A Fuzzy TODIM Approach for the Supplier Selection Problem

Ömür Tosun
Ayşe Sak School of Applied Sciences, Department of International Trade and Logistics
Akdeniz University, Antalya, Turkey
Tel: 90 242 2274454
E-mail: omurtosun@akdeniz.edu.tr

Gökhan Akyüz
Faculty of Economics and Administrative Sciences, Department of Business Administration
Akdeniz University, Antalya, Turkey
Tel: 90 242 3101913
E-mail: akyuz@akdeniz.edu.tr

Received 9 May 2013; Accepted 22 October 2014

Abstract
Supplier evaluation and selection is a kind of problem which includes multiple criteria of qualitative and quantitative properties. From different alternatives it requires to find the best option using different criteria and opinions of the decision makers. Because of the judgments or the bias of the decision makers, sometimes classical methods cannot be precise. In this paper, a relatively new decision method called TODIM (an acronym in Portuguese for iterative multi-criteria decision-making -“Tomada de Decisão Iterativa Multicritério”) is improved with fuzziness to prevent the above mentioned problems of the classical methods. A real life case study for a furniture manufacturing company is also be solved.

Keywords: Fuzzy TODIM, supplier selection, multicriteria decision making

1. Introduction
To be successful in today’s competitive environment, firms must offer their products and services with proper quality, price and speed to the customers. Achieving success not only depends on the firm’s own performance but also the performance of the other elements on the supply chain. Suppliers are the first element of this chain which also includes manufacturers, distributors and retailers. Reducing supply chain risk, decreasing production costs, optimizing the cycle time, gaining a pre-defined quality level and competitive advantage, selection of the proper supplier is one of the important decisions for any business firms.

Modern supply management approach is to maintain long term partnership with suppliers, and to use fewer but reliable suppliers. Therefore, choosing the right suppliers involves much more than scanning a series of price list, and choices will depend on a wide range of factors which includes both quantitative and qualitative. Multicriteria optimization is the process of determining the best feasible solution according to the established criteria representing different effects. Practical problems are often characterized by several non-commensurable and mostly conflicting criteria, and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision makers’ preferences.

The MCDM (multicriteria decision making) technique is a powerful tool widely used for evaluating and ranking problems containing multiple, usually conflicting criteria. Over the years different behavioral scientists, operational researchers and decision theorists have proposed a variety of methods describing how a decision maker might arrive at a preference judgment while choosing among the multiple attribute alternatives. Analytical models for supplier evaluation and selection range from simple weighted scoring models to complex mathematical programming approaches. The most common approaches and methods...
for supplier selection include different MCDM methods such as analytic hierarchy process (AHP) and analytic network process (ANP), statistical techniques such as principal components analysis and factor analysis, data analysis techniques such as cluster analysis, discriminant analysis, data envelopment analysis (DEA) and simulation.

There are at least two journal articles reviewing the literature regarding supplier evaluation and selection models. De Boer et al. identified four stages for supplier selection including definition of the problem, formulation of criteria, qualification, and final selection, respectively. They reviewed and classified MCDM approaches for supplier selection. Ho et al. described the individual approaches and integrated approaches critically, respectively. They analyzed the most prevalently used approaches, discussed the most popular evaluating criteria, and find out the limitations of the approaches.

The classical MCDM methods cannot effectively handle problems with such imprecise information. These classical methods, both deterministic and random processes, tend to be less effective in conveying the imprecision and fuzziness characteristics. This has led to the development of fuzzy set theory by Zadeh, who proposed that the key elements in human thinking are not numbers but labels of fuzzy sets.

Proposed by Zadeh in 1965, fuzzy set theory is a powerful tool to handle imprecise data and fuzzy expressions which used in different engineering and academic fields. Fuzzy set theory is applied to nearly all of the classical multi criteria decision methods. Similar situation is also seen in the supplier selection problem: fuzzy AHP and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), fuzzy Adaptive Resonance Theory (ART) algorithm, fuzzy mathematical programming, fuzzy multiobjective linear programming, fuzzy VIKOR (Vise Kriterijumsa Optimizacija i Kompromisno Resenje) and fuzzy DEMATEL.

