

## ***Decerns: A Framework for Multi-Criteria Decision Analysis***

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### **Abstract**

A new framework, *Decerns*, for multicriteria decision analysis (MCDA) of a wide range of practical problems on risk management is introduced. *Decerns* framework contains a library of modules that are the basis for two scalable systems: *DecernsMCDA* for analysis of multicriteria problems, and *DecernsSDSS* for multicriteria analysis of spatial options. *DecernsMCDA* includes well known MCDA methods and original methods for uncertainty treatment based on probabilistic approaches and fuzzy numbers. These MCDA methods are described along with a case study on analysis of multicriteria location problem.

**Keywords:** Multi-Criteria Decision Analysis; MCDA systems; Spatial Decision Support Systems; Land-use Planning; Risk Management; Uncertainty analysis.

### **1. Introduction**

Decision making on risk management, land-use planning, and many other scientific and practical problems is usually influenced by multiple potential alternative solutions and multiple factors/criteria for their evaluation. It is virtually impossible to obtain an

optimal/trade-off solution while meeting requirements for transparency without the use of decision analysis (DA). DA methods are used to review and assess value tradeoffs and identify the “best alternative(s)” from a group of policy options<sup>[1,2]</sup>. Decision Support Systems, DSSs, are particularly helpful in overcoming the limitations of ad hoc decision making<sup>[2-4]</sup>.

The term “Decision Support” is considered as an assistance for, and substantiation and corroboration of, an act or result of deciding. Typically, such a decision is directed towards determination of an “optimal” solution for problems that contain multiple criteria with multiple potential solutions. These types of problems lead naturally to the use of decision analysis tools that can evaluate the trade-off between different solutions. Decision support integrates specific objective and subjective information about a problem and general information such as legislation, guidelines, and technical expertise, to produce decision-making knowledge in a way that is transparent, consistent and reproducible.

A DSS is a tool designed specifically for supporting the users in addressing semi-structured problems<sup>[2]</sup>. An extended approach to defining DSS<sup>[5]</sup>, suggests that DSSs should meet the following requirements:

- Designed to solve semi-structured problems that upper level managers often face;
- Capable of combining analytical models with traditional data storage and retrieval functions;
- User-friendly and accessible by decision makers with minimal computer experience; and
- Flexible and adaptable to different decision-making approaches.

One branch of decision analysis that is used in decision support process and in a variety of DSSs includes Multi-Criteria Decision Analysis (MCDA)<sup>[1,6-8]</sup>. In this class of methods, MCDA affords its users with the ability to evaluate a variety of factors important to a given policy decision against the identified policy solutions in order to ascertain which of these solutions is the “optimal” for a given context. MCDA allows for the integration of quantitative and qualitative information simultaneously, and is suitable for treatment and analysis of uncertainty of objective data or/and subjective judgments<sup>[7-9]</sup>.

Spatial DSSs, SDSSs, are intended to solve semi-structured *spatial* problems<sup>[10,11]</sup>. SDSSs, in general, integrate a Geographic Information System (GIS), at least in terms of basic functions, and MCDA tools. The general approach to creating SDSSs based on integrating GIS and MCDA functions and decision rules has been presented in publications<sup>[11-16]</sup>. The history and methodology of SDSSs development and

examples of their practical implementation has also been discussed in<sup>[4,16-24]</sup>.

The main goal of this paper is to present the *Decerns* framework (section 3) focusing on the MCDA system, *DecernsMCDA* (section 5), and its implementation for analysis of a case study on comprehensive analysis of a multicriteria problem (sections 6). To provide context for the strengths and limitations of the *DecernsMCDA* system, a short description of MCDA goals (section 2) as well as a survey of MCDA software (section 4) is also provided.

## 2. MCDA goals and tasks

The central aim of MCDA is to enhance a decision maker learning and understanding of a particular decision problem in the context of their own organizational preferences, values, and objectives. In this framework, the overall goal of MCDA methods is to evaluate alternatives based on multiple criteria using systematic analyses which overcome the limitations of unstructured individual or group decision-making<sup>[1,7]</sup>. Specifically, MCDA is directed to improve the decision making process by accomplishing the following goals:

- the integration of the objective values with subjective judgments;
- the management of the decision-making process which is based on objective and subjective values; and
- the promotion of transparency of all the significant steps within the analysis of the multicriteria problem.

The general MCDA process is presented in Fig.1, including elements of decision problem definition, decision structuring, and relevant model selection. The scheme in Fig.1 may be considered as an implementation of the MCDA process adjusted to the use of a decision assistance system/framework (e.g., *DecernsMCDA*) with the user having the choice among several MCDA methods based upon the availability of certain types of information, subject experts, or desired decision output.

The flowchart in Fig.1 begins by defining the decision problem at hand and discussing the key goals that are required to be resolved in such a decision problem. The next step includes formally structuring the multicriteria problem, including an enumeration, often preliminary, of the identified decision alternatives

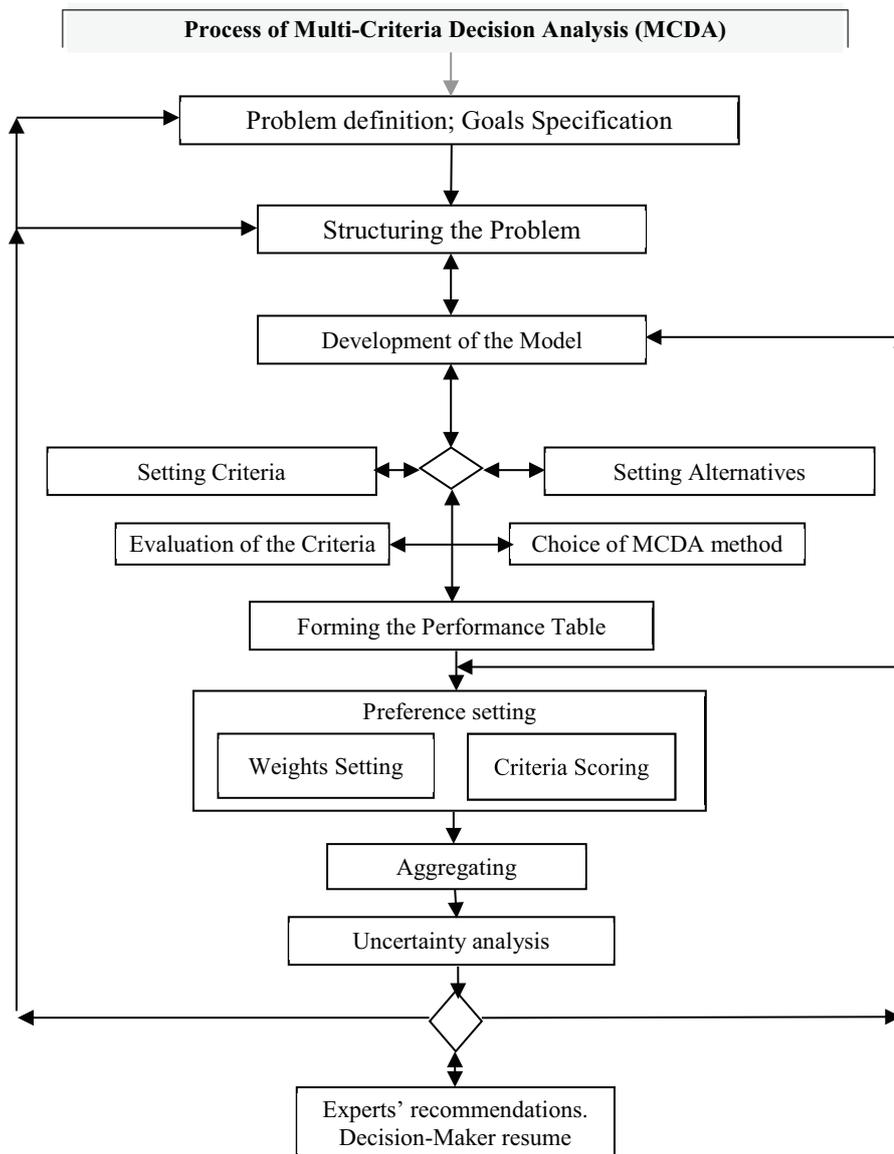


Fig.1. Aggregated scheme of the MCDA process.

and criteria by which alternatives are assessed. Based on this early model structure, the model development for multicriteria decision analysis is implemented for a particular MCDA method.

In many schemes of the MCDA process, the step “Setting Criteria” is placed after “Setting Alternatives”. In fact, this is an example of the “Alternatives-based approach” for multicriteria problem analysis<sup>[1]</sup>: several alternatives are presented for consideration, and then the criteria are selected for their analysis (e.g., for

ranking alternatives or choosing the “best” one). In other cases, however, the “Criterion based approach” can be implemented<sup>[1]</sup> in a manner where the criteria are considered for reaching the goal(s), and then alternatives are formed (e.g., several criteria are considered for analysis of a location problem, and then suitable alternatives for choosing the “best” one are formed<sup>[20]</sup>). Based upon these comments, the bars “Setting Criteria” and “Setting Alternatives” are placed on the same level to correspond to the general case.

In Fig.1, the step “Choice of MCDA method” is indicated on the same level with “Evaluation of the Criteria” (as distinct from other schemes, where the choice of MCDA method is considered implicitly or is implied at the step “Aggregating”). If experts estimate the criterion values for alternatives under consideration, they can consider, e.g., only “mean criterion values” with subsequent implementation of an appropriate method, for example, MAVT (see the section 5.1.1 below). In the cases with significant range of criterion values (uncertainties in values), experts can choose a method for uncertainty treatment. This may include a formal analysis with a probabilistic distribution of parameters or the use of fuzzy numbers. For implementation of the method for uncertainty treatment experts may obtain the corresponding uncertainty of criterion values.

The next step uses the criterion values and generates the Performance Table (that includes assessments of all the criteria for given alternatives), Fig.1, as well as implementation of the specific approaches to the weighting process (choice of a weighting method and setting weights) and scoring (e.g., defining the suitable partial value functions).

“Uncertainty analysis” is considered after aggregation of the criterion values in accordance with the chosen MCDA method. This step incorporates both uncertainty of weights and uncertainty of criterion values/scores (section 5), and reflects the reality of uncertainties in any decision problem. Uncertainty analysis allows the MCDA practitioner to determine the sensitivity of the selection of the “optimal” solution or ranking options to a range of values for each criterion and/or criterion weights. If the obtained solution is not robust, or vice versa, if it is robust and experts want to give additional arguments for Decision-Maker concerning this solution, a new decision model may be considered based on implementation of another MCDA method.

