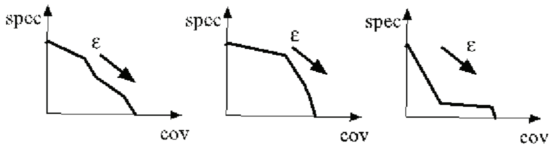


Figure 3. From fuzzy models to granular fuzzy models: a formation of a granular space of parameters

Along with the coverage criterion, one can also consider the specificity of the produced information granules. It is a non-increasing function of ε. The more general form of the optimization problem can be established by engaging the two criteria leading to the two-objective optimization problem. The problem can be re-structured in the following form in which the objective function is a product of the coverage and specificity:

- determine optimal allocation of information granularity [ε_1 ε_2,…, ε_p] so that the coverage and specificity criteria become maximized.

Plotting these two characteristics in the coverage – specificity coordinates offers a useful visual display of the nature of the granular model and possible behavior of the behavior of the granular model as well as the original model. Several illustrative plots shown in Figure 1 illustrate typical changes in the specificity/coverage when changing the values of information granularity ε. One can consider those coming as a result of the maximization of coverage while reporting also the obtained values of the specificity. There are different patterns of the changes between coverage and specificity. The curve may exhibit a monotonic change with regard to the changes in ε and could be approximated by some linear function. There might be some regions of some slow changes of the specificity with the increase of coverage with some points at which there is a substantial drop of the specificity values. A careful inspection of these characteristics helps determine a suitable value of ε – any further increase beyond this limit might not be beneficial as no significant gain of coverage is observed however the drop in the specificity compromises the quality of the granular model. Furthermore Figure 4 highlights an identification of suitable values of the level of information granularity.



(a) (b) (c)

Figure 4. Characteristics of coverage-specificity of granular models: (a) monotonic behavior of the relationship with the changes of ε, (b) increase of coverage and retention of specificity with the increase of ε, (c) rapid drop in specificity for increasing values of ε

The global behavior of the granular model can be assessed in a global fashion by computing an area under curve (AUC) of the coverage-specificity curve present in Figure 1. Obviously, the higher the AUC value, the better the granular model. The AUC value can be treated as an indicator of the global performance of the original numeric model produced when assessing granular constructs built on their basis. For instance, the quality of the original numeric models M_0 and M_0' could differ quite marginally but the corresponding values of their AUC could vary quite substantially by telling apart the models. For instance, two neural networks of quite similar topology may exhibit similar performance however when forming their granular generalizations, those could differ quite substantially in terms of the resulting values of the AUC.

As to the allocation of information granularity, the maximized coverage can be realized with regard to various alternatives as far as the data are concerned: (a) the use of the same training data as originally used in the construction of the model, (b) use the testing data, and (c) usage of some auxiliary data.

4.2. Granular input spaces in fuzzy modeling

The underlying rationale behind emergence of granular input spaces deals with an ability to capture and formalize the problem at the higher level of abstraction by adopting a granular view of the input space in which supporting system modeling and model construction are located. Granulation of input spaces is well motivated and often implied by the computing economy or a flexibility and convenience they offer to they offer when capturing the. Here we would like to highlight some illustrative examples, especially those commonly visible in some temporal or spatial domains.

Temporal domain
Values of time series recorded in several successive time moments are arranged together to form an information granule. Information granules are built over temporal interval or temporal information granules such as fuzzy sets. The size of the temporal segment could be associated with the level (scale) of temporal horizon being of interest in the problem under discussion, say a day, month, etc., see Figure 5. The numeric data falling within some time segment give rise to a single information granule. These information granules are then used later to construct a fuzzy model and use it for e.g., prediction purposes.

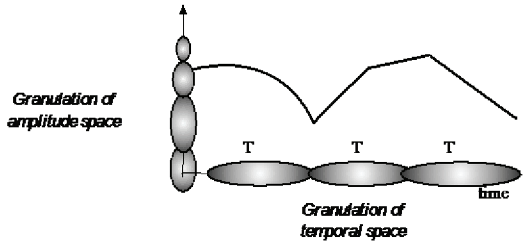


Figure 5. From numeric data of time series to information granules; T - temporal segment

Spatial domain Data located in some region (say, city, county, province) are arranged together and described in the form of a single information granule. In image, identified are regions and pixels present there form an information granule. Further modeling relies on information granules developed at this level. Again, it becomes clear that the number of data has been reduced, which facilitates the design of the fuzzy model and also supports better, more focused and problem-oriented analysis of the results.

One can encounter spatiotemporal information granules formed jointly over segments of time and regions of space. In all these cases, numeric data are transformed into a collection of information granules and result in a certain granular input space (i.e., a space of information granules).

Granular input spaces deliver an important, unique, and efficient design setting for the construction and usage of fuzzy models: (i) information granulation of a large number of data (in case of streams of data) leads to a far smaller and semantically sound entities facilitating and accelerating the design of fuzzy models, and (ii) the results of fuzzy modeling are conveyed at a suitable level of specificity suitable for solving a given problem. In the sequel, information granules used to construct a model, viz. a mapping between input and output information granules.

