

Cross-docking Location Selection in Distribution Systems: A New Intuitionistic Fuzzy Hierarchical Decision Model

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Abstract

This paper introduces a new intuitionistic fuzzy hierarchical group decision-making (IFHGDM) model for the cross-docking location selection problem. The model is based on intuitionistic fuzzy modified group complex proportional assessment and group order preference by similarity to ideal solution, structuring in a multiple-level hierarchy. A new intuitionistic relative index is then proposed by representing closeness to the ideal points to evaluate and rank the most appropriate cross-dock among a set of potential alternatives for the management application in distribution systems.

Keywords: Distribution systems, cross-docking location selection, hierarchical group decision making, multiple criteria analysis, intuitionistic fuzzy sets

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1. Introduction

Cross-docking is introduced as a new logistics concept by hybridizing intermediate nodes into a distribution system. Through cross-docking, numerous shipments between suppliers and retailers are consolidated so that economies of transportation and deliveries of just in time (JIT) are properly considered¹⁻⁵. The warehousing process in the cross-docking center, including unloading, scanning and sorting, transporting, value adding services and loading, often conduct in less than 48h. In fact, different products are not received and kept for longer periods^{2,4}.

A challenging decision problem from long-term strategic in cross-docking distribution systems concerns the cross-docks location. The issue is to determine the best cross-dock location from a discrete set of candidate locations so that logistics managers can reduce the additional investment costs of redesigning the distribution system. An inappropriate selection of cross-docking center location may result in excessive transportation and handling costs, a shortage of qualified human resource, and inability to meet demand.

There are numerous published papers on warehouse/facility location problem in the last two decades. For instance, Colson and Dorigo⁶ designed a public warehouses selection support system (PWSS) software to assist industrial users for analyzing a classical data based on public warehouses, in which different items of information are provided on each warehouse. Partovi⁷ developed an analytic model for the facility location by providing three tools, including quality function deployment (QFD), analytic hierarchy process (AHP) and analytic network process (ANP), and this study regarded both external and internal evaluation factors according to sustain competitive advantage. Tzeng and Chen⁸ introduced a location model via a fuzzy multi objective approach. The optimization model determines the number and locations of fire stations for an international airport. Chen⁹ considered a multi-criteria decision-making (MCDM) method to solve the problem of distribution center location selection in a fuzzy environment. Chao and Lin¹⁰ presented a two-phase decision method based on the fuzzy AHP to assess advanced quay cranes in container terminals. Vahdani et al.¹¹ combined the reliability model for the facility location in the network design of closed-loop supply chain.

Since the cross-docking center location is an interesting subject in long-term planning, some researchers are recently turning to this concept in the distribution systems, and they try to develop efficient techniques to facilitate the process. Ratliff et al.¹² designed a mixed-integer linear programming model, in which the ideal number and location of cross-docking are obtained in the distribution system. Gue and Kang¹³ introduced a model for staging queues in cross-docking and provided simulation results for several configurations. Bermudez and Cole¹⁴ developed a meta-heuristic approach based on the genetic algorithm for assigning doors within less-than-truck-load break bulk terminals in order to minimize the total weighted travel distance. Murty et al.¹⁵ considered several inter-related decisions made during daily operations at terminals of cross-docking centers. Jayaraman and Ross¹⁶ defined an evaluation problem of the facility location for determining the opening/operating number of cross-docking centers and origin spots with minimizing transportation costs and the fixed costs of the facilities. Chen et al.¹⁷ proposed a local search algorithm for assigning deliveries in the routes of a cross-docking distribution system based on two well-known meta-heuristics, namely simulated annealing and tabu search. Makui et al.¹⁸ provided a multi-objective nonlinear location allocation model to determine the location of cross-docking centers, the capacities and the routing of products through the system. Ross and Jayaraman¹⁹ concentrated on the previous location problem in Jayaraman and Ross¹⁶, and they presented two hybrid meta-heuristics as solving approaches on the basis of the simulated annealing.

The review of the literature in the cross-docking location indicates that the published papers take account of the mathematical modeling and heuristic approaches in this field. Also, the uncertainty issue does not consider in the previous works. In fact, the final values for the assessment of cross-docking as potential alternatives versus conflicting criteria can be fuzzy numbers with the merits of dealing with uncertain conditions. In addition, the conventional approaches to the cross-docking location selection problem tend to be less effective to tackle the imprecise or vague nature of linguistic assessments. In numerous real-life situations, values of the qualitative factors or criteria are often presented imprecisely by logistics decision makers (DMs) or experts. Due to knowledge limitations and

time constraints, the hesitancy often happens where an individual DM tries to solve such a complex logistics decision problem with uncertainty. To cope with the imprecision in information and judgments, the expression of uncertainty and vagueness can be described by the intuitionistic fuzzy set (IFS) that is characterized by three main functions, including membership, non-membership and hesitancy, which was first introduced by Atanassov in 1986²⁰. Some researchers have recommended this modern fuzzy set, and have reported more suitable results than traditional fuzzy sets to represent an individual's judgment and information in the decision-making problems in recent years²¹⁻²⁵. Indeed in real-life logistics applications, the IFS as an appropriate solution with ill-known membership grades and a generalization of the traditional fuzzy set can be properly taken into consideration for representing hesitation of the DMs or professional experts for decision-making problems in distribution systems. Xu²⁶ focused on operational laws of linguistic variables and extended some linguistic aggregation operators under uncertainty, namely LGA, LWGA, LOWGA and LHGA. Xu and Yager²³ studied some geometric aggregation operators by considering the IFSs under uncertainty, namely IFWG, IFOWG, and IFHG. They have introduced intuitionistic fuzzy numbers and the related original mathematical expressions under the intuitionistic fuzzy-environment. Xu and Xia²⁷ presented several induced generalized aggregation operators under the intuitionistic fuzzy-environment. Also, some aggregation operators were extended under uncertainty, such as the induced generalized IF-Choquet integral operators and induced generalized IF-Dempster-Shafer operators. Wei²⁸ introduced two induced intuitionistic fuzzy and interval-valued intuitionistic fuzzy aggregation operators, namely I-IFOWG and I-IIFOWG.

Xu and Yager²⁹ concentrated on dynamic MCDM problems with intuitionistic fuzzy information based on TOPSIS method, and solved complex multi-period decision-making under multiple factors. They introduced some dynamic intuitionistic fuzzy aggregation operators, namely DIFWA and UDIFWA, that could prevent losing the primary intuitionistic fuzzy information, and therefore, properly provide the rationality of these aggregations. Zhang and Xu³⁰ proposed a model based on Pythagorean fuzzy sets (PFSs) based on TOPSIS method to cope with the

uncertainty in real conditions of complex decision-making problems. A score function was defined to recognize Pythagorean fuzzy positive and negative ideal solutions. Then, a revised closeness was developed to obtain the optimal candidate. Also, some new operational laws of the PFSs as powerful fuzzy sets in the study were presented, and their desirable properties were reported. Also, Zhang and Xu³¹ focused on a soft computing approach by considering maximizing consensus and fuzzy TOPSIS method to solve complex MCDM problems by considering interval-valued intuitionistic fuzzy (IVIF)-sets and the group decision making (GDM) process. In this soft computing approach, the values of evaluation criteria utilized the form of IVIF-numbers and weights of criteria were completely known; however, weights of the experts or DMs were partially known or completely unknown. Three main advantages of this soft computing approach were as follows: 1) providing an optimization approach according to the maximizing consensus in order to obtain the weights of experts by defining the consensus index from the viewpoint of the ranking of decision information; (2) developing the conventional TOPSIS method to determine optimal candidates from the viewpoint of magnitude of decision information; and (3) hybridizing the multi-choice goal programming to provide the optimum quantities of the optimal candidates.

The selection problem of cross-docking location in distribution systems can be regarded in multi-criteria analysis framework under an intuitionistic fuzzy-environment; a set of alternatives (i.e., cross-docking locations) are provided to be evaluated versus specific criteria (attributes), including labor costs, travel time between cross-dock and suppliers (retailers), skill levels, availability of labor and government restrictions and policies. In reality, the relative importance of individual experts or DMs against a decision-making criterion is not usually equal. Also, logistics experts can assign importance weights to the identified criteria. Consequently, the selection can be conducted through the group decision-making (GDM) process along with the modern fuzzy sets theory that is affected by the degree of expertise and knowledge of such individual logistics experts or DMs. An effective method for aggregating the various influences of individual judgment, evaluation, and performance rating from several logistics DMs must be considered in the cross-

docking location selection problem involving the imprecision/vagueness inherent in linguistic assessments.