The decision analyst serves as a participant of a decision-making process, where the decision model can be continually updated and refined to include additional objective data, expert information/judgments, or an expansion of the model’s criteria or alternatives altogether.

The following are the main categories of problems that are considered to be well handled through MCDA<sup>[7]</sup>.

- *screening* alternatives – a process of eliminating those alternatives that do not appear to warrant further attention, i.e., selecting a smaller set of alternatives that likely contains the “best”/trade-off alternative;
- *sorting* alternatives into classes/categories (e.g., “unacceptable”, “possibly acceptable”, “definitely acceptable”, etc.);
- *choice /selection* – finding “the most preferred alternative” from a given set of alternatives; and
- *ranking* alternatives (from “best” to “worst” according to a chosen algorithm);
- *designing* (searching, identifying, creating) a new action/alternative to meet goals.

There also exist other categories of MCDA problems, including a portfolio problem where a choice of a subset of alternatives exists, and the decision problem is to identify the optimal subset of alternatives to meet a desired end<sup>[27,28]</sup>.

Three major categories of MCDA problems can be distinguished<sup>[7, 16]</sup>:

- multi-attribute decision making, MADM, - a finite number of alternatives which are defined explicitly, versus multi-objective decision making, MODM, - infinite or large number of alternatives which are defined, as a rule, implicitly;
- individual versus group decision making; and
- decisions under relative certainty versus decisions under uncertainty.

### 3. *Decerns* framework

Development of the *Decerns* framework has been started within the international project DECERNS (*Decision Evaluation in Complex Risk Network Systems*). The main goal of this project was development of the models and computer tools for decision-making support on *risk management* and *land-use planning* problems within remediation of contaminated territories<sup>[25]</sup>. An early prototype of the *Decerns* system, *DECERNS SDSS*, was implemented for multicriteria decision analysis on land-use planning and risk management with the use of GIS-functions and one MCDA method<sup>[20,26]</sup>.

This paper presents the *Decerns* framework - a scalable platform that contains different MCDA methods and tools for decision support when solving

multicriteria problems. A generalized scheme of the *Decerns* as a framework for decision support on risk management and land-use planning problems with the use of multicriteria DSSs is presented in Fig.2. This is the most general form of *Decerns* and applies to multicriteria analysis of spatial and non-spatial problems.

*DecernsSDSS* is a web application (Web-SDSS) built according to Java EE 6 specifications. It has three-tier architecture: presentation, the application processing, and data management. A brief description of each level of *DecernsSDSS* is presented below.

- *Presentation tier* implements the user interface. The *DecernsSDSS* user interface facilitates all of the operations connected with SDSS project creation or loading, implementation of GIS and/or MCDA functions, methods and tools, preparation of the output forms and saving the project for subsequent documentation or modification.
- *Application/logic tier* coordinates the applications, processes commands, including various functions and logic transitions, and performs calculations. It also moves and processes data between the two surrounding tiers. The main components of SDSS in this layer are
  - *GIS module/subsystem*, which includes the main GIS functions for spatial data loading, representation, editing, analysis, and storing;
  - *MCDA module/subsystem*, which includes different MCDA methods, and tools for

integration of DA and GIS (creation of spatial alternatives, data transfer from GIS to MCDA modules and vice versa);

- *Models manager* includes models integration components - an additional component that is not a part of the basic (general) version of *DecernsSDSS*. Models manager implements problem-specific models for assessing risk and other criteria. The outputs of these models are integrated with the GIS system and database. The Models Manager component, along with corresponding problem specific data, forms a customized problem-specific and site-specific version of *DecernsSDSS*; and
- different modules for management of the created projects, map manipulation, and administering the system;
- *Information/Data tier* contains data in database and xml files, and functions for retrieving the necessary information. XML files are used for storing MCDA models, map legends and spatial alternatives. DBMS PostgreSQL 9 is used for managing user projects, map data (features and attributes), user groups, user accounts, and system settings. Functions of PostGIS (GIS extension of PostgreSQL) are used to perform some spatial operations (buffering and overlay of vector layers).

The following main technologies were used for SDSS development: Servlets, Java Server Pages (JSP),

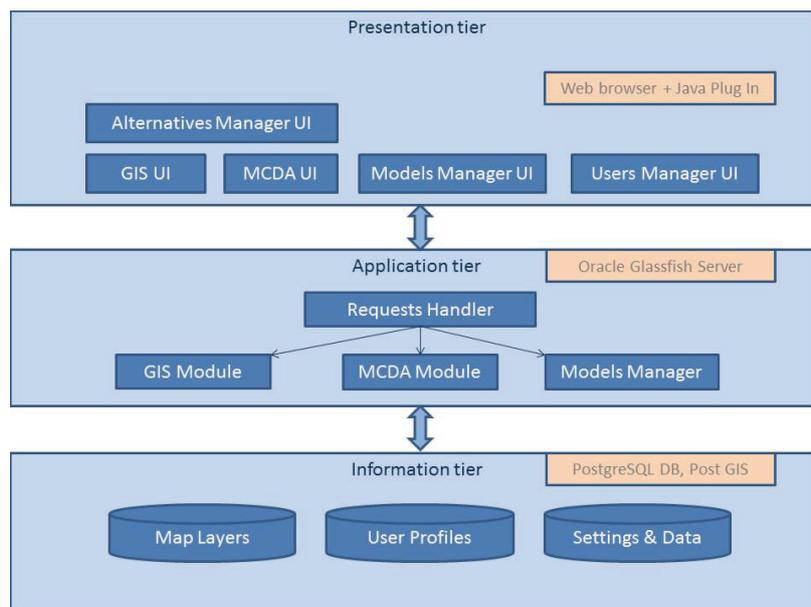


Fig.2. A generalized scheme of the *Decerns* framework.

Java Server Faces (JSF), JavaServer Pages Standard Tag Library (JSTL) and Applets. Here are the requirements for the architectural levels of the SDSS:

- *presentation* tier requires installed Java Runtime Environment (JRE) v.1.7 (or higher), and Internet browser with Java Plug-in. JRE (includes the java plug-in) is free and may be downloaded from [www.java.com](http://www.java.com). *DecernsSDSS* was tested on Windows and Linux platforms with the latest versions of the following browsers: IE, Mozilla Firefox, Google Chrome;

- *application* tier requires installed Java SDK v.1.7 and application server which supports JEE 6 specifications; *DecernsSDSS* was tested with SUN Glassfish and Apache Tomcat servers. The only hardware requirements are that the system can run java application server;

- *information/data* tier requires installed DBMS PostgreSQL 9 with PostGIS (GIS extension for PostgreSQL) v.1.4 or higher. The only hardware requirements are that the system can run PostgreSQL 9.

Table 1. Computer systems for decision analysis (with implementation of MADM methods).

Software	Brief description, Features
Systems for structuring multicriteria problems: Decision Explorer - <a href="http://www.banxia.com">www.banxia.com</a> Mind Manager 4.0 - <a href="http://www.mind-map.com">www.mind-map.com</a>	Provide tools for structuring multicriteria problems, including visual support of the process of problem analysis, setting criteria and alternatives with corresponding interconnections using various graphical means. These systems do not include any MCDA method.
Criterium DecisionPlus <a href="http://www.infoharvest.com">www.infoharvest.com</a>	Implementation of MAVT and AHP methods; provides problem structuring with the use of value tree, weight sensitivity and (restricted) uncertainty analysis; versions for web and group decision support are also available.
Decision Lab <a href="http://www.visualdecision.com">www.visualdecision.com</a>	Realization of PROMETHEE method; support of geometrical data analysis in the criterion space using GAIA tool; weight sensitivity analysis; provides analysis of several scenarios, and group decision support.
Expert Choice <a href="http://www.expertchoice.com">www.expertchoice.com</a>	Classical implementation of AHP method, including support of hierarchical structure; provides weight sensitivity analysis, and group decision support ( <i>Team Expert Choice</i> )
HIPRE <a href="http://www.hipre.hut.fi">www.hipre.hut.fi</a>	Implementation of MAVT and AHP methods using interval criterion values; web-version ( <i>Web-HIPRE</i> ) for support of group analysis is also available.
SANNA <a href="http://nb.vse.cz/~jablon/sanna.htm">http://nb.vse.cz/~jablon/sanna.htm</a>	Application for MS Excel; includes TOPSIS, ELECTRE I, PROMETHEE-II methods.
Decision Deck <a href="http://www.decision-deck.org">www.decision-deck.org</a>	Within the <i>Decision Deck</i> project a framework for creating open source software which provides a process of developing and using various MCDA methods and tools.
JSMAA <a href="http://www.smaa.fi">www.smaa.fi</a>	JSMAA includes SMAA-2 and SMAA-TRI methods (stochastic realization of MAVT and ELECTRE-TRI methods based on acceptability concept).
<i>DecernsMCDA</i> <a href="http://www.deesoft.ru">www.deesoft.ru</a>	<i>DecernsMCDA</i> contains the main MADM methods for choice, ranking, and sorting alternatives, including weight sensitivity and value function sensitivity analysis, and uncertainty treatment (with the use of probabilistic methods and fuzzy numbers). Methods include MAVT, AHP, TOPSIS, PROMETHEE, FlowSort, MAUT, ProMAA, FMAA, F-MAVT.
FLINTSTONES <a href="http://serezade.ujacn.es/flintstones/">http://serezade.ujacn.es/flintstones/</a>	FLINTSTONES: software tool to solve linguistic decision-making problems under uncertainty and contains extensions to deal with complex frameworks such as multi-granular linguistic frameworks, heterogeneous frameworks, and unbalanced linguistic frameworks.
Other MCDA systems: <a href="http://www.isy.vcu.edu/~hweistro/mcdmchapter.htm">www.isy.vcu.edu/~hweistro/mcdmchapter.htm</a> <a href="http://www.cs.put.poznan.pl/ewgmcda">www.cs.put.poznan.pl/ewgmcda</a> <a href="http://www.inescc.pt/~ewgmcda/Software.html">www.inescc.pt/~ewgmcda/Software.html</a> <a href="http://www.orms-today.org/surveys/das/das.html">www.orms-today.org/surveys/das/das.html</a>	ELECTRE IS, III-IV, TRI; Equity; HIVIEW; MACBETH; MIIDAS; MINORA; NAIADE; PRIAM; REMBRANDT; UTA Plus; IRIS; PREFDIS; TOMASO; AGAP; MEDIATOR; SCDAS; GMCR; VISA; ACADEA; DIMITRA; ESY; INVEX, MARKEX; MEDICS; SANEX; FINCLASS;...