In this study, fuzzy TODIM method will be applied to the supplier selection problem of a furniture manufacturing company. Proposed by Gomes and Rangel, TODIM is a relatively new MCDM based method. Only a few papers exist in the literature and this study is one of the first fuzzy approaches integrated with TODIM and simulation. Also a real life case study is used in the paper. In the second section of the study a brief literature survey about supplier selection is given. TODIM and Fuzzy TODIM methodologies are given in section three and four. Application and solution steps are given step by step in section five. Finally in the Conclusion section general discussions are given.

2. Literature Survey

The very first studies regarding supplier selection are done by Dickson in 1966 which determines the criteria about supplier selection with questionnaires. According to the Weber et al. (1991), which investigates the supplier selection studies between 1966 and 1990, price, delivery time and quality are the most important criteria chosen in these studies. Although this three criteria are the most chosen ones flexibility, process capability, geographical distance, technological capability, product development possibilities and financial situation can become important based on the problem studied.

In Table 1, examples based on the supplier selection and evaluation studies are given. It's seen from Table 1 that in some studies more than one approach is used together. Also classic logic and fuzzy logic methods are used (solely or together) in some of the studies based on the decision making under certainty or uncertainty.

Akarte et al., Tam and Tummala, Özdemir used classic AHP for their supplier selection models. Günner and Mutlu used fuzzy AHP for the selection of suppliers from different sectors. Ozaki et al. used minor AHP for the supplier selection problem in fine casting production. In Pang and Bai for supplier selection of the high technology producer and in Kang et al. for the evaluation of IC packaging firms of the semi-conductive industry, authors used fuzzy ANP. Min used Multi Attribute Utility Theory (MAUT) which combines qualitative and quantitative factors based on risk and uncertainty in international supplier selection problem. Dagdeviren and Eraslan solved supplier selection problem of a new designed semi-manufactured product with PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluation) method. Hong et al. propose an effective supplier selection method to maintain a continuous supply-relationship with suppliers and used mixed integer programming to solve the model. Liu et al. and Shaen both used DEA to evaluate the relative importance of the alternatives. For automotive supplier
selection problem Kumar et al.38 proposed fuzzy mixed integer goal programming whereas Keskin et al.39 proposed ART. Sanayei et al.40, Cheng and Wang41 used fuzzy VIKOR method supplier selection in automotive industry and subcontractor evaluation in information systems project, respectively.

<table>
<thead>
<tr>
<th>Table 1. Examples of supplier selection studies and their methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHP</td>
</tr>
<tr>
<td>Kasirian and Yusuff42</td>
</tr>
<tr>
<td>Pang and Bai31</td>
</tr>
<tr>
<td>Kang et al.32</td>
</tr>
<tr>
<td>Ozaki et al.30</td>
</tr>
<tr>
<td>Shaw et al.43</td>
</tr>
<tr>
<td>Li et al.44</td>
</tr>
<tr>
<td>Li et al.45</td>
</tr>
<tr>
<td>Kubat and Yüce46</td>
</tr>
<tr>
<td>Luo et al.48</td>
</tr>
<tr>
<td>Sanayei et al.40</td>
</tr>
<tr>
<td>Özdemir27</td>
</tr>
<tr>
<td>Keskin et al.39</td>
</tr>
<tr>
<td>Lee29</td>
</tr>
<tr>
<td>Chen and Wang41</td>
</tr>
<tr>
<td>Luo et al.49</td>
</tr>
<tr>
<td>Li et al.45</td>
</tr>
<tr>
<td>Kang et al.32</td>
</tr>
<tr>
<td>Liu et al.36</td>
</tr>
<tr>
<td>Tam and Tummala26</td>
</tr>
<tr>
<td>Min33</td>
</tr>
<tr>
<td>F: Fuzzy - C:Classic</td>
</tr>
</tbody>
</table>