This paper presents a new intuitionistic fuzzy hierarchical group decision-making (IFHGDM) model for the cross-docking location selection problem. The proposed model is based on modified complex proportional assessment (COPRAS) and order preference by similarity to ideal solution (TOPSIS) methods, which are introduced by a group of logistics experts or DMs to tackle cross-docking location decision-making and/or selection problems based on the IFS-theory. The characteristics of the alternatives and conflicting criteria are represented by the IFSs. The proposed IFHGDM model utilizes the truth-membership function and non-truth-membership function to denote and provide the degrees of satisfiability and non-satisfiability of each cross-docking center (alternative) versus a set of criteria, respectively. In addition, it allows the logistics experts or DMs to have two degrees of membership and non-membership for the relative importance of each criterion. In the proposed model, linguistic variables are considered and applied to capture fuzziness in decision information and the GDM process by means of an IFS-decision matrix by considering the hierarchical structure. In the GDM process under uncertainty, an aggregation of the logistics experts' judgments is important to appropriately conduct the evaluation process. Thus, intuitionistic fuzzy weighted averaging (IFWA) operator is employed to aggregate all individual DMs' judgments for the performance rating of the cross-docking centers (alternatives) versus each criterion and the relative importance of the criteria. Then, a new intuitionistic relative index is introduced on the basis of the concept of the closeness to the ideal solutions under an intuitionistic fuzzy-environment. It also overcomes the difficulties arising from the classical COPRAS method in the intuitionistic fuzzy environment. The proposed IFHGDM model provides a useful way to assist the logistics experts or DMs to make his/her decision of cross-docking location in distribution systems. Finally, a real application in chemical industry is presented to report the applicability of the presented IFHGDM model via group decision-making process for the cross-docking location selection in distribution systems under uncertainty.

The main innovations to distinguish efforts of this paper from those already published on the subject in distribution systems are as follows:

- An intuitionistic fuzzy hierarchical analysis is presented to solve the location selection problem of large-sized cross-docking in order to make the solutions of the complex logistics decision problem easy and understandable.
- A relative importance of each cross-dock location as well as a utility degree of each cross-dock location as potential alternative is introduced under an intuitionistic fuzzy environment to obtain the best alternative's ranking among potential alternatives and to indicate, as a percentage, the extent to which one cross-docking center location is better or worse than other location alternatives regarded for the comparison.
- A new intuitionistic fuzzy relative index is introduced for evaluating and ranking the location alternatives of cross-docking centers by representing closeness to the ideal points.
- Proposed IFHGDM model enables the logistics experts or DMs to provide a reduced criterion determining the overall efficiency of each cross-docking center location under an intuitionistic fuzzy environment. Generalized criterion can be directly proportional to an intuitionistic fuzzy relative effect of the values and intuitionistic fuzzy weights of criteria on the efficiency of each cross-docking center location.
- Proposed IFHGDM model has the ability to determine a complete ranking of cross-docking centers under an intuitionistic fuzzy environment by indicating the position of each center location, and the ability to deal with logistics evaluation criteria by considering positive and negative influences and those of quantitative and qualitative natures.

The remainder of this paper is organized as follows. Sections 2 to 4 discuss the definitions and main steps of classical COPRAS method, TOPSIS method and IFSs. The proposed IFHGDM model and the procedure are described in Section 5. Section 6 presents a real application in chemical industry for the cross-docking location selection problem in distribution systems. Conclusions and implications for future research are provided in Section 7.

2. Multiple criteria complex proportional assessment (COPRAS) method

The COPRAS method was first introduced and developed by Zavadskas and Kaklauskas³², which is the method of multiple criteria complex proportional evaluation. It is an effective evaluation approach that tries to rank each alternative described in terms of a number of criteria, their significance and utility degree. In fact, this method selects the best decision by considering both the positive and negative ideal solutions. The positive ideal solution (PIS) is regarded as a solution that makes an attempt to minimize the cost criteria and makes an attempt to maximize the benefit criteria; whereas, the negative ideal solution (NIS) makes an attempt to maximize the cost criteria and makes an attempt to minimize the benefit criteria. The so-called benefit evaluation factors are those for maximization, while the cost evaluation factors are those for the minimization. The best alternative is the first candidate, which is closest to the PIS and farthest from the NIS. The procedure of the complex proportional evaluation method includes the following steps³²:

Step 1. Select an available set of important criteria, which describes alternatives or candidates.

Step 2. Prepare the decision-making matrix (X) for an MCDM problem, in which A_1, A_2, \dots, A_m are m possible alternatives and C_1, C_2, \dots, C_n are n criteria.

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \tag{1}$$

Step 3. Take each relative importance (weight) of factors or criteria q_j .

Step 4. Normalize the decision-making matrix \bar{X} . Normalized values of this matrix are calculated by:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \tag{2}$$

After this step, we have the normalized decision-making matrix as follows:

$$\bar{X} = \begin{bmatrix} \bar{x}_{11} & \bar{x}_{12} & \dots & \bar{x}_{1n} \\ \bar{x}_{21} & \bar{x}_{22} & \dots & \bar{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{x}_{m1} & \bar{x}_{m2} & \dots & \bar{x}_{mn} \end{bmatrix} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \tag{3}$$

Step 5. Compute the weighted normalized decision matrix \hat{X} . The weighted normalized values \hat{x}_{ij} are determined by:

$$\hat{x}_{ij} = \bar{x}_{ij} \cdot q_j \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \tag{4}$$

After this step, we have the weighted normalized decision-making matrix as:

$$\hat{X} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \dots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \dots & \hat{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \dots & \hat{x}_{mn} \end{bmatrix} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \tag{5}$$

Step 6. Compute sums (P_i) of factors or criteria values. The larger values are more preferable for each alternative or candidate as follows:

$$P_i = \sum_{j=1}^u \hat{x}_{ij}. \tag{6}$$

In Eq. (6), u is number of benefit criteria.

Step 7. Compute sums (R_i) of criteria values. The smaller values are more preferable for each alternative or candidate as follows:

$$R_i = \sum_{j=u+1}^n \hat{x}_{ij}. \tag{7}$$

Step 8. Obtain the minimum value of R_i .
 $R_{\min} = \min_i R_i \quad i = 1, 2, \dots, m. \tag{8}$

Step 9. Compute the relative weight of each alternative (Q_i) by:

$$Q_i = P_i + \frac{R_{\min} \sum_{i=1}^m R_i}{R_i \sum_{i=1}^m \frac{R_{\min}}{R_i}}. \tag{9}$$

Step 10. Provide the priority of each alternative or candidate. The greater significance (relative weight of alternative) Q_i , the higher is regarded as the priority of the alternative. In the case of Q_{\max} , the satisfaction degree is regarded as the highest.

Step 11. Compute the utility degree of each alternative or candidate.

$$N_i = \frac{Q_i}{Q_{\max}} 100\%, \quad (10)$$

where Q_i and Q_{\max} are the significance of alternatives given from Eq. (9).

3. Order preference by similarity to ideal solution (TOPSIS) method

The TOPSIS method is first introduced and developed by Hwang and Yoon³³. The PIS is regarded as a solution that tries to minimize the cost criteria and tries to maximize the benefit criteria whereas the NIS tries to maximize the cost criteria and tries to minimize the benefit criteria. The so-called benefit evaluation factors are those for maximization, while the cost evaluation factors are those for minimization. The best alternative is regarded as the first one that could be closest to the PIS and farthest from the NIS.

Suppose that a MCDM problem has m alternatives or candidates (A_1, A_2, \dots, A_m) and n decision criteria (C_1, C_2, \dots, C_n) . Each alternative is evaluated versus n factors or criteria. All the ratings assigned to the alternatives in terms of each criterion form a decision matrix denoted by $X = (x_{ij})_{m \times n}$. Let $W = (w_1, w_2, \dots, w_n)$ be the relative weight vector about the factors or criteria, satisfying $\sum_{j=1}^n w_j = 1$. Then, the main steps of

TOPSIS method can be given as follows:

Step 1. Normalize the decision matrix $X = (x_{ij})_{m \times n}$ by the following relation:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (11)$$

where, r_{ij} is the normalized rating of each factor.

Step 2. Compute the weighted normalized decision matrix $V = (v_{ij})_{m \times n}$.

$$v_{ij} = w_j r_{ij}, i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

where, w_j is the relative importance (weight) of the j -th criterion or factor, and $\sum_{j=1}^n w_j = 1$.