Based on the *Decerns* framework, a desktop system (cross-platform software) *DecernsMCDA* for multicriteria analysis of various scientific and practical problems on choosing, ranking, and sorting alternatives with the possibility to implement different MCDA methods has been developed (section 5).

#### 4. MCDA software

MCDA methods and tools provide a wide spectrum of approaches to various multicriteria problems with involvement of objective values and subjective judgments. In this paper, computer systems/software which implement MADM models are considered. Currently, free and commercial systems (software) have been developed that use various MCDA methods and tools<sup>[6,7,49]</sup>. Table 1 presents some of the more common and popular programs. Among those, the following systems/software are highlighted due to their widespread use: Expert Choice, Decision Lab, and Criterium DecisionPlus.

Expert Choice represents a classical approach in the construction of the Analytic Hierarchy Process (AHP) model and contains a range of tools, including hierarchical structuring of attributes/criteria for multicriteria problems, and weight sensitivity analysis.

Criterium DecisionPlus is a system used to structure a multicriteria problem by implementing a value tree tool with subsequent use of MAVT or AHP methods. This system provides a limited capability for uncertainty analysis concerning criteria values and sensitivity analysis by reviewing the variation of weight coefficients that shows their influence on ranking alternatives.

Decision Lab implements the PROMETHEE method. The interface of the system provides a toolkit for setting values for model parameters and conducting sensitivity analysis to weight variation of weighting procedures. Additionally, the GAIA application (as a 'geometrical support' of multicriteria analysis) is used within the system to present vector values of criteria for comparison of alternatives. Decision Lab also supports an examination of several scenarios for multicriteria problem analysis.

(All of the MCDA methods mentioned above are described in section 5 below.)

The popularity of different MCDA software tools for analysis of *risk management problems* for 2009-2014 is presented in Table 2 according to a search in sciencedirect.com.

Table 2. Number of *articles* in ScienceDirect database for 2009-2014 in which indicated software *AND* words associated with risk management were mentioned in full text of an article.

	<b>Expert Choice</b>	<b>Decision Lab</b>	<b>Criterium DecisionPlus</b>	<b>Decision-Deck</b>	<b>JSMAA</b>	<b><i>Decerns</i></b>
<i>Sci.Direct</i>	143	16	8	8	7	7

#### 5. *DecernsMCDA* computer system

A general scheme of *DecernsMCDA* is presented in Fig.3 (for the current version of the *DecernsMCDA*; an application for group analysis, *DecernsMCDA-G*, is not considered in this paper). The explanation of this scheme, including the indicated MCDA methods, tools, and weighting algorithms, is given below in sections 5.1-5.3.

The following versions of *DecernsMCDA* are currently available: desktop system, and as a component, fully embedded in web-system *DecernsSDSS*. Both systems are capable of analysis of various problems on risk management and land-use

planning<sup>[20,26]</sup>. *DecernsMCDA* methods and tools allow for an effective implementation of the MCDA process within a multicriteria decision problem in accordance with the scheme of Fig.1. Additionally, users/experts can select one or several MCDA models for analysis of a specific problem.

*DecernsMCDA* and *DecernsSDSS* (section 3) are based on the same *Decerns* codebase. User interface and components were implemented with the Java SE technology (with Swing for the user interface) to build a standalone desktop application in case of *DecernsMCDA* and web-application with embedded Java applet in case of *DecernsSDSS*.

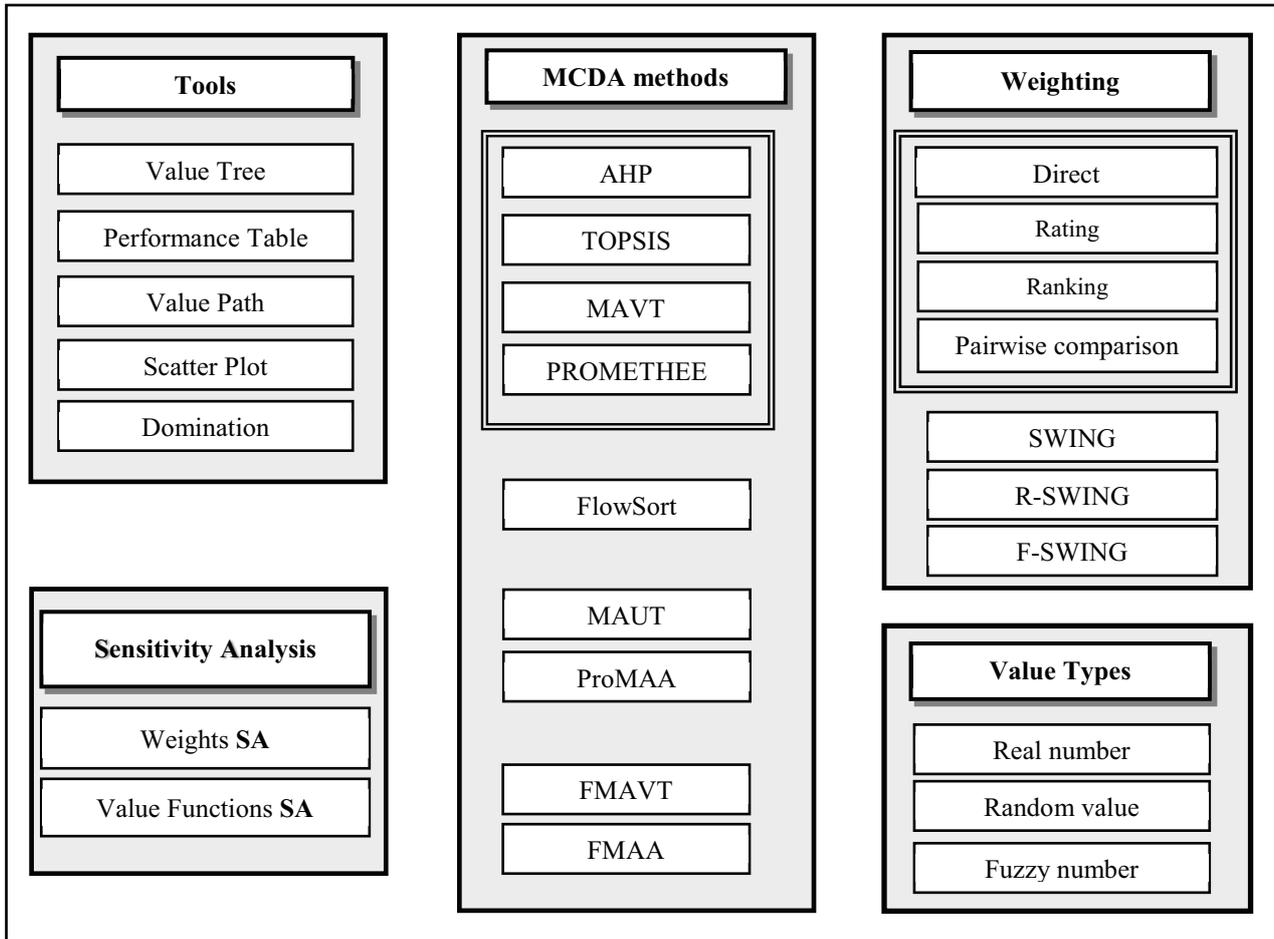


Fig.3. A general scheme of the *DecernsMCDA*.

These applications differ in terms of file systems (web versus desktop), but they are similar in terms of user experience due to reuse of the same components. The biggest limitation for such systems is file size (built Java jar-file). To diminish the size of files and system response in the case of web (applet based) applications, the *Decerns* framework includes several original libraries, including *jSimpleCharts* (library for building colorful charts) and *jSimpleViz* (library for building interactive 2D components). Both libraries are compact, 40-50Kb each, and are based on Java2D technology.

For mathematical computations, the original libraries *jrandomlib* (for the distributions of functions of random variables), *jfuzzylib* (for assessing functions of fuzzy numbers), and *jfuzzyrate* (implementation of several methods for ranking fuzzy numbers) were developed. MCDA related algorithms and relations are

implemented with the original *dsmlib* library, which is coupled with above libraries.

### 5.1. Basic MADM methods

In this section, we briefly discuss the most widely used MCDA/MADM methods that are available in *DecernsMCDA*, which have a ‘simple’ structure for the input information. In our case, these are methods where ‘distributed values’ and, correspondingly, probabilistic methods or fuzzy sets, are not used.

#### 5.1.1. MAVT (Multi-Attribute Value Theory)

The objective of MAVT<sup>[1,6,7]</sup> is to model and represent the decision maker’s preferential system into an ‘integrated’ value function  $V(\mathbf{a})$ ,

$$V(\mathbf{a}) = F(V_1(a_1), \dots, V_m(a_m)); \quad (1)$$

where alternative  $\mathbf{a}$  is presented as a vector of the evaluation criteria  $\mathbf{a}=(a_1, \dots, a_m)$ ;  $a_j$  is an estimate of this alternative against a criterion  $C_j, j=1, \dots, m$ ; and  $V_j(a_j)$  is the value score of the alternative reflecting its performance on criterion  $C_j$  via use of a value function  $V_j(x)$  ( $0 \leq V_j(x) \leq 1$ ). MAVT allows ranking alternatives based on assessing overall scores for alternatives under consideration.

The most widely used form of the function  $F()$  is an additive model:

$$V(\mathbf{a}) = w_1 V_1(a_1) + \dots + w_m V_m(a_m), \quad (2)$$

$$w_j > 0, \sum w_j = 1, \quad (3)$$

where  $w_j, j=1, \dots, m$ , are the criterion weights reflecting the scaling factors<sup>[6]</sup>.

However, it should be stressed that for a justified implementation of the additive model (2), some requirements concerning the problem under investigation should be held, especially the *preferential independence* requirements<sup>[1,6]</sup> (i.e., in a form for practical checking, preference between alternatives which differ only in two criteria, say  $C_1$  and  $C_2$ , does not depend on the level of fixed values for other criteria  $C_3, \dots, C_m$ ).

The additive model of MAVT has been implemented in *DecernsMCDA* with the use of different types of partial value functions (linear, non-linear, and piecewise linear) for each criterion along with *swing* weighting method for setting weights as *scaling coefficients* (relative importance of criteria taking into account the range of values for each criterion)<sup>[7]</sup>.