There are many papers using combination of two or more methods together. Kasirian and Yusuff42 presents a comprehensive approach to find the best ranking among the alternative suppliers of a typical product. A hybrid modified TOPSIS is integrated with a preemptive goal programming model in the study and then compared to AHP and ANP to show the effect of considering interdependencies in the supplier selection process. Shaw et al.43 used fuzzy AHP with fuzzy multi-criteria programming for the supplier selection of a textile company based on carbon emitting criterion. Li et al.44 used fuzzy AHP to determine criteria weights and DEA to determine alternative suppliers, then using axiomatic fuzzy sets method to make the final evaluation. In Li et al.45 authors classified suppliers with axiomatic fuzzy sets methods, determine criteria weights with fuzzy AHP and used TOPSIS for the final evaluation. Kubat and Yüce46 used fuzzy AHP to calculate each suppliers’ weights and then used genetic algorithm (GA) to find the best supplier. Soner and Onüt47 combined AHP and ELECTRE (ELimination Et Choix Tradusiant la REalite), where they used AHP to calculate criteria weights and ELECTRE for the selection.

Luo et al.48 and Kuo et al.49 are the examples of papers using artificial intelligence for the supplier selection. In Ref. 48 artificial neural networks (ANN) and genetic algorithms for an agile supply chain
network, whereas in Ref 49 DEA, ANP and artificial neural networks are used for supplier selection in green supply chain.

3. The TODIM method

The TODIM method (an acronym in Portuguese of Interactive and Multi-criteria Decision Making) is a discrete multi-criteria method based on Prospect Theory\textsuperscript{50} which has awarded the Nobel Prize for Economics in 2002\textsuperscript{51}.

Consider a set of \( m \) alternatives to be ordered in the presence of \( n \) quantitative or qualitative criteria, and assume that one of these criteria can be considered as the reference criterion. After the definition of these elements, experts are asked to estimate, for each one of the qualitative criteria \( c \), the contribution of each alternative \( I \) to the objective associated with the criterion. This method requires the values of the evaluation, of the alternatives in relation to the criteria, to be numerical and to be normalized; consequently the qualitative criteria evaluated in a verbal scale are transformed into a cardinal scale\textsuperscript{19} and \textsuperscript{52}.

The evaluation of the alternatives in relation to all the criteria produces the matrix of evaluation, where the values are all numerical. Their normalization is then carried out for each criterion, the division of the value of one alternative by the sum of all the alternatives. This normalization is carried out for each criterion and obtains a matrix with all the values between zero and one. It is called the matrix of normalized alternatives' scores against criteria. \( P = [P_{mi}] \), with \( m \) indicating the number of alternatives and \( n \) the number of criteria, as shown in Table 2.

<table>
<thead>
<tr>
<th>( A_i )</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>…</th>
<th>( C_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>( P_{11} )</td>
<td>( P_{12} )</td>
<td>…</td>
<td>( P_{1n} )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( P_{21} )</td>
<td>( P_{22} )</td>
<td>…</td>
<td>( P_{2n} )</td>
</tr>
<tr>
<td>… | | | | |</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_m )</td>
<td>( P_{m1} )</td>
<td>( P_{m2} )</td>
<td>…</td>
<td>( P_{mn} )</td>
</tr>
</tbody>
</table>

After the attribution of the weights of the criteria and their normalization, the partial matrices of dominance and the final matrix of dominance must be calculated. The decision makers must indicate which criterion \( r \) is to be chosen as the reference criterion for the calculations according to the relative importance assigned to each criterion. In this way, the criterion with the highest value accorded to its importance will usually be chosen as the reference criterion. The weight of each criterion is determined by the decision makers on a numerical scale and is then normalized. Thus, \( w_{rc} \) is the weight of criterion \( c \) divided by the weight of the reference criterion \( r \). Using \( w_{rc} \) allows all pairs of differences between performance measurements to be translated into the same dimension, i.e. that of the reference criterion. The final measurement of dominance of each alternative \( A_j \), over each alternative \( A_i \), now incorporated to Prospect Theory, is given by the mathematical expression (1). That measurement is given by a sum of relative gains and losses. Expression (2a) describes the gain part of the value function, while expression (2c) describes its loss part. Expression (2b) applies where there exists neither a gain nor a loss.