Step 3. Take the positive ideal and negative ideal solutions by:

$$A^* = \{v_1^*, \dots, v_n^*\} = \left\{ \left(\max_i v_{ij} \mid j \in \Omega_b \right), \left(\min_i v_{ij} \mid j \in \Omega_c \right) \right\} \quad (12)$$

and

$$A^- = \{v_1^-, \dots, v_n^-\} = \left\{ \left(\min_i v_{ij} \mid j \in \Omega_b \right), \left(\max_i v_{ij} \mid j \in \Omega_c \right) \right\} \quad (13)$$

where, Ω_b and Ω_c are the sets of benefit evaluation factors and cost evaluation factors, respectively.

Step 4. Compute the Euclidean distances of each alternative from the PIS and the NIS, respectively.

$$D_i^* = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^*)^2}, \quad i = 1, 2, \dots, m \quad (14)$$

and

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, \quad i = 1, 2, \dots, m. \quad (15)$$

Step 5. Compute the relative closeness of each alternative or candidate to the ideal solution. The relative closeness of the alternative A_i in terms of A^* is defined by:

$$RC_i = \frac{D_i^-}{D_i^* + D_i^-}, \quad i = 1, 2, \dots, m \quad (16)$$

Step 6. Rank the alternatives or candidates by considering their relative closeness to the ideal solution. The bigger the RC_i , the better the alternative A_i is. The best alternative is the candidate with the greatest relative closeness to the ideal solution.

4. Intuitionistic fuzzy sets

Let X be a universe of discourse. Concept and definition of traditional fuzzy sets introduced by Zadeh in 1965:

$$F = \{(x, \mu_F(x) | x \in X)\}, \quad (17)$$

whose basic component is regarded as a membership degree $\mu_F(x)$ with the non-membership degree being $1 - \mu_F(x)$. However, in real-life situations when DMs or experts are asked to describe his/her preference degree to an object, there may exist an uncertainty or may consider a hesitation about the degree, and there is no means to include the uncertainty or hesitation in a traditional fuzzy set³⁴. To solve this issue, Atanassov²⁰,²¹ has generalized Zadeh's fuzzy set to the IFS by adding an uncertainty (or hesitation) degree. The IFS is defined as follows. IFS A in a finite set X can be written as³⁵:

$$A = \{(x, \mu_A(x), v_A(x) | x \in X)\}, \quad (18)$$

where $\mu_A(x), v_A(x): X \rightarrow [0,1]$ are membership function and non-membership function, respectively, such that

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \quad (19)$$

A third parameter of the IFS is $\pi_A(x)$, known as the intuitionistic fuzzy index or hesitation degree of whether x belongs to A or not

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x). \quad (20)$$

It is observed that for every $x \in X$:

$$0 \leq \pi_A(x) \leq 1. \quad (21)$$

If the $\pi_A(x)$ is small, knowledge about x is more certain. If $\pi_A(x)$ is great, knowledge about x is more uncertain. Obviously, when $\mu_A(x) = 1 - v_A(x)$ for all elements of the universe, the ordinary fuzzy set concept is recovered³⁶.

Let A and B denote two IFSs of the universe of discourse X , where $A = \{(x, \mu_A(x), v_A(x) | x \in X)\}$, $B = \{(x, \mu_B(x), v_B(x) | x \in X)\}$. Burillo and Bustince³⁷ defined the following relations and expressions:

Definition 1. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$.

Definition 2. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \leq v_B(x)$ for all $x \in X$. In addition, $A \geq B$ if and only if $B \leq A$; $A \succ B$ if and only if $B \leq A$.

Atanassov²¹ and Atanassov et al.²² defined addition and multiplication operations as follows:

Definition 3. Let A and B be two IFSs. Addition operation of them can be defined as:

$$A + B = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), v_A(x) \cdot v_B(x) | x \in X)\}. \quad (22)$$

Definition 4. Let A and B be two IFSs. Multiplication operation between A and B can be defined as:

$$A \cdot B = \{(x, \mu_A(x) \cdot \mu_B(x), v_A(x) + v_B(x) - v_A(x) \cdot v_B(x) | x \in X)\}. \quad (23)$$

Also, Chen³⁸ defined subtraction and division operations as follows:

Definition 5. Let A and B be two IFSs. Subtraction operation between A and B can be defined as:

$$A - B = \left\{ \left(x, \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)}, \frac{v_A(x)}{v_B(x)} \right) | x \in X \right\}. \quad (24)$$

Condition: $A \geq B, \mu_B(x) \neq 1, v_B(x) \neq 0$ and $\mu_A(x) \cdot v_B(x) - \mu_B(x) \cdot v_A(x) \leq v_B(x) - v_A(x)$

Definition 6. Let A and B be two IFSs. Division operation between A and B can be defined as:

$$\frac{A}{B} = \left\{ \left(x, \frac{\mu_A(x)}{\mu_B(x)}, \frac{v_A(x) - v_B(x)}{1 - v_B(x)} \right) | x \in X \right\}. \quad (25)$$

Condition: $A \leq B, \mu_B(x) \neq 0, v_B(x) \neq 1$ and $\mu_A(x) \cdot v_B(x) - \mu_B(x) \cdot v_A(x) \geq \mu_A(x) - \mu_B(x)$

Definition 7. The IFS nA for any positive integer n as follows³⁹:

$$nA = \{(x, \mu_{nA}(x), v_{nA}(x) | x \in X)\}, \quad (26)$$

where $\mu_{nA}(x) = 1 - (1 - \mu_A(x))^n, v_{nA}(x) = [v_A(x)]^n$.

Definition 8. The definition of distance presented in Ref. 40 is as follows:

$$d(A, B) = \frac{1}{n} \sum_{j=1}^n \max\{|\mu_A(x_j) - \mu_B(x_j)|, |v_A(x_j) - v_B(x_j)|\}. \quad (27)$$

Notably that intuitionistic fuzzy numbers and their mathematical expressions have been used in the following section, originally introduced by Xu and Yager²³.

5. Proposed IFHGDM model for the cross-docking location selection problem

In this section, the proposed IFHGDM model is presented for the cross-docking location selection problem in distribution networks. The proposed model is structured in a multiple-level hierarchy under uncertainty by a group of logistics experts or DMs which is based on the COPRAS and TOPSIS methods with the new IFS theory.

5.1. Proposed intuitionistic fuzzy group modified COPRAS

Let A be a set of cross-docking center location (alternatives) and let C be a set of logistics assessment factors or criteria for the cross-docking location decision problem, where

$$A = \{A_1, A_2, \dots, A_m\}, C = \{C_1, C_2, \dots, C_n\}. \quad (28)$$

Assume that the characteristics and features of the cross-docking center location or candidate A_i are given by the IFS shown as below:

$$A_i = \{(C_1, \mu_{i1}, v_{i1}, \pi_{i1}), (C_2, \mu_{i2}, v_{i2}, \pi_{i2}), \dots (C_n, \mu_{in}, v_{in}, \pi_{in})\}, i = 1, 2, \dots, m, \quad (29)$$

where μ_{ij} indicates the degree which the cross-docking center location or candidate A_i satisfies logistics assessment factor or criterion C_j , v_{ij} indicates the degree to which the candidate or alternative A_i does not satisfy factor or criterion, and π_{ij} indicates the intuitionistic fuzzy index or hesitation degree of the alternative A_i versus factor C_j ($(C_j, \mu_{ij}, v_{ij}, \pi_{ij}), i = 1, 2, \dots, m; j = 1, 2, \dots, n$). The procedure of proposed intuitionistic fuzzy group COPRAS method is given for the cross-docking location selection problem as follows:

Step 1. Determine a relative importance or weight of each expert for the cross-docking location selection problem.

Assume that a group of k experts or DMs for the cross-docking center location selection problem. The importance of the DMs described in linguistic terms can be represented in intuitionistic fuzzy numbers.

Let $D_k = [\mu_k, v_k, \pi_k]$ be an intuitionistic fuzzy number to express the performance rating of k th DM. Then the relative importance (weight) of k th DM can be computed as ⁴¹:

$$\lambda_k = \frac{\left(\mu_k + \pi_k \left(\frac{\mu_k}{\mu_k + v_k}\right)\right)}{\sum_{k=1}^l \left(\mu_k + \pi_k \left(\frac{\mu_k}{\mu_k + v_k}\right)\right)}, \quad (30)$$

and $\sum_{k=1}^l \lambda_k = 1$.

Step 2. Construct aggregated intuitionistic fuzzy decision matrix regarding to opinions of the logistics experts or DMs.