Some extensions of MAVT have been developed. For example, inclusion of MAVT in the Monte Carlo process for assessing the distribution of overall values of alternatives taking into account the uncertainty of criterion values and weight coefficients (SMAA-2 method<sup>[29,30]</sup>), and utilization of fuzzy criterion values and fuzzy weights for determination of (fuzzy) overall values of alternatives under consideration (FMAVT, see section 5.2.4).

### 5.1.2. AHP (Analytic Hierarchy Process)

AHP<sup>[31]</sup> presents an integration of the additive model (2) with a pre-defined system for determination of the decision matrix,  $V_j(a)$ , and criteria weights,  $w_j, j=1, \dots, m$ . Within AHP, a systematic pairwise comparison of alternatives with respect to each criterion is used based

on a special ratio scale: for a given criterion, alternative  $i$  is preferred to alternative  $j$  with the strength of preference given by  $a_{ij}=s, 1 \leq s \leq 9$ , correspondingly,  $a_{ji}=1/s$ . Then, the same procedure is implemented for pairwise comparisons of  $m$  criteria. The derived matrices are processed (by extracting the eigenvector corresponding to the maximum eigenvalue of the pairwise comparison matrix), and yield the (normalized) values  $V_{j,a}$  and weights  $w_j$  for subsequent use with the model when preferences are aggregated across different criteria according to (2).

Modifications of the classical<sup>[31]</sup> AHP method, such as ANP (Analytic Network Process)<sup>[8,32]</sup>, stochastic AHP<sup>[33]</sup>, and Fuzzy AHP<sup>[34-38]</sup> also are used for multicriteria decision analysis including various types of problems on risk management.

### 5.1.3. TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution)

TOPSIS<sup>[39]</sup> is based on using criterion values from a user defined performance table. So called “ideal” and “anti-ideal” points are defined, and (weighted) distances from each alternative to these points in the criterion space (with normalized criterion values) are assessed. The “ideal” and “anti-ideal” points represent hypothetical alternatives that consist of the most desirable (ideal) and the least desirable (anti-ideal) levels of each criterion across the alternatives under consideration. For ranking alternatives, an integrated measure for each alternative is determined. Within TOPSIS, the following distance from the alternative  $\mathbf{a}_i$  to the “ideal point”,  $\mathbf{x}_+$ , is used:

$$s_{i+} = \left( \sum_j w_j^p (x_{ij} - x_{+j})^p \right)^{1/p} \quad (4)$$

where  $w_j$  is a weight assigned to the  $j$ -th criterion,  $x_{ij}$  is the standardized criterion value of the  $i$ -th alternative,  $x_{+j}$  is the ideal value for the  $j$ -th criterion,  $p$  is a parameter; in *DecernsMCDA* parameter  $p=2$  (as the most often used) is implemented. For transformation of criterion values  $C_{ij}$  ( $C_{ij}$  is the estimation of alternative  $\mathbf{a}_i$  for criterion  $j$ ) to corresponding standardized value,  $x_{ij}$ , the following approach is used in *DecernsMCDA*:

$$x_{ij} = C_{ij} / \left( \sum_{i=1}^n C_{ij}^2 \right)^{1/2}$$

The distance,  $s_{i-}$ , from the alternative  $\mathbf{a}_i$  to negative (anti) ideal point,  $\mathbf{x}_-$ , are defined similarly to (4).

The following decision rule is most often used within TOPSIS (implemented in *DecernsMCDA*):

$$c_{i+} = s_{i-} / (s_{i+} + s_{i-})$$

The alternative with the highest  $c_{i+}$  is considered as the “best” one.

Different approaches to assigning weight coefficients in TOPSIS, including pairwise comparison of criteria and setting “objective/entropy” weights, are used<sup>[39-41]</sup>.

TOPSIS is very attractive method to decision problems when the dependency among criteria is difficult to test or verify. That is especially true in case of spatial decision problems, which typically involve complex interdependencies among attributes. TOPSIS and different modifications, including Fuzzy TOPSIS<sup>[35,36,38,41,42]</sup>, have been used for analysis of various multicriteria problems<sup>[43]</sup>.

**5.1.4. PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations)**

PROMETHEE is a method from the family of ORT (Outranking Relation Theory) methods<sup>[7,8,44]</sup>. PROMETHEE<sup>[44]</sup> is based on the use of a performance matrix  $\{z_i(\mathbf{a})\}$  (where  $z_i(\mathbf{a})$  is an evaluation of alternative  $\mathbf{a}$  against criterion  $i$ ) and a chosen preference function  $p_j(d)$ ,  $0 \leq p_j(d) \leq 1$ , with specified indifference ( $q_j$ ) and preference ( $p_j$ ) thresholds, where  $d = z_j(\mathbf{a}) - z_j(\mathbf{b})$  for criterion  $j$ . The intensity of preference for alternative  $\mathbf{a}$  over alternative  $\mathbf{b}$ ,  $P_j(\mathbf{a}, \mathbf{b}) = p_j(z_j(\mathbf{a}) - z_j(\mathbf{b}))$ , and the preference index,  $P(\mathbf{a}, \mathbf{b})$ , are assessed:

$$P(\mathbf{a}, \mathbf{b}) = \sum w_j P_j(\mathbf{a}, \mathbf{b}),$$

where weights  $w_j$  reflect the relative importance of the criteria. According to the features of preference functions  $p_j(x)$ , if  $P_j(\mathbf{a}, \mathbf{b}) > 0$ , then  $P_j(\mathbf{b}, \mathbf{a}) = 0$ . Preference indices are used for determination of positive outranking flow  $Q^+(\mathbf{a})$ :

$$Q^+(\mathbf{a}) = \sum_b P(\mathbf{a}, \mathbf{b}) / (n-1)$$

and negative outranking flow  $Q^-(\mathbf{a})$ :

$$Q^-(\mathbf{a}) = \sum_b P(\mathbf{b}, \mathbf{a}) / (n-1),$$

summed over all alternatives  $\mathbf{b} \neq \mathbf{a}$ ,  $n$  is the number of alternatives under consideration.

According to the PROMETHEE I method,  $\mathbf{a}$  outranks  $\mathbf{b}$  if  $Q^+(\mathbf{a}) \geq Q^+(\mathbf{b})$  and  $Q^-(\mathbf{a}) \leq Q^-(\mathbf{b})$ ;

$\mathbf{a}$  is indifferent to  $\mathbf{b}$  if  $Q^+(\mathbf{a}) = Q^+(\mathbf{b})$  and  $Q^-(\mathbf{a}) = Q^-(\mathbf{b})$ ;  $\mathbf{a}$  and  $\mathbf{b}$  are incomparable if  $Q^+(\mathbf{a}) > Q^+(\mathbf{b})$  and  $Q^-(\mathbf{b}) < Q^-(\mathbf{a})$ , or  $Q^+(\mathbf{b}) > Q^+(\mathbf{a})$  and  $Q^-(\mathbf{a}) < Q^-(\mathbf{b})$ . Thus, PROMETHEE I admits partial ranking, since some alternatives may be incomparable.

The PROMETHEE-2 method is based on the “net flow”  $Q(\mathbf{a})$  for alternative  $\mathbf{a}$ :

$$Q(\mathbf{a}) = Q^+(\mathbf{a}) - Q^-(\mathbf{a}),$$

and it may be used for a complete ranking of alternatives: alternative  $\mathbf{a}$  outranks  $\mathbf{b}$  if  $Q(\mathbf{a}) > Q(\mathbf{b})$ .

Weight coefficients in PROMETHEE are interpreted as voting weights<sup>[8,44]</sup>. PROMETHEE also allows using other weighting methods, including pairwise comparison of criteria, and swing weights, taking into account their compliance with expert judgments.

Modifications of the PROMETHEE method, including stochastic PROMETHEE and Fuzzy PROMETHEE<sup>[45-47]</sup>, are used within multicriteria risk management problems and strategic evaluations.

**5.1.5. FlowSort (Sorting method with the use of Net Flows)**

FlowSort<sup>[48]</sup> is a method that sorts alternatives according to predefined multicriteria categories or classes (e.g., zones of high, medium, low, and negligibly small risk). The algorithms implemented in FlowSort are similar to those in PROMETHEE with corresponding modification to multicriteria sorting. In the first step, the experts define the categories to which the alternatives will be assigned. Within FlowSort, the distinction from PROMETHEE is that the decision maker can express a *transitive preference relation* on the categories, where the categories can be ordered from the best to the worst (e.g., low risk zone, medium risk zone, and high risk zone). In order to define the meaning of the categories, the analyst needs to specify *limiting profiles* which completely characterize the categories. These limiting profiles are created by defining each category by an *upper* and a *lower* boundary: the category  $C_h$  ( $h=1, \dots, K$ ) is thus defined by the upper limiting profile  $r_h$  and the lower profile  $r_{h+1}$  of the limiting profiles set  $R = \{r_1, \dots, r_{K+1}\}$ . Since the categories are completely ordered, each limiting profile *dominates* all the successive ones:  $r_1 \succ^D r_2 \succ^D \dots \succ^D r_{K+1}$ . The profiles respect the following condition

if we suppose that the  $m$  criteria (noted  $f_l$ ,  $l=1, \dots, m$ ) have to be maximized:

$$\forall h = 1, \dots, K; l = 1, \dots, q : r_h \succ^D r_{h+1} : f_l(r_h) \geq f_l(r_{h+1}) \text{ and } \exists j : f_j(r_h) > f_j(r_{h+1})$$

In FlowSort, an alternative  $a_i$  that is to be sorted is compared solely to the reference profiles by means of the PROMETHEE-2 ranking method. The category, to which the alternative will be assigned to is deduced from its relative position with respect to the reference profiles (further details can be found in<sup>[48,49]</sup>).

The approaches to setting weight coefficients in FlowSort are the same as for PROMETHEE.

Extensions of FlowSort<sup>[50,51]</sup> and other methods for multicriteria sorting, such as ELECTRE-Tri<sup>[52]</sup> and its modifications for uncertainty treatment<sup>[53]</sup>, have also been used for analysis of problems on strategic planning and risk management.

## 5.2. Advanced MADM methods

Advanced methods make use of distributed input data and these methods have been implemented in *DecernsMCDA*. In this case, criterion values or/and weight coefficients may be considered as random variables or fuzzy numbers. If decision maker(s) and/or experts do not have enough knowledge in advanced MADM methods, an MCDA analyst may participate in the decision analysis process to treat corresponding uncertainties in cooperation with experts in the analysis of the multicriteria problem.