\[
\Phi_c = \begin{cases} 
\frac{\sum w_{rc}(P_{ic} - P_{jc})}{\sum w_{rc}P_{jc}} & \text{if } (P_{ic} - P_{jc}) > 0, \ (a) \\
0 & \text{if } (P_{ic} - P_{jc}) = 0, \ (b) \\
-\frac{1}{\theta} \sqrt{\frac{\sum w_{rc}P_{ic} - P_{jc}^2}{\sum w_{rc}P_{jc}}} & \text{if } (P_{ic} - P_{jc}) < 0, \ (c)
\end{cases}
\]

\[
\delta(A_i, A_j) = \sum_{c=1}^{m} \Phi_c(A_i, A_j), \forall (i, j)
\]

\( \delta(A_i, A_j) \) represents the measurement of dominance of alternative \( A_i \), over alternative \( A_j \);

\( n \) is the number of criteria;

\( c \) is any criterion, for \( c = 1, …, n \);

\( w_{rc} \) is equal to \( w_r \) divided by \( w_c \), where \( r \) is the reference criterion;

\( P_{ic} \) and \( P_{jc} \) are, respectively, the performances of the alternatives \( A_i \) and \( A_j \) in relation to \( c \);

\( \theta \) is the attenuation factor of the losses; different choices of \( \theta \) lead to different shapes of the prospect theoretical value function in the negative quadrant.

The expression \( \Phi_c(A_i, A_j) \), represents the contribution of criterion \( c \) to function \( \delta(A_i, A_j) \), when comparing alternative \( i \) with alternative \( j \). If the value of \( (P_{ic} - P_{jc}) \) is positive, it will represent a gain for the function \( \delta(A_i, A_j) \) and, therefore the expression \( \Phi_c(A_i, A_j) \) will be used in the Eq. (2a). If \( (P_{ic} - P_{jc}) \) is zero, the value zero will be assigned to \( \Phi_c(A_i, A_j) \) by applying the Eq. (2b). If \( (P_{ic} - P_{jc}) \) is negative, \( \Phi_c(A_i, A_j) \) will be represented by the Eq. (2c). The construction of function \( \Phi_c(A_i, A_j) \) in fact permits an
adjustment of the data of the problem to the value function of Prospect Theory, thus explaining the aversion and the propensity to risk.

After the diverse partial matrices of dominance have been calculated, one for each criterion, the final dominance matrix of the general element \( \delta(A_i, A_j) \) is obtained, through the sum of the elements of the diverse matrices.

Expression (3) is used to determine the overall value of alternative \( i \) through normalization of the corresponding dominance measurements. The rank of every alternative originates from the ordering of their respective values.

\[
\xi_i = \frac{\sum_{j=1}^{m} \delta(A_i, A_j) - \min_{j=1}^{m} \delta(A_i, A_j)}{\max_{j=1}^{m} \delta(A_i, A_j) - \min_{j=1}^{m} \delta(A_i, A_j)} \quad (3)
\]

Therefore, the global measures obtained computed by (3) permit the complete rank ordering of all alternatives. A sensitivity analysis can be applied to verify the stability of the results based on the decision makers' preferences. The sensitivity analysis should therefore be carried out on \( \theta \) as well as on the criteria weights, the choice of the reference criterion, and performance evaluations\(^{19} \text{ and } 52\).

### 4. Fuzzy TODIM

To prevent the effects of decision makers’ prejudice and bias in the ranking of alternatives, fuzziness has integrated to the original TODIM. Triangular fuzzy numbers are used for expressing the linguistic variables for the attribute values. Using these fuzzy numbers and according to the concept of the TODIM method, gain and loss of each one of the alternatives relative to another are assessed. Then, by calculating the dominance degree of each alternative over the others, the overall value of each alternative is obtained and alternatives are ranked. The proposed fuzzy TODIM method is discussed below.