Let $R^{(k)} = (r_{ij}^{(k)})_{m \times n}$ be an intuitionistic fuzzy decision matrix of each expert or DM. $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_l\}$ is considered as the relative importance of each DM and $\sum_{k=1}^l \lambda_k = 1, \lambda_k \in [0, 1]$. In the GDM process, all the individual decision opinions need to be combined into group judgments to obtain aggregated intuitionistic fuzzy decision matrix. In order to do that, IFWA operator proposed by Xu ⁴² is employed. $R = (r_{ij})_{m \times n}$, where

$$\begin{aligned} r_{ij} &= IFWA_{\lambda}(r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(l)}) \\ &= \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \dots \oplus \lambda_l r_{ij}^{(l)} \\ &= \left[1 - \prod_{k=1}^l (1 - \mu_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^l (v_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^l (1 - \mu_{ij}^{(k)})^{\lambda_k} - \prod_{k=1}^l (v_{ij}^{(k)})^{\lambda_k}\right]. \end{aligned} \quad (31)$$

Here, $r_{ij} = (\mu_{A_i}(x_j), v_{A_i}(x_j), \pi_{A_i}(x_j)), i = 1, 2, \dots, m; j = 1, 2, \dots, n$.

The aggregated intuitionistic fuzzy decision matrix based on the opinions of logistics experts can be provided as follows:

$$R = \begin{bmatrix} (\mu_{A_1}(x_1), v_{A_1}(x_1), \pi_{A_1}(x_1)) & (\mu_{A_1}(x_2), v_{A_1}(x_2), \pi_{A_1}(x_2)) & \dots & (\mu_{A_1}(x_n), v_{A_1}(x_n), \pi_{A_1}(x_n)) \\ (\mu_{A_2}(x_1), v_{A_2}(x_1), \pi_{A_2}(x_1)) & (\mu_{A_2}(x_2), v_{A_2}(x_2), \pi_{A_2}(x_2)) & \dots & (\mu_{A_2}(x_n), v_{A_2}(x_n), \pi_{A_2}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_m}(x_1), v_{A_m}(x_1), \pi_{A_m}(x_1)) & (\mu_{A_m}(x_2), v_{A_m}(x_2), \pi_{A_m}(x_2)) & \dots & (\mu_{A_m}(x_n), v_{A_m}(x_n), \pi_{A_m}(x_n)) \end{bmatrix}$$

Step 3. Obtain weights of evaluation factors or criteria for the cross-docking location selection problem.

All factors or criteria may not be assumed in the GDM process of the cross-docking location selection problem to be equal importance. W represents a set of grades regarding to the relative importance. In order to calculate W , all judgments of logistics experts for the importance of each evaluation factor or criterion need to be hybridized.

Let $w_j^{(k)} = [\mu_j^{(k)}, v_j^{(k)}, \pi_j^{(k)}]$ be an intuitionistic fuzzy number related to evaluation factor or criterion C_j by the k th DM. Then, according to the IFWA operator the

relative importance of these factors or criteria are calculated as below:

$$w_j = IFWA_{\lambda}(w_j^{(1)}, w_j^{(2)}, \dots, w_j^{(l)})$$

$$= \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \dots \oplus \lambda_l w_j^{(l)}$$

$$= \left[1 - \prod_{k=1}^l (1 - \mu_j^{(k)})^{\lambda_k}, \prod_{k=1}^l (v_j^{(k)})^{\lambda_k}, \prod_{k=1}^l (1 - \mu_j^{(k)})^{\lambda_k} - \prod_{k=1}^l (v_j^{(k)})^{\lambda_k} \right], \quad (32)$$

$$W = [w_1, w_2, \dots, w_j].$$

Here, $w_j = [\mu_j, v_j, \pi_j]$, $j = 1, 2, \dots, n$.

Step 4. Provide aggregated weighted intuitionistic fuzzy decision matrix.

After determining the weights of evaluation factors or criteria (W) and aggregated intuitionistic fuzzy decision matrix for the cross-docking location selection problem, R' as aggregated weighted intuitionistic fuzzy decision matrix is provided as follows:

$$R' = \begin{pmatrix} (\mu_{A_1W}(x_1), v_{A_1W}(x_1), \pi_{A_1W}(x_1)) & (\mu_{A_1W}(x_2), v_{A_1W}(x_2), \pi_{A_1W}(x_2)) & \dots & (\mu_{A_1W}(x_n), v_{A_1W}(x_n), \pi_{A_1W}(x_n)) \\ (\mu_{A_2W}(x_1), v_{A_2W}(x_1), \pi_{A_2W}(x_1)) & (\mu_{A_2W}(x_2), v_{A_2W}(x_2), \pi_{A_2W}(x_2)) & \dots & (\mu_{A_2W}(x_n), v_{A_2W}(x_n), \pi_{A_2W}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_nW}(x_1), v_{A_nW}(x_1), \pi_{A_nW}(x_1)) & (\mu_{A_nW}(x_2), v_{A_nW}(x_2), \pi_{A_nW}(x_2)) & \dots & (\mu_{A_nW}(x_n), v_{A_nW}(x_n), \pi_{A_nW}(x_n)) \end{pmatrix}$$

So that $(\mu_{A_i}(x_j), v_{A_i}(x_j), \pi_{A_i}(x_j)) = r'_{ij}$

Step 5. Calculate values of sums (P_i) for evaluation factors as follows:

$$P_i = \sum_{j=1}^u r'_{ij} \quad (33)$$

In Eq. (33), u donates number of benefit factors or criteria; in the decision-making matrix columns of the cross-docking location selection problem, it is assumed that first of all columns are placed benefit evaluation factors or criteria and ones which cost evaluation factors or criteria are placed after.

Step 6. Calculate values of sums (N_i) for evaluation factors, in which smaller values can be more preferable and suitable for each cross-docking center location (alternative) as follows:

$$N_i = \sum_{j=u+1}^n r'_{ij}. \quad (34)$$

Step 7. Calculate the minimum value of N_i :

$$N_{min} = ((\min_i \mu_{N_i}), (\max_i v_{N_i})). \quad (35)$$

Step 8. Compute the proposed modified intuitionistic fuzzy relative weight of each alternative(Q_i) as follows:

$$Q_i = P_i + \frac{N_i \sum_{i=1}^m (\frac{N_i - N_{min}}{N_i})}{N_{min} \sum_{i=1}^m N_i}. \quad (36)$$

The IFSs have two different concepts unlike conventional fuzzy sets: membership function and non-membership function. With utilizing the operations of the IFS, particularly minus operation through the classical COPRAS to extend a new version of this method in an intuitionistic fuzzy environment, the calculation of the Q_i as the relative weight of each cross-docking center location (alternative) will face some difficulties in the calculations. For instance, this means that the value of R_i/R_{min} according to Eq. (9) in an intuitionistic fuzzy environment leads to unreasonable ranking of the cross-docking center location. In other words, the most utility is obtained for the worst alternative, which is not logical; therefore, the classical relative index in an intuitionistic fuzzy environment has not enough efficiency. Hence, in this paper a new modified intuitionistic fuzzy relative index is introduced under uncertainty that dissolves the above-mentioned difficulties under the intuitionistic fuzzy environment, and affects the ranking of alternatives desirably with respect to the evaluation factors or criteria.

Step 9. Determine the maximum value of Q_i .

$$Q_{max} = ((\max_i \mu_{Q_i}), (\min_i v_{Q_i})). \quad (37)$$

Step 10. Determine the priority of each center location candidate or alternative. The greater significance or relative importance of candidate Q_i , the higher is the priority of the potential center location candidate. In the case of Q_{max} , the satisfaction degree is the highest for the cross-docking location selection problem.

Step 11. Compute the utility degree of each center location candidate or alternative.

$$U_i = \frac{Q_i}{Q_{max}}, \quad (38)$$

where Q_i and Q_{max} are denoted as the significance of center location alternatives, computed by Eqs. (36) and (37).

5.2. Proposed intuitionistic fuzzy group modified TOPSIS

The procedure of proposed intuitionistic fuzzy group TOPSIS method is given for the cross-docking location selection problem as follows. The GDM process of the proposed group TOPSIS method with intuitionistic fuzzy setting can be performed by steps 1 to 4, presented in sub-section 5.1. The next steps are described as below:

Step 5. Provide intuitionistic fuzzy positive-ideal solution and intuitionistic fuzzy negative-ideal solution.