### 5.2.1. MAUT (Multi-Attribute Utility Theory)

MAUT<sup>[1,6-8]</sup> may be considered as an extension or generalization of MAVT for the cases when criterion values (all or some of them) are *random variables* with prescribed probability distributions. The additive MAUT model has been implemented in *DecernsMCDA*:

$$U(\mathbf{a}) = w_1 U_1(a_1) + \dots + w_m U_m(a_m), \quad (5)$$

where weight coefficients  $\{w_j, j=1, \dots, m\}$  satisfy (3).  $U(\mathbf{a})$  is an overall utility for the alternative  $\mathbf{a}=(a_1, \dots, a_m)$  and here  $a_j$  is an estimate of this alternative against a criterion  $C_j$ ,  $j=1, \dots, m$ .  $U_j(a_j)$  is an assessment of alternative  $\mathbf{a}$  in a utility scale with the use of a partial utility function  $U_j(x)$  for criterion  $C_j$ ,  $0 \leq U_j(x) \leq 1$ .

The type of MAUT model (function  $F$  in (1)) depends on the corresponding requirements (preferential independence, utility independence, and

additive independence), which determine the implementation of the appropriate function  $F$  (additive or multiplicative)<sup>[6]</sup>. In<sup>[7]</sup> (section 4.3.5), the significance of using additive MAUT model for analysis of multicriteria problems is discussed, and the author points out that, in “most practical cases”, the uncertainties involved in constructing the partial utility functions, which need to be addressed by careful sensitivity analysis, are likely to far outweigh any distinctions between the additive and multiplicative models.

Uncertainty of the criterion value  $a_j$  are represented in MAUT by a random variable  $X_j = X_j(\mathbf{a})$  with density of distribution  $\varphi_j(x)$ ,  $j=1, \dots, m$  (the types of possible distributions available for setting in *Decerns* are the same as for ProMAA, see below). Thus, the overall utility (5) for the alternative  $\mathbf{a}$  is considered, in the general case, as a random variable  $U(\mathbf{a})$ .

Ranking of alternatives within MAUT is based on the comparison of expected utilities. The alternative  $\mathbf{a}_1$  exceeds the alternative  $\mathbf{a}_2$ , if and only if

$$E(U(\mathbf{a}_1)) > E(U(\mathbf{a}_2))$$

where  $E(X)$  is the mathematical expectation of random variable  $X$ . According to (5),

$$E(U(\mathbf{a})) = w_1 E(U_1(X_1)) + \dots + w_m E(U_m(X_m)).$$

Weight coefficients  $w_j$  may also be interpreted as scaling factors.

Some modifications of MAVT/MAUT are also used within multicriteria risk evaluation<sup>[54,55]</sup> (see also sections 5.2.2- 5.2.4 below).

### 5.2.2. ProMAA (Probabilistic Multi-criteria Acceptability Analysis)

Despite extensive use of the expected utility concept (in MAUT, and, indirectly and implicitly, using mean criterion values and weights in MAVT), its application is not the only approach within decision analysis, and other approaches which do not use expected utility methods have been developed and implemented<sup>[29,30]</sup>.

ProMAA<sup>[56]</sup> is based on the modification of additive MAVT/MAUT models with transition to the acceptability concept. In this application, acceptability means that the choice of alternative(s), and ranking (if appropriate), is based on the probability/measure to be accepted. ProMAA is intended for treatment of uncertainty in criterion values and weight coefficients with the use of probabilistic methods.

Within the ProMAA, probabilities  $P_{ik} = P\{S_{ik}\}$  of “likely rank events”  $S_{ik}$  are determined, where event  $S_{ik}$  is defined as follows:

$S_{ik} = \{\text{Alternative } a_i \text{ has the rank } k\}$ ,  $i, k = 1, \dots, n$ , (i.e.,  $k-1$  alternatives are/may be considered as “better”  $a_i$  in a chosen scale). For probabilities  $P_{ik} = P\{S_{ik}\}$  the term “rank acceptability indices” are used<sup>[29]</sup>. For aggregation of the indicated probabilities, a weighted sum may also be used (weighted or holistic acceptability indices):

$$R_i = \sum_{k=1}^n \alpha_k P_{ik} \quad (6)$$

where  $\alpha_k$  are weights of relative importance of ranks.

Based on an analysis of the matrix  $\{P_{ik}\}$ ,  $i, k = 1, \dots, n$ , choosing the “best” alternative among  $\{a_i, i = 1, \dots, n\}$ , screening alternatives or, in some cases, ranking alternatives can be implemented within ProMAA. Ranking alternatives can also be obtained based on the “holistic acceptability indices”  $R_i$ ;  $i = 1, \dots, n$ , however, the recommendations concerning implementation of such a secondary ranking (6) are restricted<sup>[30]</sup>. Within ProMAA, both values/utilities  $U_j(a_i)$  and weights  $w_j$  may be considered as random variables with the given probability distributions. The following distributions may be chosen by users in *DecernsMCD*A: delta function, uniform, (truncated) normal, and lognormal distributions.

Implementation of ProMAA is based on numerical approximation of functions of random variables and numerical assessment of integrals (for approximate determination of probabilities  $P_{ik} = P\{S_{ik}\}$ ).

The user interface of the ProMAA module and corresponding functions allow the user to: specify the probability distribution of  $C_j(a_i)$  for criterion  $C_j$ ,  $j = 1, \dots, m$ , and the set of alternatives  $\{a_i, i = 1, \dots, n\}$ ; specify the utility functions  $U_j(x)$  (from the class of linear, exponential, and piecewise-linear functions); and specify the probability distribution for the weight coefficients  $w_j$ ,  $j = 1, \dots, m$ . Using this information, the distributions of random variables  $\eta_i = U(a_i)$ ,  $i = 1, \dots, n$ , and rank acceptability indices  $P_{ik}$ ,  $i, k = 1, \dots, n$ , are calculated by the *Decerns* software. The users can analyze graphical and tabular representations of the output results for subsequent decision-making. Users have the ability to implement *sensitivity analysis* of the model results to changing utility functions (through changing one or several selected partial utility functions  $U_j(x)$ ).

Within MAVT/MAUT and other classical multicriteria methods, weight coefficients are considered as constant positive numbers. However, extended uncertainty treatment, when weights are considered as distributed in the intervals given by experts, is justified for most practical multicriteria problems.

The recommended approach for setting weight coefficients in ProMAA is a natural one and corresponds to the steps for assignment of scaling factors as in the *swing weighting method*, adapted for setting distributed weights (R-Swing):

- weight coefficient  $w_1 = 1$  is assigned for the most highly weighted criterion (let us denote this criterion as  $C_1$ ), taking into account evaluation of the increase in the overall value as a result of the swing from worst to best for each criterion;
- the variation interval  $[w_2^{min}, w_2^{max}]$ ,  $0 < w_2^{min} \leq w_2^{max} \leq 1$ , is assigned for the weight coefficient  $w_2$  of the second ranked criterion (we denote it as  $C_2$ ) based on evaluation of a range for the relative value of the swing from worst to best for this criterion in comparison with the corresponding value of the swing for the most highly weighted criterion;
- the previous step is repeated for the third, and subsequent criteria;
- the probability distributions (as subjective probabilities or as a result of statistical analysis of expert judgments) for (independent) weight coefficients  $w_j$  in the given interval  $[w_j^{min}, w_j^{max}]$ ,  $j = 2, \dots, m$ , are assigned by experts.

ProMAA is similar to the SMAA-2<sup>[29]</sup> in the use of the acceptability principle, although they differ conceptually in the probabilistic approach to interpretation of weights (and criterion values) in ProMAA and the stochastic implementation, using Monte Carlo simulation, in SMAA (where the sum of simulated weights equals 1). Modification of ProMAA to include fuzzy numbers is described below.

### 5.2.3. FMAA (Fuzzy Multicriteria Acceptability Analysis)

In many cases, when we use vague/uncertain values within a multicriteria problem, the application of fuzzy numbers and fuzzy logic may be considered as justified and more natural than the use of (subjective) probability distributions. The use of fuzzy sets in such cases can assist with uncertainty/imprecision assimilation for both

criterion values and weight coefficients. In connection with this, modifications of some classical methods<sup>[34-38,42,46,47,50]</sup>, as well as development and applications of new methods for decision analysis based on fuzzy sets<sup>[57-62]</sup> have a place.

FMAA is a fuzzy analog of ProMAA described above. Within FMAA, criterion values  $a_{ij}=X_j(a_i)$ , scores  $V_j(a_{ij})$ , and weights  $w_j$  are considered as fuzzy numbers,  $i=1,\dots,n$ ,  $j=1,\dots,m$ , and overall (fuzzy) value  $V(a_i)$  is determined by the expression (2). The partial value functions,  $V_j(x)$ , are considered here as the usual/crisp functions, defined by experts on the variation interval of the criterion  $C_j$ ,  $j=1,\dots,m$ , for alternatives under consideration.

Within FMAA, the measure  $\mu(S_{ik})$  of the event  $S_{ik}$  as a *degree of confidence* that alternative  $i$  has rank  $k$ , is determined with the use of fuzzy logic and fuzzy calculations. Using the matrix  $\{\mu_{ik} = \mu(S_{ik})\}$ , experts/decision-makers can select the most acceptable alternative(s). For aggregation of these measures, a weighted sum (6) may also be used. The approach, presented in ProMAA, for assigning distributed weights based on the modified swing method (F-Swing) is similarly adjusted for assigning fuzzy weights  $w_j$  in FMAA.

#### 5.2.4. FMAVT (Fuzzy MAVT)

FMAVT is an extension of the classical additive MAVT with the use of fuzzy criterion values and fuzzy weights (and is a basis for subsequent implementation of the acceptability concept in FMAA, presented above). The FMAVT model is intended for imprecision treatment when solving multicriteria problems with the use of the "value function concept".

Within FMAVT, the additive model (2) is used, where criterion values  $a_{ij}$ , scores  $V_j(a_{ij})$ , and weights  $w_j$  are considered as fuzzy numbers,  $i=1,\dots,n$ ,  $j=1,\dots,m$ ; partial value functions  $V_j(x)$  are the usual/crisp functions. The approach for assigning fuzzy weights  $w_j$  in FMAVT is a modified swing weighting process F-Swing as for FMAA.

Ranking alternatives  $\{a_i; i=1,\dots,n\}$  within FMAVT is based on comparison of overall fuzzy values  $V(a_i)$  based on their visual representation, and using several methods for ranking fuzzy numbers<sup>[63]</sup>.