From the examples provided above, it is apparent that a construction of input granular spaces has to be supported by a sound mechanism of building information granules. The principle of justifiable granularity offers here a sound alternative.

5. Rule-based models – schemes of allocation of information granularity

These functional rules (Takagi-Sugeno format of the conditional statements) link any input space with the corresponding local model whose relevance is confided to the region of the input space determined by the fuzzy set standing in the input space (A_i). The local character of the conclusion makes an overall development of the fuzzy model well justified: we fully adhere to the modular modeling of complex relationships. The local models (conclusions) could vary in their diversity; in particular local models in the form of constant functions (m_i) are of interest

-if $\boldsymbol{x}$ is A_i then y is m_i

These models are then equivalent to those produced by the Mamdani-like rules with a weighted scheme of decoding (defuzzification). There has been a plethora of design approaches to the construction of rule-based models, cf. [1,2,5,6,7,18,19].

Information granularity emerges in fuzzy models in several ways by being present in the condition parts of the rules, their conclusion parts and both. In a concise way, we can describe this in the following way (below the symbol $G(.)$ underlines the granular expansion of the fuzzy set construct abstracted from their detailed numeric realization or a granular expansion of the numeric mapping).

(i) *Information granularity associated with the conditions of the rules*. We consider the rules coming in the format

-if $G(A_i)$ then f_i

where $G(A_i)$ is the information granule forming the condition part of the i-th rule. An example of the rule coming in this format is the one where the condition is described in terms of a certain interval-valued fuzzy set or type-2 fuzzy set, $G(A_i)$.

(ii) *Information granularity associated with the conclusion part of the rules*. Here the rules take on the following form

-if $\boldsymbol{x}$ is A_i then $G(f_i)$

with $G(f_i)$ being the granular local function. The numeric mapping f_i is made more abstract by admitting their parameters being information granules. For instance, instead of the numeric linear function f_i, we consider $G(f_i)$ where $G(f_i)$ is endowed with parameters regarded as intervals or fuzzy numbers. In this way, we have $f_i(A_0, A_1, \ldots, A_n) = A_{i0} + A_{i1}x_1 + \ldots A_{in}x_n$ with the algebraic operations carried out on information granules (in particular adhering to the algebra of fuzzy numbers).

(iii) *Information granularity associated with the condition and conclusion parts of the rules.* This forms a general version of the granular model and subsumes the two situations listed above. The rules read now as follows

-if $G(A_i)$ then $G(f_i)$

The augmented expression for the computations of the output of the model generalizes the expression used in the description of the fuzzy models (3). We have

$$Y = \underset{\oplus}{\sum_{i=1}^{c}} (G(A_i(\boldsymbol{x}) \otimes G(f_i))$$

where the algebraic operations shown in circles $\otimes$ and $\oplus$ reflect that the arguments are information granules instead of numbers (say, fuzzy numbers). The detailed calculations depend upon the formalism of information granules being considered. Let us stress that Y is an information granule. Obviously, the aggregation presented by (9) applies to (i) and (ii) as well; here we have some simplifications of the above stated formula. The two commonly used formalisms already reported in the literature are interval-valued fuzzy sets and type 2 fuzzy sets [14].

The role of fuzzy sets of higher type becomes visible when compressing the originally available collection of fuzzy rules with intent to make the collection of rules easier to interpret. For instance, it is easier to comprehend 10 rules than a collection of 40-50 rules present in the original model. To compensate for their lower number, the reduced rules are made more abstract by admitting information granules in their formation. This is one of the compelling reasons behind the emergence of rule-based models of granular character, refer to Figure 6.

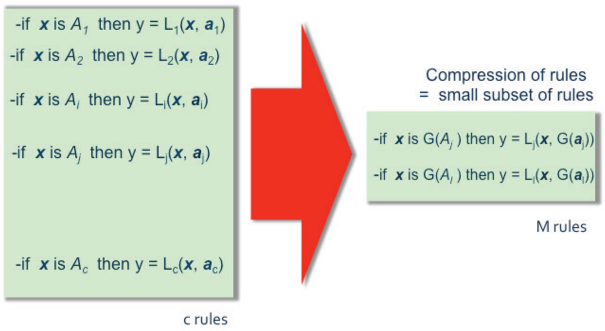


Figure 6. From fuzzy rules to granular fuzzy rules: a process of granular abstraction of rule base resulting as a result of rule compression

6. Conclusions

We have proposed a new direction of granular modeling engaging information granules and effectively resulting in the emergence of granular parameter spaces and granular input spaces. Granular fuzzy models constitute an intuitively appealing generalization of fuzzy models and are developed on their basis. We offered a number of compelling reasons behind the formation of granular counterparts of existing models.

There are a number of promising directions of granular modeling. In particular, one can envision hierarchical granular system modeling. In this mode of modeling, one is engaged in system modeling in a successive way leading to the formation of granular parameters of type-1, type-2 etc. and successively producing models of type M_1, M_2,…. The containment relationship holds in the design of the series of models – starting from M_0 they are developed to enhance the functionality of the successively constructed models by engaging information granules of higher type. Along with the realization of the models, one can also identify potential outliers, which depending at the level of modeling they arise, can be labeled as type-0, type-1, type-2 (granular) outliers.

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