Let J_1 and J_2 be benefit evaluation factors or criteria and cost evaluation factors or criteria, respectively. A^* denotes intuitionistic fuzzy positive-ideal solution and A^- denotes intuitionistic fuzzy negative-ideal solution. Then A^* and A^- are computed by the following relations:

$$A^* = (\mu_{A^*W}(x_j), \nu_{A^*W}(x_j)) \text{ and } A^- = (\mu_{A^-W}(x_j), \nu_{A^-W}(x_j)) \tag{39}$$

where

$$\begin{aligned} \mu_{A^*W}(x_j) &= \left(\left(\max_i \mu_{A_iW}(x_j) \mid j \in J_1 \right), \left(\min_i \mu_{A_iW}(x_j) \mid j \in J_2 \right) \right) \\ \nu_{A^*W}(x_j) &= \left(\left(\min_i \nu_{A_iW}(x_j) \mid j \in J_1 \right), \left(\max_i \nu_{A_iW}(x_j) \mid j \in J_2 \right) \right) \\ \mu_{A^-W}(x_j) &= \left(\left(\min_i \mu_{A_iW}(x_j) \mid j \in J_1 \right), \left(\max_i \mu_{A_iW}(x_j) \mid j \in J_2 \right) \right) \\ \nu_{A^-W}(x_j) &= \left(\left(\max_i \nu_{A_iW}(x_j) \mid j \in J_1 \right), \left(\min_i \nu_{A_iW}(x_j) \mid j \in J_2 \right) \right) \end{aligned}$$

Step 6. Compute the intuitionistic separation measures for the cross-docking location alternatives.

In order to measure separation between center location alternatives on the IFS, distance measures according Definition 8 is employed. After selecting the distance measure, the separation measures, S_{i^+} and S_{i^-} , of each center location candidate or alternative from

intuitionistic fuzzy positive-ideal and negative-ideal solutions are computed. We have:

$$S^+ = \frac{1}{n} \sum_{j=1}^n \max \{ |\mu_{A_iW}(x_j) - \mu_{A^*W}(x_j)|, |\nu_{A_iW}(x_j) - \nu_{A^*W}(x_j)| \} \tag{40}$$

and

$$S^- = \frac{1}{n} \sum_{j=1}^n \max \{ |\mu_{A_iW}(x_j) - \mu_{A^-W}(x_j)|, |\nu_{A_iW}(x_j) - \nu_{A^-W}(x_j)| \} \tag{41}$$

Step 7. Compute the relative closeness coefficient of each cross-docking center location to the intuitionistic ideal solution.

The relative closeness coefficient of each center location candidate A_i versus the intuitionistic fuzzy positive-ideal solution A^* is defined as follows:

$$C_{i^*} = \frac{S_{i^-}}{S_{i^+} + S_{i^-}} \tag{42}$$

where $0 \leq C_{i^*} \leq 1$

Step 8. Rank the potential center location candidates or alternatives.

After obtaining the relative closeness coefficient of each alternative, potential center location alternatives are ranked regarding to descending order of C_{i^*} .

5.3. Proposed intuitionistic fuzzy hierarchical group model

The canonical COPRAS and TOPSIS methods do not regard a hierarchical structure between main criteria and sub-criteria. These methods assess and rank the potential alternatives versus only main factors or criteria with a single level. The proposed IFHGDM model is presented for the cross-docking location selection problem to take the advantages of the hierarchical structure of the AHP method and easiness of implementation for the compromise solution methods (i.e., COPRAS and TOPSIS). The proposed IFHGDM model is easy to implement, and its calculations are faster than canonical MCDM methods to cope with uncertain conditions in distribution systems. Assume that there are n main criteria, m sub-criteria, k candidates or alternatives, and s respondents for the cross-docking location selection problem. Each main criterion has rsc_i sub-criteria where the total number of sub-criteria m is equal to $\sum_{i=0}^n rsc_i$.

The first intuitionistic fuzzy matrix (\tilde{I}_{MC}), provided by Eq. (43), is built regarding to the intuitionistic fuzzy

weights of the main criteria versus the goal of the cross-docking location selection problem.

$$\tilde{I}_{MC} = \begin{matrix} MC_1 \\ MC_2 \\ \vdots \\ MC_p \\ \vdots \\ MC_n \end{matrix} \begin{matrix} Goal \\ w_1 \\ w_2 \\ \vdots \\ w_p \\ \vdots \\ w_n \end{matrix} \quad (43)$$

where w_p is the arithmetic mean of the intuitionistic fuzzy weights assigned by the respondents and is obtained by Eq. (44):

$$w_p = \frac{\sum_{i=1}^s \tilde{q}_{pi}}{s}, \quad p = 1, 2, \dots, n \quad (44)$$

where \tilde{q}_{pi} denotes the intuitionistic fuzzy evaluation score of p th main criterion versus goal evaluated by the i th respondent in the GDM process of the center location selection problem. The second matrix (\tilde{I}_{SC}) represents the relative importance of the sub-criteria versus the main criteria. The intuitionistic fuzzy weights vector provided by \tilde{I}_{MC} are regarded as above this \tilde{I}_{SC} as illustrated in Eq. (45).

$$\tilde{I}_{SC} = \begin{matrix} & \tilde{w}_1 & \tilde{w}_2 & \dots & \tilde{w}_p & \dots & \tilde{w}_n \\ & MC_1 & MC_2 & \dots & MC_p & \dots & MC_n \\ SC_{11} & \tilde{w}_{11} & 0 & \dots & 0 & \dots & 0 \\ SC_{12} & \tilde{w}_{12} & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ SC_{1r_1} & \tilde{w}_{1r_1} & 0 & \dots & 0 & \dots & 0 \\ SC_{21} & 0 & \tilde{w}_{21} & \dots & 0 & \dots & 0 \\ SC_{22} & 0 & \tilde{w}_{22} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ SC_{2r_2} & 0 & \tilde{w}_{2r_2} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ SC_{pl} & 0 & 0 & \dots & \tilde{w}_{pl} & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ SC_{n1} & 0 & 0 & \dots & 0 & \dots & \tilde{w}_{n1} \\ SC_{n2} & 0 & 0 & \dots & 0 & \dots & \tilde{w}_{n2} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ SC_{nr_n} & 0 & 0 & \dots & 0 & \dots & \tilde{w}_{nr_n} \end{matrix} \quad (45)$$

where \tilde{w}_{pl} is the arithmetic mean of the intuitionistic fuzzy weights assigned by the respondents in the GDM process of the center location selection problem; it is computed by Eq. (46).

$$w_{pl} = \frac{\sum_{i=1}^s \tilde{q}_{pli}}{s} \quad (46)$$

where \tilde{q}_{pli} is the weight of i th evaluation sub-criterion versus p th main evaluation criterion evaluated by the i th respondent.

The third matrix (\tilde{I}_A) is established by the scores of the potential center location candidates or alternatives versus the evaluation sub-criteria. The weights vector provided by \tilde{I}_{SC} is presented above this \tilde{I}_A as in Eq. (47).

$$\tilde{I}_{SC} = \begin{matrix} & \tilde{w}_{11} & \tilde{w}_{12} & \dots & \tilde{w}_{1r_1} & \dots & \tilde{w}_{pl} & \dots & \tilde{w}_{nr_n} \\ SC_{11} & SC_{12} & \dots & SC_{1r_1} & \dots & SC_{pl} & \dots & SC_{nr_n} \\ A_1 & \tilde{c}_{111} & \tilde{c}_{112} & \dots & \tilde{c}_{11r_1} & \dots & \tilde{c}_{1pl} & \dots & \tilde{c}_{1nr_n} \\ A_2 & \tilde{c}_{211} & \tilde{c}_{212} & \dots & \tilde{c}_{21r_1} & \dots & \tilde{c}_{2pl} & \dots & \tilde{c}_{2nr_n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ A_q & \tilde{c}_{q11} & \tilde{c}_{q12} & \dots & \tilde{c}_{q1r_1} & \dots & \tilde{c}_{qpl} & \dots & \tilde{c}_{qnr_n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ A_{k'} & \tilde{c}_{k'11} & \tilde{c}_{k'12} & \dots & \tilde{c}_{k'1r_1} & \dots & \tilde{c}_{k'pl} & \dots & \tilde{c}_{k'nr_n} \end{matrix} \quad (47)$$

where

$$\tilde{W}_{pl} = \sum_{j=1}^n \tilde{w}_j \tilde{w}_{pj} \quad (48)$$

Since $\tilde{w}_{pj} = 0$ for $j \neq l$, Eq. (49) is employed instead of Eq. (48):

$$\tilde{W}_{pl} = \tilde{w}_j \tilde{w}_{pj} \quad (49)$$

In \tilde{I}_A , \tilde{c}_{qpl} is the arithmetic mean of the intuitionistic fuzzy scores in the GDM process of the center location selection problem, assigned by the respondents; it is computed by the following relation:

$$\tilde{c}_{qpl} = \frac{\sum_{i=1}^s \tilde{q}_{qpli}}{s}, \quad (50)$$

where \tilde{q}_{qpli} is the intuitionistic fuzzy evaluation score of q th center location alternative versus i th sub-criterion under p th main criterion in the GDM process, assessed by i th respondent.