Probabilistic algorithms in *DecernsMCDA* (in MAUT and ProMAA) are based on implementation of a library *jrandomlib* for assessing functions of random

variables (with numerical calculation of corresponding integrals without Monte Carlo algorithms). Similarly, a library *jfuzzylib* for assessing functions of fuzzy numbers (with implementation of the transformation method) is used in FMAVT and FMAA.

### 5.3. DecernsMCDA Tools

The following tools of *DecernsMCDA* are also used for effective implementation of the included MCDA methods and the decision-making process, Fig.1:

- *Value Tree* is a tool for structuring multicriteria problem through developing a hierarchical set of criteria and alternatives with a possibility of subsequent editing, Fig.4; the value tree is also used for loading, representation, and editing model data (criterion values for the defined alternatives, and weight coefficients);

- *Performance Table*, Fig.5, is intended for loading, representation, and editing model data for the problem under consideration. Within *DecernsMCDA*, an extended *performance table* is implemented and it includes not only the matrix of criterion data against alternatives, but also a description of criterion, data dimension, and weight coefficients;

- *Value Path* provides graphics of criterion values for alternatives, and a Scatter Plot represents the criterion values for alternatives on a two-dimensional plane according to any two chosen criteria; a special tool has been developed for analysis of domination among alternatives (alternative a1 is dominated by a2, if for each of the criteria given, a2 is not worse than a1, and at least for one of the criteria a2 exceeds a1);

- Tools for *weight sensitivity analysis* (for MAVT, MAUT, AHP, TOPSIS, PROMETHEE); Figs.8,10, and tools for

- *Value/utility function sensitivity analysis* (for MAVT, MAUT, ProMAA, FMAA, F-MAVT), Figs.9 and 13.

*Uncertainty analysis* in *DecernsMCDA* is carried out through sensitivity analysis of the results to changing weights and/or value/utility functions, and through uncertainty treatment (when using ProMAA, FMAA and FMAVT) based on probabilistic methods and fuzzy sets. For analysis of a specific multicriteria problem, *DecernsMCDA* users can compare several MCDA methods provided corresponding data are available. Such an approach may be considered as uncertainty analysis to the choice of MCDA model.

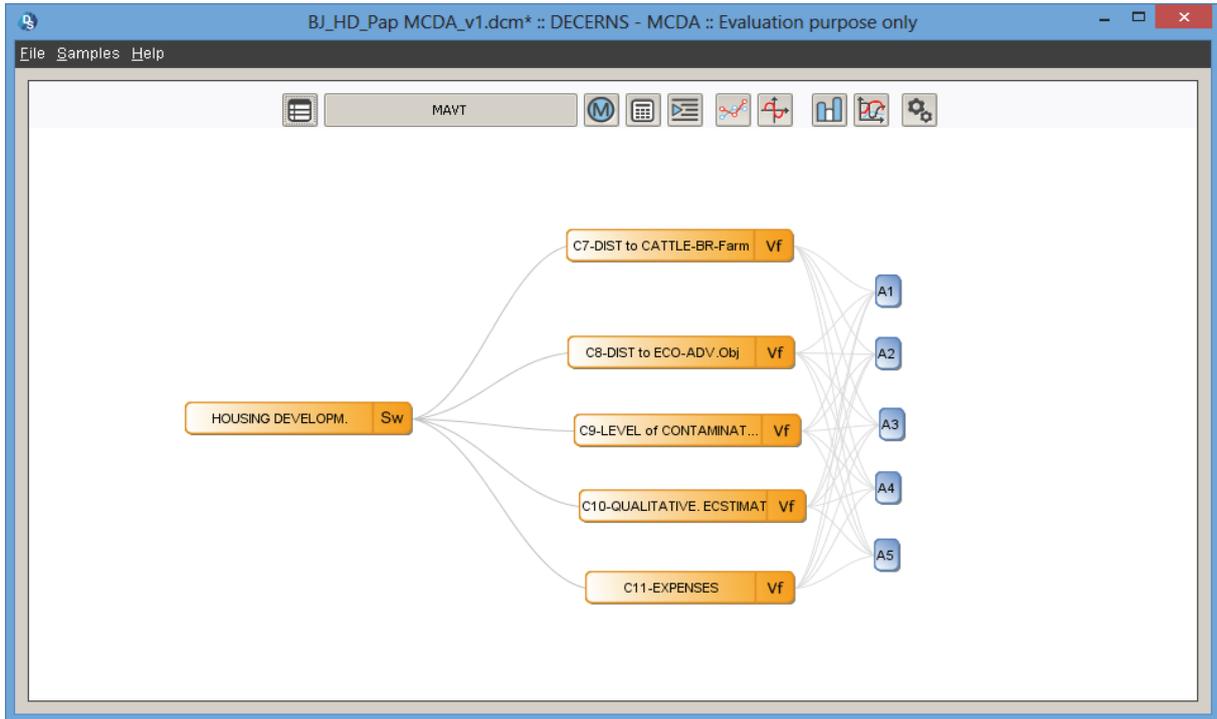


Fig.4. DecernsMCDA: Value Tree.

Criteria						
	C7	C8	C9	C10	C11	
Name	C7	C8	C9	C10	C11	
Description	Distance to cattle-brid. farm	Distance to Ecol. Adverse objects	Level of Radioactive contamination	General qualitative estimate of the Site	Total expenses	
Scale	local \ km \ maximize \ vf: linear	local \ km \ maximize \ vf: linear	local \ Cj/km2 \ minimize \ vf: linear	local \ Sc: 1-5 \ maximize \ vf: linear	local \ \$ \ minimize \ vf: line	
Weight	0.05	0.15	0.45	0.15	0.20	
Performance Table						
Alternatives / Criteria	C7	C8	C9	C10	C11	
A1	2.80	3.20	9.50	2.00	60.00	
A2	2.70	2.90	4.70	5.00	98.00	
A3	2.50	3.10	7.50	1.00	55.00	
A4	2.30	3.00	1.20	4.50	88.00	
A5	1.90	4.50	2.60	3.50	82.00	

Fig.5. DecernsMCDA: Performance Table.

The value tree for the problem under consideration is independent of the MCDA method, while the performance table is dependent on the method chosen.

In addition, the tools for choosing a particular multicriteria method and setting weights allow creation of the different scenarios for analysis and comparison.

For example, the user may create scenarios with the names MAVT-Ecological-W-S1 and MAVT-Economical-W-S2. This means using MAVT with different scenarios for weight coefficients (W-S1 and W-S2) and with different partial value functions. The list of weight scenarios can be used for selecting weights when choosing an additional multicriteria method (e.g., initially using MAVT, and then TOPSIS), and selected weights can then be corrected and stored with a new name.

## 6. Application of DecernsMCDA

Decerns systems (*DecernsSDSS* and *DecernsMCDA*) because of their ease of use and ability to form and conduct analysis of spatial alternatives and implement different MCDA methods on a single platform are effective tools for multicriteria analysis of a wide range of scientific and practical problems.

The *Decerns* framework was used for multicriteria analysis of remediation measures for contaminated sites<sup>[26]</sup>. In this project, extensive implementation of *DecernsSDSS* GIS functions, models for assessment of the risk values/criteria, and MCDA method (MAVT) were used for analysis of six alternatives/protective measures for restoration of radioactively contaminated territories after the Chernobyl accident with the use of five criteria (cost, dose to the local population, two criteria connected with contamination of agricultural products, and socio-economic criterion).

More than 10 topical problems on risk management, including risk-based environmental protection and sediment management, restoration of contaminated sites, choice of nanotechnologies and others have been considered in<sup>[64]</sup> based on the *Decerns* software with the use of different MCDA methods.

This section presents an application of several MCDA methods, included in *DecernsMCDA*, for analysis of the case study on housing development<sup>[20]</sup> in a region of Russia that has contaminated land from the Chernobyl accident<sup>[65]</sup>.

Six stakeholders, including an MCDA expert to guide the process, took part in the workshop on the use of multicriteria decision analysis for this problem in accordance with the scheme of Fig.1. The criterion-based approach was implemented within this problem: six criteria (proximity to major roads, distance from railroads, proximity to major rivers or lakes/ponds,

distance from wetlands, proximity to the towns),  $C_1$ - $C_6$ , were used solely for the screening process based on application of GIS-tools of *DecernsSDSS*<sup>[20]</sup> to find the suitable areas. Then, the following five criteria,  $C_7$  -  $C_{11}$ , were used for multicriteria analysis of five suitable areas (polygons) determined through the screening process,  $A_1$ - $A_5$ :

- $C_7$  - distance from stockyards/cattle-breeding farms or manufactures (*maximize*;  $C_7 \geq 1.5$  km);
- $C_8$  - distance from ecologically adverse objects (*maximize*;  $C_8 \geq 2.5$  km];
- $C_9$  - level of radioactive contamination (*minimize*,  $C_9 \leq 10$  Ci/km<sup>2</sup>);
- $C_{10}$  - general (qualitative) assessment of the local landscape/site quality (*maximize*);
- $C_{11}$  - expenses (*minimize*), (land cost and all the expenses associated with building housing).

The Value Tree and Performance Table for this multicriteria problem are presented in Fig.4 and Fig.5.

The initial analysis used the MAVT method; weight coefficients were suggested after discussion using the swing weighting method, Fig.6. Partial value functions were considered as linear for distance-criteria  $C_7$ ,  $C_8$ , and site quality criterion  $C_{10}$ . Criteria  $C_9$  (level of radioactive contamination) and  $C_{11}$  (total expenses) were set as non-linear/exponential with a weak decrease for "small values" and subsequent more abrupt decrease for values to the right part of criterion interval.

Taking into account the MCDA experts' remarks and discussion concerning vagueness of setting weights and value functions, the group decided that extensive sensitivity analysis to the change in weights and value functions should be performed in accordance with the steps of decision-making process, Fig.1.

Ranking of alternatives according MAVT is presented in Fig.7. The evaluation of all alternatives with the MAVT analysis demonstrated a superiority of the alternative  $A_5$  with alternative  $A_4$  ranked slightly lower. Weight sensitivity analysis confirms a superiority of  $A_5$  against  $A_4$ , Fig.8, for values of the weight for  $C_9$  (radioactive contamination) less than  $\sim 0.75$ . Above this limit  $A_4$  becomes the preferred alternative.