#### 4.1. Proposed fuzzy TODIM method

In this chapter Fuzzy TODIM approach of Zhang and Fan\(^{53}\) will be discussed. Let \( M = \{1, 2, ..., m\} \) and \( N = \{1, 2, ..., n\} \). Let \( A = \{A_1, A_2, ..., A_m\} \) be a finite alternative set, where \( A_i \) denotes the \( i \) th alternative; \( C = \{C_1, C_2, ..., C_n\} \) be a finite attribute set, where \( C_j \) denotes the \( j \) th attribute. Let \( w = (w_1, w_2, ..., w_m)^T \) be an attribute weight vector, where \( w_j \) denotes the weight or the importance degree of attribute \( C_j \), such that \( \sum_{j=1}^{n} w_j = 1 \) and \( 0 \leq w_j \leq 1 \), \( j \in N \). \( S = \{s_f \mid f = 0, 1, ..., T\} \) is the linguistic term set, \( X = [\tilde{x}_{ij}]_{m \times n} \) be a decision matrix, where \( \tilde{x}_{ij} \) is the attribute value, i.e., the linguistic assessment of alternative \( A_i \) with respect to attribute \( C_j \), \( \tilde{x}_{ij} \in S, i \in M, j \in N \).

**Step 1: Define the linguistic variables.**

Linguistic variables are given with linguistic terms. Let \( S = \{s_0 = \text{VL: very low}, s_1 = \text{L: low}, s_2 = \text{M: medium}, s_3 = \text{H: high}, s_4 = \text{VH: very high}\} \).

A linguistic variable \( s_f \) can be represented by a triangular fuzzy number \( \tilde{A} = (l, m, u) \) using the formula given below:

\[
\tilde{A} = (l, m, u) = \left( \max \left( \frac{f}{T}, 0 \right), \frac{T}{T}, \min \left( \frac{f+1}{T}, 1 \right) \right) \quad (4)
\]

**Step 2: Evaluate the criteria and alternatives**

Linguistic variables are used in evaluating the criteria and alternatives. To lessen the subjectivity, more than one decision maker (expert) must be selected. After decision maker’s evaluation, their scores are integrated. Integrated fuzzy weights of each criteria and integrated fuzzy evaluation of each alternatives according to each criteria can be calculated with the given equations. In the formulas, \( d \) is the number of decision makers.

\[
cw_j = \frac{1}{d} \left[ \frac{d}{\sum_{e=1}^{d} cw_j^e} \right] \quad j = 1, 2, ..., n \quad (5)
\]

\[
\tilde{x}_{ij} = \text{integrated fuzzy evaluation of } i. \text{ alternative according to } j. \text{ criteria}
\]

\[
\tilde{x}_{ij} = \frac{1}{d} \left[ \frac{d}{\sum_{e=1}^{d} \tilde{x}_{ij}^e} \right] \quad i = 1, 2, ..., m \quad (6)
\]

fuzzy \( \tilde{x}_{ij} \) values are used as triangular fuzzy numbers in generating the loss and gain matrices.
Step 3: Fuzzy criteria weights \((cw_j)\) are defuzzificated. From different methods of normalization, method of Abdel-Kader and Dugdale\(^5^4\) is used in this study.

In this method, the three parameters of triangular fuzzy numbers (full memberships, partial memberships located in the right-hand side and left-hand side) for the fuzzy estimates are used. Also, and index of optimism \((\alpha)\) is used in the ranking process. Bigger values in \(\alpha\) represents an optimistic decision maker, whereas smaller values represents a pessimistic decision maker.

\(\alpha\) parameter is used to reflect the decision makers’ characteristics, risk taking attitude and different environment conditions. For example, in a high uncertainty environment a decision maker with a risk avoiding attitude prefers a lower index of optimism. On the other hand, the calculations can be repeated for avoiding attitude prefers a higher index of optimism to balance between pessimism and optimism.