After obtaining \tilde{c}_{qpl} , the proposed hierarchical structure for the cross-docking location selection problem is provided to be integrated with the proposed intuitionistic fuzzy group modified TOPSIS or the proposed intuitionistic fuzzy group modified COPRAS explained in sub-sections 5.1 and 5.2, respectively.

6. A real application for hierarchical group decision making in cross-docking location selection

In this section, the proposed IFHGDM model is applied to a real cross-docking location selection problem in distribution systems for chemical industry in Iran. The problem of cross-docking location selection involves a

strategic investment decision on long-term planning and business profitability for the chemical company. Selecting the most proper cross-dock among potential alternatives has great importance for logistics managers in the distribution systems. However, it is difficult to select the most suitable center location by considering different features under conflicting evaluation factor or criteria. To assess and select the best cross-dock location, the proposed IFHGDM model is employed in detail.

Studied company is the leading producer of chemical products. Top managers in the company are trying to be pioneer of innovation in the chemical industry. The studied company introduces more than nine types of products and then distributed in several cities. In this paper, we take into account five possible cross-dock locations for the chemical company as new warehousing systems. We do not report more chemical products' details since the chemical company has reserved the information as confidential. Logistics experts of the studied company have participated voluntary for the evaluations based on the proposed IFHGDM model in this research. The interviewees have a minimum of fourteen years of experience in the chemical industry, and include marketing, R&D, and financial managers.

6.1. Implementation

As an initial step, a group of logistics experts or DMs is established for the GDM process of the cross-docking location selection problem under an intuitionistic fuzzy environment (DM₁, DM₂ and DM₃). With a preliminary work, the professional group determines five possible cross-dock locations by considering the needs of the distribution systems. Then, five main evaluation criteria, namely costs (C₁), markets (C₂), government influence (C₃), infrastructure (C₄) and labor resource (C₅) are taken into account in the evaluation and selection process in an intuitionistic fuzzy environment. An overall view of the hierarchy of the GDM problem for the cross-docking location is presented in Fig. 1.

Linguistic variables are utilized for the relative importance of selected evaluation criteria and the DMs for the cross-docking location selection problem as given in Table 1. The importance degree of the logistics experts or DMs and the weight of evaluation factor or criteria are provided for the location decision-making process in Tables 2 to 4, respectively. Then, three DMs express the linguistic variables illustrated in Table 5 in

order to assess the performance rating of five cross-docking centers as potential modern warehousing alternatives versus five selected criteria; their results are presented in Table 6.

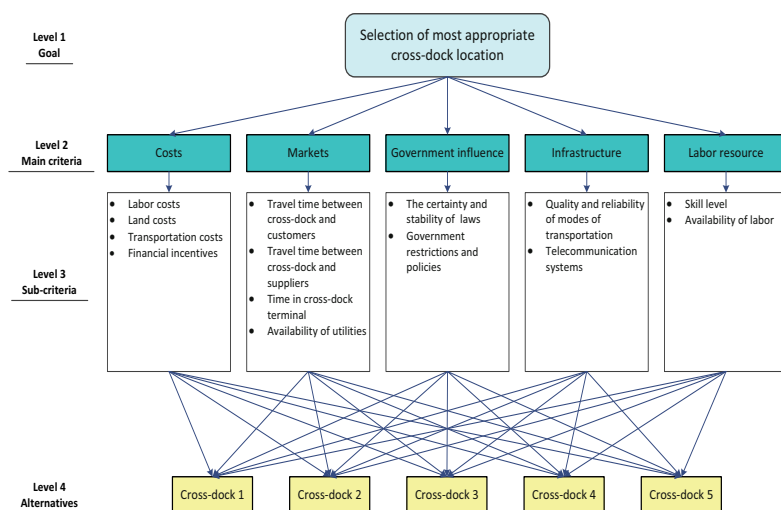


Fig. 1. Decision hierarchy of the cross-docking location selection problem

Table 1. Linguistic variables for rating the importance of criteria and decision makers for the cross-docking location selection problem

Linguistic variables	Intuitionistic fuzzy numbers
Very important (VI)	[0.90, 0.10]
Important (I)	[0.75, 0.20]
Medium (M)	[0.50, 0.45]
Unimportant (UI)	[0.35, 0.60]
Very unimportant (VUI)	[0.10, 0.90]

Table 2. The importance of decision makers for the cross-docking location selection problem

	DM ₁	DM ₂	DM ₃
Linguistic variables	Important	Medium	Very important
Intuitionistic fuzzy numbers	[0.75, 0.20, 0.05]	[0.50, 0.45, 0.05]	[0.90, 0.10, 0]
Weights	0.356	0.238	0.406

Table 3. Weights of the sub-criteria in linguistic variables for the cross-docking location selection problem

DMs Sub-criteria	DM1	DM2	DM3
C1-1	UI	UI	VUI
C1-2	UI	UI	VUI
C1-3	UI	UI	UI
C1-4	M	VUI	UI
C2-1	M	M	UI
C2-2	VI	I	VI
C2-3	UI	UI	M
C2-4	UI	UI	UI
C3-1	VI	VI	I
C3-2	VI	I	VI
C4-1	VI	VI	I
C4-2	VI	VI	VI
C5-1	VUI	UI	UI
C5-2	VI	I	VI

Table 4. Weights of the main criteria and sub-criteria in intuitionistic fuzzy numbers for the cross-docking location selection problem

	Weights of criteria	Weights of sub-criteria	Final weights
C1	[0.2581,0.7074,0.0345]		
C2	[0.3141,0.6474,0.0385]		
C3	[0.8756,0.1178,0.0066]		
C4	[0.8549,0.1325,0.0126]		
C5	[0.2581,0.7074,0.0345]		
C1-1		[0.258,0.707,0.035]	[0.067,0.914,0.019]
C1-2		[0.199,0.779,0.022]	[0.051,0.935,0.014]
C1-3		[0.35,0.6,0.05]	[0.09,0.883,0.027]
C1-4		[0.36,0.596,0.044]	[0.093,0.882,0.025]
C2-1		[0.389,0.56,0.051]	[0.122,0.845,0.033]
C2-2		[0.876,0.118,0.006]	[0.275,0.689,0.036]
C2-3		[0.258,0.707,0.035]	[0.081,0.897,0.022]
C2-4		[0.35,0.6,0.05]	[0.11,0.859,0.031]
C3-1		[0.855,0.133,0.012]	[0.749,0.235,0.016]
C3-2		[0.876,0.118,0.006]	[0.767,0.222,0.011]
C4-1		[0.9,0.1,0]	[0.769,0.219,0.012]
C4-2		[0.876,0.118,0.006]	[0.749,0.235,0.016]
C5-1		[0.27,0.693,0.037]	[0.07,0.91,0.02]
C5-2		[0.861,0.128,0.011]	[0.222,0.745,0.033]

Table 5. Linguistic variables for the performance rating of cross-docking center location alternatives

Linguistic variables	Intuitionistic fuzzy numbers
Extremely good (EG)/extremely high (EH)	[1.00, 0.00]
Very very good (VVG)/very very high (VVH)	[0.90, 0.10]
Very good (VG)/very high (VH)	[0.80, 0.10]
Good (G)/high (H)	[0.70, 0.20]
Medium good (MG)/medium high (MH)	[0.60, 0.30]
Fair (F)/medium (M)	[0.50, 0.40]
Medium bad (MB)/medium low (ML)	[0.40, 0.50]
Bad (B)/low (L)	[0.25, 0.60]
Very bad (VB)/very low (VL)	[0.10, 0.75]
Very very bad (VVB)/very very low (VVL)	[0.10, 0.90]

Table 6. Performance ratings of cross-docking location center alternatives in linguistic variables

Criteria	Alternatives	Decision makers		
		DM ₁	DM ₂	DM ₃
C1-1	A ₁	VVB	VVB	VVB
	A ₂	VVB	VVB	VB
	A ₃	VB	VB	VB
	A ₄	B	VB	VB
	A ₅	VB	VB	VB
C1-2	A ₁	VVB	VVB	VB
	A ₂	B	VB	VVB
	A ₃	VVB	VVB	VB
	A ₄	B	B	VB
	A ₅	VB	VB	VVB
C1-3	A ₁	VVB	VB	VB
	A ₂	VB	B	B
	A ₃	VB	VVB	VB
	A ₄	VB	VB	VB
	A ₅	VVB	VVB	VB
C1-4	A ₁	VB	VB	VB
	A ₂	VB	B	VVB
	A ₃	VB	MB	B
	A ₄	VVB	VVB	VB
	A ₅	VB	VB	VB
C2-1	A ₁	VB	B	VVB
	A ₂	VVB	VVB	VB
	A ₃	VB	VVB	VVB
	A ₄	VB	VB	B
	A ₅	VB	B	VB