To examine the sensitivity to the value function, the partial value functions for radioactive contamination,  $C_9$ , was changed such that it is close to linear over the range, the partial value function for

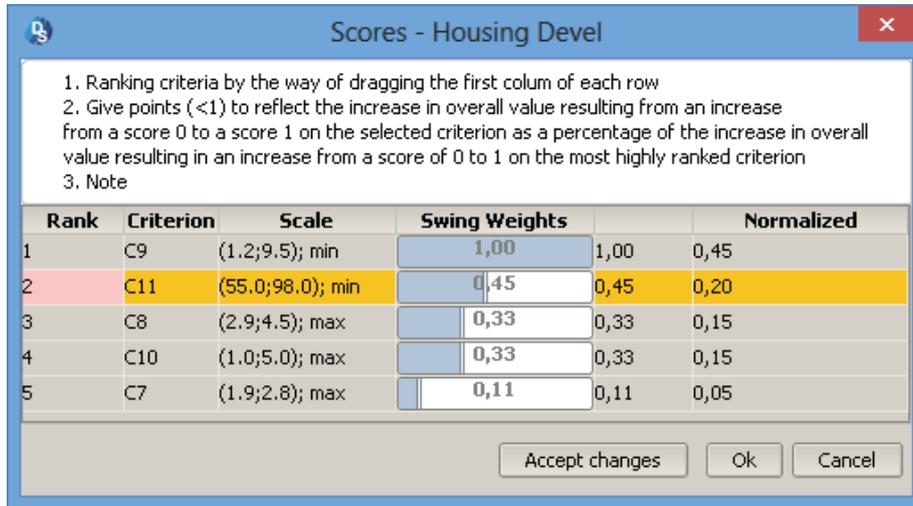


Fig.6. DecernsMCDA: weighting criteria using swing method.

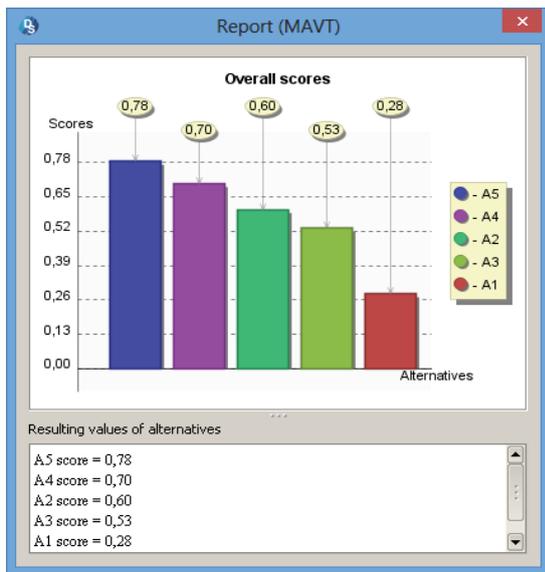


Fig.7. DecernsMCDA: MAVT ranking.

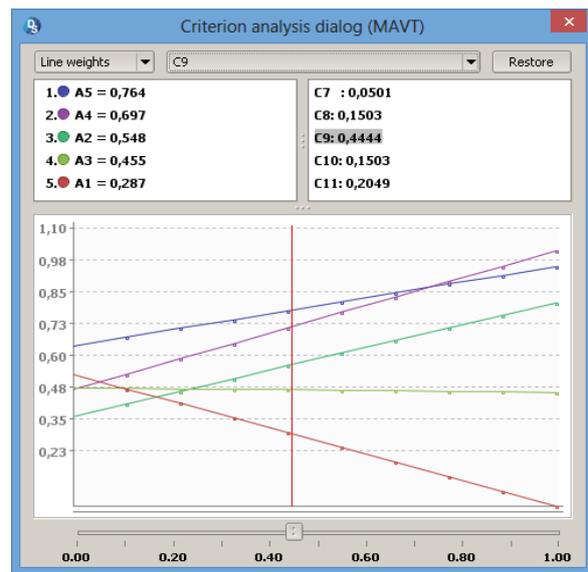


Fig.8. DecernsMCDA: weight sensitivity analysis.

criterion of site quality,  $C_{10}$ , was considered as nonlinear; in this case the difference between alternatives  $A_5$  and  $A_4$  is negligible, Fig.9.

Other methods were also used for analysis of this problem. In these analyses the weight coefficients were the same as for MAVT (for ProMAA, FMAA, and FMAVT they were considered as average values for distributed/fuzzy weights), and extensive weight sensitivity analysis and value function sensitivity analysis were conducted. According to the TOPSIS method, the alternative  $A_4$  score exceeds that of  $A_5$ , and

weight sensitivity analysis demonstrates a robust superiority of  $A_4$  against  $A_5$ , Fig.10. Thus, TOPSIS ranking of alternatives is not in agreement with MAVT ranking.

In PROMETHEE-I methods these two alternatives are considered as “the best” and are incomparable, Fig.11.  $A_5$  is the preferred alternative using the PROMETHEE-II method.

Extensive uncertainty treatment and analysis both for criterion values and weight coefficients was performed with the use of ProMAA, FMAVT, and FMAA.



Fig.9. DecernsMCDA: value function sensitivity analysis: right hand picture – Initial ranking, left picture – ranking according to changed partial value function(s).

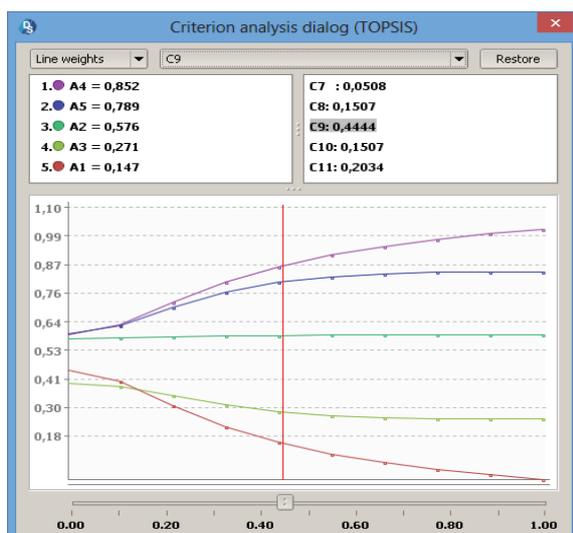


Fig.10. Weight sensitivity analysis in TOPSIS.

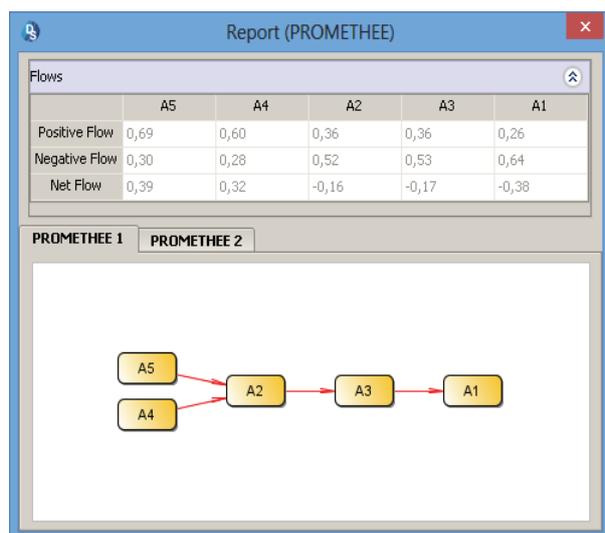


Fig.11. Ranking alternatives in PROMETHEE-I.

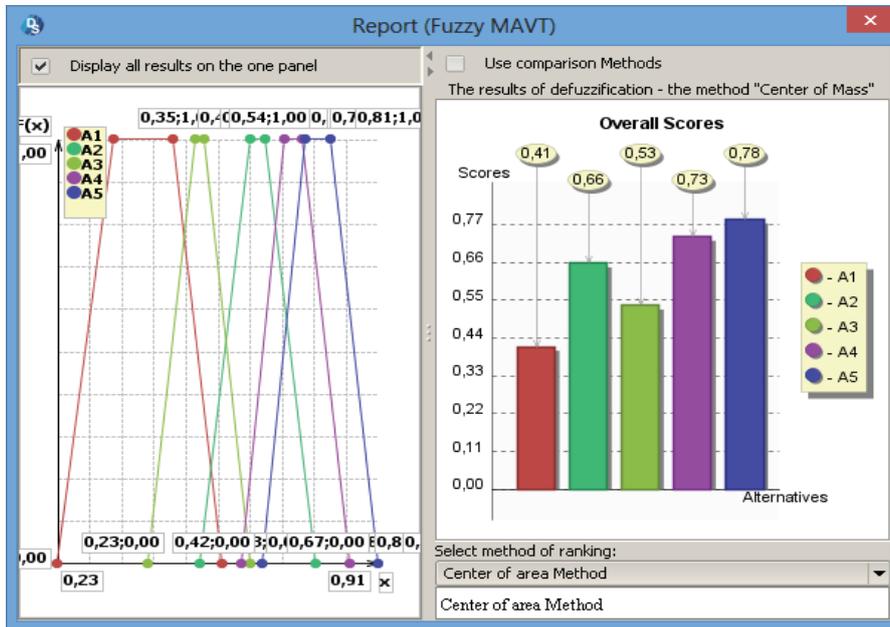


Fig.12. DecernsMCDA: analysis of alternatives with FMAVT.

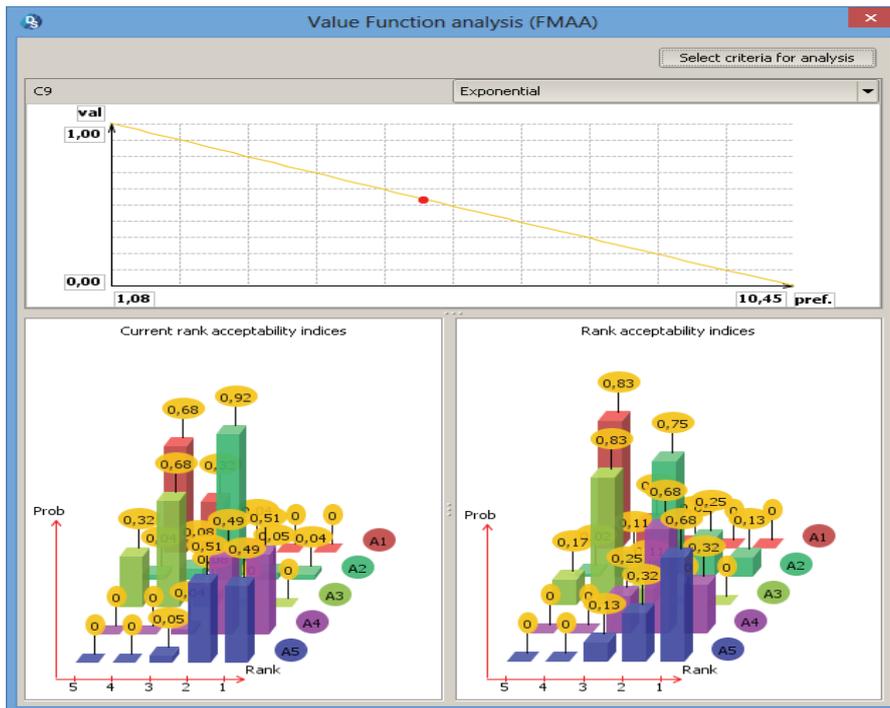


Fig.13. DecernsMCDA: value function sensitivity analysis in FMAA.