Let \(\varepsilon \in [0, 1]\) will be index of optimism. For a fuzzy number \(\tilde{A}_j=(l_j, m_j, u_j)\) \((j=1, 2, ..., n)\);

Let \(V(\tilde{A}_j)\) will be the value of \(\tilde{A}_j\). In this situation, ordering can be calculated as;

\[
V(\tilde{A}_j) = m_j \left\{ \begin{array}{c}
\alpha \left[ \frac{-u_j-x_{\min}}{x_{\max}-x_{\min}+u_j-m_j} \right] \\
(1-\alpha) \left[ \frac{-x_{\max}-l_j}{x_{\max}-x_{\min}+m_j-l_j} \right]
\end{array} \right.
\]

(7)

Here;

\[
x_{\min} = \inf S
\]

\[
x_{\max} = \sup S
\]

\[
S = \bigcup_{j=1}^{n} S_j
\]

\[
S_j=(l_j, m_j, u_j, ..., l_m, m_m, u_m) \ j=1, 2, ..., n
\]

(8)

(9)

(10)

Calculated weights with the ordering method are standardized (normalized) with the given formula:

\[
w_j = \frac{V(\tilde{A}_j)}{\sum_{j=1}^{n} V(\tilde{A}_j)}
\]

(11)

Step 4: Calculation of Gains and Loses

To calculate the gain and loss of each alternative relative to the others, first the attribute values of alternatives must be compared by pair. Let \(\tilde{x}_{ij}\) and \(\tilde{x}_{kj}\) be the alternative value of alternative \(A_i\) and \(A_k\) concerning attribute \(C_j\), \(i, k \in M, j \in N\). Let \(s_f\) and \(s_g\) be the linguistic assessments of alternative \(A_i\) and \(A_k\) with respect to attribute \(C_j\), \(f, g = 0, 1, ..., T\). Then \(\tilde{x}_{ij}\) and \(\tilde{x}_{kj}\) are compared as follows\(^2^0\) and\(^5^3\).

1. If \(s_f > s_g\), then \(\tilde{x}_{ij} > \tilde{x}_{kj}\)
2. If \(s_f < s_g\), then \(\tilde{x}_{ij} < \tilde{x}_{kj}\)
3. If \(s_f = s_g\), then \(\tilde{x}_{ij} = \tilde{x}_{kj}\), where \(i, k \in M, j \in N\).

Here \(\tilde{x}_{ij}\) and \(\tilde{x}_{kj}\) are represented by triangular numbers given in Eq. (4). The distance between them are used to measure the separation between two fuzzy numbers:

\[
d(\tilde{x}_i, \tilde{x}_k) = \frac{1}{2} [ (x_{ij} - x_{kj})^2 + (x_{ij} - x_{kj})^2 + (x_{ij} - x_{kj})^2 ]
\]

(12)

Then, gain and loss of alternatives \(A_i\) relative to \(A_k\) concerning attribute \(C_j\) can be given as:

For benefit attribute:

\[
G^{ij}_{ik} = \begin{cases} 
0, & \tilde{x}_{ij} \leq \tilde{x}_{kj} \\
n(\tilde{x}_i, \tilde{x}_k), & \tilde{x}_{ij} > \tilde{x}_{kj}
\end{cases}
\]

(13)

\[
L^{ij}_{ik} = \begin{cases} 
0, & \tilde{x}_{ij} \geq \tilde{x}_{kj} \\
n(\tilde{x}_i, \tilde{x}_k), & \tilde{x}_{ij} < \tilde{x}_{kj}
\end{cases}
\]

(14)

For cost attribute:

\[
G^{ij}_{ik} = \begin{cases} 
0, & \tilde{x}_{ij} \geq \tilde{x}_{kj} \\
n(\tilde{x}_i, \tilde{x}_k), & \tilde{x}_{ij} < \tilde{x}_{kj}
\end{cases}
\]

(15)

\[
L^{ij}_{ik} = \begin{cases} 
0, & \tilde{x}_{ij} \leq \tilde{x}_{kj} \\
n(\tilde{x}_i, \tilde{x}_k), & \tilde{x}_{ij} > \tilde{x}_{kj}
\end{cases}
\]

(16)

Using the above equations gain matrix \(G_j = [G^{ij}_{ik}]_{max}\) and loss matrix \(L_j = [L^{ij}_{ik}]_{max}\) for attribute \(C_j\) can be constructed\(^2^0\) and\(^5^3\).