C2-2	A_1	MG	G	MG
	A_2	EG	VVG	VG
	A_3	G	VG	VG
	A_4	G	MG	G
	A_5	MG	MG	G
C2-3	A_1	VB	B	VVB
	A_2	VVB	VVB	VB
	A_3	VVB	VVB	VB
	A_4	VB	B	B
	A_5	VB	VB	B
C2-4	A_1	VB	B	VVB
	A_2	VB	B	B
	A_3	VVB	VB	MB
	A_4	B	VB	VB
	A_5	MB	B	VB
C3-1	A_1	F	F	F
	A_2	G	G	VVG
	A_3	MG	MG	G
	A_4	MG	G	MB
	A_5	MB	F	F
C3-2	A_1	F	MB	F
	A_2	MG	VG	VG
	A_3	G	MG	G
	A_4	MB	F	MB
	A_5	F	MB	F
C4-1	A_1	MG	F	MG
	A_2	MG	G	VVG
	A_3	MG	MG	VG
	A_4	G	G	F
	A_5	MG	MG	F
C4-2	A_1	F	MG	MB
	A_2	G	MG	MG
	A_3	MG	VG	F
	A_4	MG	MG	F
	A_5	F	MG	F
C5-1	A_1	MB	B	VB
	A_2	VB	VVB	VVB
	A_3	VB	VB	VB
	A_4	VB	VB	MB
	A_5	B	VB	VB
C5-2	A_1	MG	MG	G
	A_2	VG	VG	EG
	A_3	G	VG	VG
	A_4	MG	F	G
	A_5	G	F	F

After rating each cross-docking center location alternative with respect to each factor or criterion by three DMs, the aggregated intuitionistic fuzzy decision matrix is obtained based on the DMs' judgments in Table 7. The weighted intuitionistic fuzzy decision matrix is then obtained for the cross-docking location selection problem as illustrated in Table 8.

Table 7. Aggregated intuitionistic fuzzy decision matrix for the cross-docking location selection problem

Criteria	Alternatives				
	A_1	A_2	A_3	A_4	A_5
C1-1	[0.1, 0.9, 0]	[0.1, 0.8358, 0.0642]	[0.1, 0.75, 0.15]	[0.1566, 0.6927, 0.1507]	[0.1, 0.75, 0.15]
	C1-2	[0.1, 0.8358, 0.0642]	[0.1566, 0.7459, 0.0975]	[0.1, 0.8358, 0.0642]	[0.1924, 0.6569, 0.1507]
C1-3		[0.1, 0.8003, 0.0997]	[0.1997, 0.6497, 0.1506]	[0.1, 0.7832, 0.1168]	[0.1, 0.75, 0.15]
	C1-4	[0.1, 0.75, 0.15]	[0.1381, 0.766, 0.0959]	[0.241, 0.6221, 0.1369]	[0.1, 0.8358, 0.0642]
C2-1		[0.1381, 0.766, 0.0959]	[0.1, 0.8358, 0.0642]	[0.1, 0.8434, 0.0566]	[0.1642, 0.685, 0.1508]
	C2-2	[0.6264, 0.2725, 0.1011]	[1,0,0]	[0.7689, 0.128, 0.1031]	[0.6788, 0.2202, 0.101]
C2-3		[0.1381, 0.766, 0.0959]	[0.1, 0.8358, 0.0642]	[0.1, 0.8358, 0.0642]	[0.1997, 0.6497, 0.1506]
	C2-4	[0.1381, 0.766, 0.0959]	[0.1997, 0.6497, 0.1506]	[0.2367, 0.6788, 0.0845]	[0.1566, 0.6927, 0.1507]
C3-1		[0.5, 0.4, 0.1]	[0.808, 0.1509, 0.0411]	[0.6441, 0.2544, 0.1015]	[0.5595, 0.3353, 0.1052]
	C3-2	[0.4779, 0.4218, 0.1003]	[0.744, 0.1479, 0.1081]	[0.6788, 0.2202, 0.101]	[0.4254, 0.4742, 0.1004]
C4-1		[0.5782, 0.3212, 0.1006]	[0.7873, 0.1744, 0.0383]	[0.6982, 0.192, 0.1098]	[0.6308, 0.265, 0.1042]
	C4-2	[0.4894, 0.409, 0.1016]	[0.639, 0.2596, 0.1014]	[0.6285, 0.2597, 0.1118]	[0.5621, 0.3372, 0.1007]
C5-1		[0.2541, 0.6156, 0.1303]	[0.1, 0.8434, 0.0566]	[0.1, 0.75, 0.15]	[0.2367, 0.6361, 0.1272]
	C5-2	[0.6441, 0.2544, 0.1015]	[1,0,0]	[0.7689, 0.128, 0.1031]	[0.6247, 0.2724, 0.1029]

Table 8. Weighted intuitionistic fuzzy decision matrix for the cross-docking location selection problem

Criteria	Alternatives				
	A_1	A_2	A_3	A_4	A_5
C1-1	[0.007,0.991, 0.002]	[0.007,0.986, 0.007]	[0.007,0.979, 0.014]	[0.01,0.974, 0.016]	[0.007,0.979, 0.014]
C1-2	[0.005,0.989, 0.006]	[0.008,0.984, 0.018]	[0.005,0.989, 0.006]	[0.01,0.978, 0.012]	[0.005,0.988, 0.007]
C1-3	[0.009,0.977, 0.014]	[0.018,0.959, 0.023]	[0.009,0.975, 0.016]	[0.009,0.971, 0.02]	[0.009,0.981, 0.01]
C1-4	[0.009,0.97, 0.021]	[0.013,0.972, 0.015]	[0.022,0.955, 0.023]	[0.009,0.981, 0.01]	[0.009,0.97, 0.021]
C2-1	[0.017,0.964, 0.019]	[0.012,0.975, 0.013]	[0.012,0.976, 0.012]	[0.02,0.951, 0.029]	[0.017,0.955, 0.028]
C2-2	[0.172,0.774, 0.054]	[0.275,0.689, 0.036]	[0.212,0.729, 0.059]	[0.187,0.758, 0.055]	[0.177,0.768, 0.055]
C2-3	[0.011,0.976, 0.013]	[0.008,0.983, 0.009]	[0.008,0.983, 0.009]	[0.016,0.964, 0.02]	[0.013,0.968, 0.019]
C2-4	[0.015,0.967, 0.018]	[0.022,0.951, 0.027]	[0.026,0.955, 0.019]	[0.017,0.957, 0.026]	[0.028,0.946, 0.026]
C3-1	[0.374,0.541, 0.085]	[0.605,0.35, 0.045]	[0.482,0.429, 0.089]	[0.419,0.491, 0.09]	[0.349,0.566, 0.085]
C3-2	[0.366,0.55, 0.084]	[0.57,0.337, 0.093]	[0.521,0.393, 0.086]	[0.326,0.591, 0.083]	[0.366,0.55, 0.084]
C4-1	[0.445,0.47, 0.085]	[0.606,0.355, 0.039]	[0.537,0.369, 0.094]	[0.485,0.426, 0.089]	[0.3,0.483, 0.218]
C4-2	[0.366,0.548, 0.086]	[0.478,0.433, 0.089]	[0.471,0.434, 0.095]	[0.421,0.493, 0.086]	[0.394,0.521, 0.085]
C5-1	[0.018,0.966, 0.016]	[0.007,0.986, 0.007]	[0.007,0.978, 0.015]	[0.017,0.967, 0.016]	[0.011,0.972, 0.017]
C5-2	[0.143,0.81, 0.047]	[0.222,0.745, 0.033]	[0.171,0.778, 0.051]	[0.139,0.814, 0.047]	[0.130,0.825, 0.046]

Table 9 shows the computational results by the proposed intuitionistic fuzzy group modified COPRAS by considering the P_i , N_i , the modified relative weight of each cross-docking center location alternative (Q_i) and U_i for each alternative versus selected evaluation criteria. The ranking of five cross-docking center location alternatives is finally taken based on Q_i as follows:

$$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5.$$

Table 9. Computational results of multiple criteria analysis in the cross-docking location problem by the intuitionistic fuzzy modified COPRAS

Alter-natives	P_i	N_i	Q_i	U_i	Ran-king
A_1	[0.9011, 0.0480, 0.0508]	[0.0876, 0.8163, 0.0960]	[0.9494, 0.0165, 0.0340]	[0.9587, 0.0134, 0.0277]	4
A_2	[0.9803, 0.0093, 0.0103]	[0.0911, 0.8119, 0.0969]	[0.9903, 0.0030, 0.0066]	[1,0,0]	1
A_3	[0.9602, 0.0153, 0.0244]	[0.0928, 0.8070, 0.1001]	[0.9808, 0.0047, 0.0144]	[0.9903, 0.0017, 0.0079]	2
A_4	[0.9182, 0.0376, 0.0441]	[0.1036, 0.7689, 0.1273]	[0.9655, 0.0065, 0.0279]	[0.9749, 0.0035, 0.0215]	3
A_5	[0.8746, 0.0495, 0.0758]	[0.0951, 0.7818, 0.1229]	[0.9411, 0.0109, 0.0478]	[0.9503, 0.0079, 0.0417]	5

The above-mentioned results for the cross-docking location selection problem is also obtained by the proposed intuitionistic fuzzy group modified TOPSIS. The computational results are provided in Table 10.