When using these advanced methods ranges of changing criterion values and weight coefficients were considered as  $\pm 10-30\%$ . Uniform distributions for

weight coefficients, and uniform and normal distributions (and non-random values for  $C_{10}$ ) for criterion values were taken for ProMAA; triangular and

trapezoidal fuzzy numbers were used within FMAA and FMAVT. The results of rank analysis in FMAVT and value function sensitivity analysis in FMAA are presented in Fig.12 and Fig.13. They demonstrate that  $A_5$  and  $A_4$  are the preferred alternatives with the preference of  $A_5$  in comparison with  $A_4$ .

Implementation of the MCDA methods was carried out by the MCDA expert with participation of experts/stakeholders in all steps of multicriteria problem analysis in accordance with the scheme of Fig.1.

The results of ranking alternatives  $A_1, \dots, A_5$  based on implementation of different MCDA methods, included in *DecernsMCDA*, are presented in Table 3 (in

each cell of the table the rank of the alternative for the MCDA method is indicated).

According to these results, alternative  $A_5$  is preferable for all the methods which use the concept of value/utility function. Only for TOPSIS, which is based on the concept of (weighted) distances in the m-dimensional space, the output ranking of alternatives differs from all other analyses.

After implementation of the indicated MCDA methods, the experts recommended further study of two alternatives/sites,  $A_4$  and  $A_5$ . The studies included more precise evaluation of the criterion  $C_{10}$  (site quality) as well as analysis of local roads quality and other impressions of the site.

Table 3. Ranks of alternatives  $A_1$ - $A_5$  for different MCDA methods.

Method/Alternative	A1	A2	A3	A4	A5
MAVT	5	3	4	2	1
TOPSIS	5	3	4	1	2
PROMETHEE-I	5	3	4	1-2	1-2
PROMETHEE-II	5	3	4	2	1
MAUT	5	3	4	2	1
ProMAA	5	3	4	2	1
FMAA	5	3	4	2	1
FMAVT	5	3	4	2	1

## 7. Discussion

Implementation of the MCDA methods was conducted by the MCDA analyst with participation of experts/stakeholders in all the steps of multicriteria problem analysis in accordance with the scheme of Fig.1. Although, for this case study, the results of ranking alternatives for advanced methods generally agree with the ranking for MAVT, experts considered implementation of different methods as a very useful exercise. The different analyses let experts take part in the practical use of the different MCDA methods, and allowed them to compare the results and analyze the causes that impacted the ranking. The use of multiple MCDA methods and sensitivity analysis reduced or eliminated many disagreements between experts concerning weight coefficients for different criteria and the shape of partial value functions. Additionally, implementation of different MCDA methods and the transparency of their use helped participants of the

workshop to reach a trade-off in setting all the key parameters of the model for the ranking suitable sites.

Based on the results of using different MCDA methods, the group made the decision to recommend (to the stakeholders and decision maker) implementation of either  $A_4$  or  $A_5$  alternatives depending on the additional analysis of the local site characteristics.

One of the arguments in favor of using several MCDA methods when analyzing a multicriteria problem is demonstrated by this case study with a comparison of the preferred alternatives identified by TOPSIS, Fig.10, Table 3, and other methods. In this example, if the experts used only TOPSIS for analysis of the multicriteria problem, alternative  $A_4$  would have been chosen as the preferred choice, Table 4, and this choice would be considered as robust based on the weight sensitivity analysis (Fig. 10) for this method. However, this choice is not confirmed by other methods, Table 3, and the "best" alternative according to other methods is  $A_5$ .

An advantage of using the TOPSIS method includes limited input (weight coefficients) from the experts/decision maker and quick identification of “the best” alternative. According to some investigations, TOPSIS criteria weights typically affect the outcomes less than the number of alternatives or criteria<sup>[40]</sup>. However, the conclusion that TOPSIS outcomes are robust (in comparison with other methods) concerning changing weights is incorrect in general.

Experts in MCDA stress that there is not a “best” MCDA method, and different methods may be considered appropriate depending on the problems to be solved.

According to a search using Scopus (June 22-25, 2014), 16,480 *articles* published from 2000 through June 2014, were connected with MCDA (both MADM and MODM). Among them, 6,995 articles (43%) dealt with risk problems (risk management, and associated

terms such as remediation, rehabilitation, restoration, cleaning, waste management). The application of some MCDA/MADM methods, presented in section 5 above, for analysis of risk management problems for the same period is presented in Table 4. The search looked for words associated with risk and the different MCDA methods. The first value in the Table is the number of articles that a match was found in the TAK (Title-Abstract-Keywords), and the second value, in the TAK for risk terms and in the full text for the MCDA method.

According to Table 4, AHP is the most widely used method and its use exceeds that of TOPSIS by a factor of four, and the MAVT/MAUT methods by ~20 times. This order in popularity is confirmed by the same analysis in ScienceDirect and Web of Science (although the corresponding numbers differ).

Table 4. Usage of MCDA methods. Number of *articles* in Scopus database for 2000-2014 in which MCDA method was mentioned: Numerator - the method and words associated with risk management are mentioned in TAK (Title-Abstract-Keywords); Denominator - words associated with risk management are mentioned in TAK and the method is mentioned in the full text of an article.

	AHP	TOPSIS	PROMETHEE	MAVT/MAUT
Scopus	762/ 1841	152/ 486	38/ 173	29/ 102

There are several reasons that AHP has a significant advantage in popularity compared with other MCDA methods (according to Table 4). One of them is there are several existing software packages with effective implementation of AHP. The second important reason is the relative simplicity of using the AHP method. Many AHP users often attempt to minimize or avoid the work with “complex and vague” steps of MCDA methods (e.g., implementation of swing-weighting process along with evaluation of the types of partial value functions) and restrict these steps of MCDA process by a “simple comparison of alternatives and criteria in the given verbal/ratio scale”. The third reason for the level of AHP usage is the positive feedback caused by the existing high level of AHP popularity.

*DecernsMCDA* is also used within the educational courses (Decision Analysis, and Methods and Systems for Decision Making Support) at the Obninsk University IATE NRNU MEPHI. The first author of this article, teaching MCDA to students for several years, carried out the analysis of students’ preferences

for MCDA methods (each student is required to solve his/her MCDA problem using *DecernsMCDA* software, and then present his/her analysis at an open seminar). Students from the Economic and Management department preferred TOPSIS (about 45%), because “it is the most simple and quick method”, and AHP (~30%), as “it allows the user to compare criteria and alternatives in a simple verbal scale without problems found in other methods requiring weighting and value functions setting”. Students from the Information Systems department gave the opposite results: about 40% of students prefer MAVT/MAUT, ~30% - FMAVT/FMAA/ProMAA, ~15% – PROMETHEE, and 15% - TOPSIS and AHP. The reason for such a difference may be explained by the following factors: the Information Systems students have a better understanding of probability and fuzzy theory; disinclination to use a “black box /AHP” or “very simple TOPSIS method”; willingness to spend more time for a more involved analysis of the multicriteria problem (swing weighting, value functions setting, extended sensitivity analysis, and uncertainty

treatment); a wish to use “more complicated” methods to present more thorough analysis was also a factor for the Information Systems students.

The analysis of the “popularity of different MCDA methods” for the students cannot be directly compared with data presented in Tables 4. In the case with the students, the effects of availability of different methods in one software package (*DecernsMCDA*), the influence of “group preferences”, and several other reasons have an influence on the results of this analysis. However, the conclusion that the choice of MCDA method for analysis of the multicriteria problem depends on the user’s degree of expertise and profession, as well as on the availability of corresponding software has a place.

In our opinion, the presence of several methods in one computer system is effective for decision analysis and decision-making support and can enhance the role of the analyst (expert in MCDA) when solving multicriteria problem. This is a positive factor in improving the quality of the decision-making process and outcomes.

## 8. Conclusion

*Decerns* is a framework for multicriteria decision analysis of spatial and non-spatial alternatives with the use of corresponding scalable decision support systems.

*DecernsSDSS* is a spatial decision support system and includes basic GIS functions for spatial data representation and analysis, and several MCDA methods for multicriteria decision analysis of spatial alternatives. This system, including extended customized versions with problem oriented site-specific models, is flexible enough to permit analysis of a wide range of problems on land-use planning and risk management.

*DecernsMCDA* is a (scalable) DSS for analysis of multicriteria problems. One of the key differences of *DecernsMCDA* from other systems, presented in Table 1, is the inclusion of several popular MADM methods (MAVT, TOPSIS, AHP, PROMETHEE, MAUT) in a single framework. Additionally, *DecernsMCDA* contains a range of original MCDA methods (ProMAA, FMAA, F-MAVT) for uncertainty treatment, including uncertainty of criterion values and weight coefficients with the use of probabilistic and fuzzy sets methods. *DecernsMCDA* tools allow uncertainty analysis through sensitivity analysis of the results to changing weight coefficients and value/utility functions. The architecture

of the system allows a flexible transition from one multicriteria method to another one, and comparison of the results for subsequent decision-making. This approach may be considered as uncertainty analysis to the choice of the MCDA model. The indicated methods for uncertainty analysis allow users/decision makers to refine the decision-making process, in accordance with the scheme in Fig.1, to obtain a robust trade-off decision.

*DecernsMCDA* has been shown to be an effective tool for decision support when analyzing a wide range of multicriteria problems on risk management for various types of environmental problems involving environmental protection, remediation of contaminated sites, and risk based land-use planning. The strength of *DecernsMCDA* is the wide range of MCDA analysis tools contained on a single software platform.

The *DecernsMCDA* software can be used on a desktop system or a web based system, which facilitates its use for education and training. The use of MCDA techniques can lead to improved transparency and acceptance of the recommended decisions based on increased understanding of the trade-offs and consideration of different views.

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