According to Tables 9 and 10, the best cross-docking center location alternative is A_2 , and its utility degree and relative closeness coefficient have the highest values. It means that the needs of the DMs and the logistics managers are satisfied the best. In fact, the second cross-docking center location for the chemical company assessed by using the proposed IFHGDM model, based intuitionistic fuzzy group modified TOPSIS and COPRAS methods, is more desirable to the DMs' predefined objectives and is better than other available cross-docking center locations. The logistics managers in the chemical distribution systems also adjust choices in accordance with their knowledge, experience and preference by taking into consideration the acquired results.

Table 10. Computational results of multiple criteria analysis in the cross-docking location problem by the intuitionistic fuzzy modified TOPSIS

Alternatives	S^+	S^-	C_i^*	Preference order ranking
A_1	0.068	0.024	0.259	4
A_2	0.004	0.088	0.954	1
A_3	0.031	0.062	0.668	2
A_4	0.065	0.028	0.302	3
A_5	0.082	0.01	0.109	5

6.2. Discussion of results

The computational results illustrate that the proposed IFHGDM model based intuitionistic fuzzy group COPRAS and TOPSIS is able to solve the complex multi-criteria group problem for evaluating and ranking the cross-docking center location alternatives. The proposed IFHGDM can provide a new applicable framework to utilize the MCDM within cross-docking location selection problems under uncertain environment by a group of logistics experts. The model takes the advantages of the intuitionistic fuzzy logic and gives a systematic approach under uncertain conditions. Proposed IFHGDM model introduces a new intuitionistic fuzzy relative index based on the measure of closeness to the ideal solution, and avoids the difficulties and errors arising from the extension of classical COPRAS method in the intuitionistic fuzzy environment. Also, the proposed IFHGDM model can evaluate and rank the potential cross-docking center location alternatives based on a relative closeness to the intuitionistic ideal solution.

A comparative analysis is performed between the proposed IFHGDM models based on intuitionistic fuzzy group modified COPRAS method and intuitionistic fuzzy group modified TOPSIS method. Computational results of the intuitionistic fuzzy group TOPSIS are given in Table 10, according to the separation measures and relative closeness coefficient of each cross-docking center location alternative. By considering Tables 9 and 10, it is observed that the ranking of five cross-docking center location alternatives versus five selected criteria are the same, where A_2 is the first rank and A_5 is the fifth rank in the location selection problem for the cross-docking distribution systems under multiple criteria. Proposed intuitionistic fuzzy group modified COPRAS and TOPSIS methods have relatively simple computations and straightforward under uncertain conditions in order to select the best center location alternative in the complex group decision-making process. Both methods can handle multiple conflicting criteria properly. Two intuitionistic fuzzy modified group decision-making methods are based on the concept of ideal solutions with intuitionistic fuzzy setting. They evaluate and select the logistics decisions by considering both the fuzzy-positive and negative-ideal solutions. The compromise solution obtained by these methods can help the logistics managers or DMs to reach a final decision for a complex decision-making

and/or selection problem versus conflicting evaluation factors. The compromise solution provided by these two methods is an appropriate solution, which could simultaneously satisfy the closest to the intuitionistic fuzzy positive-ideal solution and the farthest from the intuitionistic fuzzy negative-ideal solution. The logistics experts or DMs can easily utilize the proposed intuitionistic fuzzy group modified COPRAS and TOPSIS methods in a multiple-level hierarchy to assess the center location alternatives and can easily make the best decision in an intuitionistic fuzzy environment. Finally, the comparison among previous methods^{9, 33, 43} and the proposed IFHGDM model is summarized in Table 11.

Table 11. The comparison among the proposed IFHGDM model and the previous decision methods

Characteristics	COPRAS ⁴³	TOPSIS ³³	Fuzzy TOPSIS ⁹	Proposed IFHGDM model
Multiple hierarchical structure				√
Membership function to incorporate the uncertainty			√	√
Hesitancy function to incorporate the full uncertainty				√
Evaluating and ranking algorithm	√	√	√	√
Providing objective criteria' weights			√	√
Possibility of graphical interpretation	√			√
Simple computations	√	√	√	√

7. Conclusion

The problem of selecting cross-docking location can be considered as one of the important decision in the logistics management. The decision can be effectively

made under uncertain conditions within the multiple criteria analysis framework. Complex group decision making by considering the multi-criteria copes with uncertain and insufficient information, and the modern fuzzy sets theory is suitable way to deal with the conditions of cross-docking distribution systems. Being a generalization of the fuzzy set, the intuitionistic fuzzy set (IFS) allows logistics experts or decision makers (DMs) to provide an additional possibility in order to represent imperfect knowledge by two functions of truth-membership and non-truth-membership in order to simultaneously indicate the degrees of satisfiability and non-satisfiability. This paper presents a new intuitionistic fuzzy hierarchical group decision-making (IFHGDM) model for the alternative' evaluating and ranking of the cross-docking center location in the complex logistics decision-making and/or selection problems for the distribution systems. The IFHGDM model is based on the proposed intuitionistic fuzzy group modified COPRAS and TOPSIS methods. This new intuitionistic fuzzy model in a multiple-level hierarchy through the group decision-making process is more adequate to deal with uncertainty than conventional approaches in the cross-docking networks. In the complex assessment process, the rating of each cross-docking center location alternative with respect to each evaluation factor or criterion and the weight of each factor or criterion are linguistic variables as characterized by intuitionistic fuzzy numbers. First, intuitionistic fuzzy weighted averaging (IFWA) operator is utilized to aggregate judgments of logistics experts or DMs. Second, a new intuitionistic fuzzy relative index is presented based on the incorporated fuzzy approach and concepts of positive-ideal and negative-ideal solutions in the distribution systems to solve the group decision-making problem of cross-docking distribution systems. It avoids the difficulties and errors arising from the extension of classical COPRAS method in the intuitionistic fuzzy environment. Then, the utility degree of each center location alternative under uncertainty is obtained. The proposed IFHGDM model can satisfy the closest to the intuitionistic fuzzy positive-ideal solution and the farthest from the intuitionistic fuzzy negative-ideal solution, and finally the center location alternatives are ranked based on the main intuitionistic fuzzy operations. Finally, the intuitionistic fuzzy hierarchical analysis is utilized to solve the large-sized cross-docking location

selection problem in order to make the solutions of the problem easy and understandable. Furthermore, an attempt has been made to explore the applicability and capability of proposed IFHGDM model in the chemical distribution systems. A real application in chemical industry is presented through the cross-docking location selection problem for the multi-echelon distribution network. The IFHGDM model leads to an indisputable preference order through the method of multiple criteria group complex proportional evaluation and group order preference by similarity to ideal solution. For future research, the proposed fuzzy hierarchical group decision-making model can be developed to handle uncertain conditions, (1) based on Xu and Yager²⁹ study to solve dynamic logistics decision problems in distribution networks with intuitionistic fuzzy information, (2) based on Zhang and Xu³⁰ study by considering Pythagorean fuzzy sets (PFSs) as new powerful fuzzy sets, and finally, (3) based on Zhang and Xu³¹ study to take advantages of an optimization approach according to the maximizing consensus to obtain weights of logistics experts or managers, determining the optimal candidates from the viewpoint of magnitude of decision information, and hybridizing the multi-choice goal programming with interval-valued intuitionistic fuzzy (IVIF) sets.

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