Step 5: Calculate each criterion’s \((C_j)\) relative weights \((w_{Cj})\) based on the reference criterion \((C_r)\)

TODIM method is based on a projection of the differences between the consequences of any two alternatives to a reference attribute. Attribute with the highest weight is chosen as the reference attribute to
translate all pairs of differences between performance measurements into the same dimension. Let \( C_r \) denote the reference attribute, then the relative weight \( w_{jr} \) of attribute \( C_j \) to the reference attribute \( C_r \) can be given as\(^{20,53}\).

\[
w_{jr} = w_j / w_r, \forall N \text{ where } w_r = \max \{w_j | j \in N\} \quad (17)
\]

**Step 6: Construct dominance degree matrix for each criterion (\( C_j \))**

First of all, calculate the dominance degree of alternative \( A_i \) over alternative \( A_k \) for attribute \( C_j \). The dominance degree for gain (\( \Phi_{ik}^{( + )} \)) and dominance degree for loss (\( \Phi_{ik}^{( - )} \)) can be calculated as:

\[
\Phi_{ik}^{( + )} = \frac{\sqrt{G_{ik}^j w_{jr} / \left( \sum_{j=1}^n w_{jr} \right)}}{\omega_j} \quad (18)
\]

\[
\Phi_{ik}^{( - )} = -\frac{1}{\theta} \sqrt{-L_{ik}^j \left( \sum_{j=1}^n w_{jr} \right)} / w_{jr}, \quad (19)
\]

Where \( \theta \) is the attenuation factor of the loss. Then the dominance degree for the gain and loss (\( \Phi_{ik} \)) can be found:

\[
\Phi_{ik} = \Phi_{ik}^{( + )} + \Phi_{ik}^{( - )} \quad (20)
\]

By using it, dominance degree matrix for attribute \( C_j \), \( \Phi_j = [\Phi_{ik}]_{mn} \), can be constructed.

**Step 7: Calculate overall dominance degree matrix (\( \Delta = [\delta_{ik}]_{mnm} \))**

\[
\delta_{ik} = \sum_{j=1}^n \Phi_{ik}^j \quad (21)
\]

**Step 8: Calculate overall value of each alternative and rank the alternatives.**

Based on matrix \( \Delta \), overall value of alternative \( A_i \), \( \xi(A_i) \), can be calculated:

\[
\xi(A_i) = \frac{\sum_{k=1}^m \delta_{ik} - \min_{k \in M} \{ \sum_{k=1}^m \delta_{ik} \}}{\max_{k \in M} \{ \sum_{k=1}^m \delta_{ik} \} - \min_{k \in M} \{ \sum_{k=1}^m \delta_{ik} \}} \quad (22)
\]

Obviously, \( 0 \leq \xi(A_i) \leq 1 \), and the greater \( \xi(A_i) \) is, the better alternative \( A_i \) will be. Therefore, in accordance with a descending order of the overall values of all the alternatives, one can determine the ranking of all alternatives or select the desirable alternative(s).

### 5. Application

Proposed methodology is going to use for the packaging supplier selection problem of a furniture manufacturing company. The company was established in 1984 to serve in the sector. Today, all kinds of furniture accessories sector needs to be heard (profile, paneling and parts of sub-groups of these products) production in Antalya Organized Industrial Zone is 400,000 m² premises realizes.

Production of more than 50% is exported to nearly 50 countries, also on the basis production quality and capacity of the sector, the company is one of the world’s largest. Using all the possibilities of modern technology to 900 qualified workforce, company produces 100 million per running meter in a year of production. With the great share of R & D budget, company offers unique opportunities to its customers.

Working with the weekly production schedules, it’s very important for the company to get the packaging on time with the desired quality and specifications to prevent the shipping delays. For this reason, choosing the proper supplier is very critical for the decision makers. In this study, current suppliers of the firms are evaluated with different suppliers from the market.

A group of five decision makers are selected from the company for the study. According to their expectations and experiences, five alternative suppliers based on seven criteria are selected. These criteria are selected with the help of decision makers’ experiences and literature surveys: \( C_1 \)-Price, \( C_2 \)-Quality, \( C_3 \)-Delivery on time, \( C_4 \)-Technical capability, \( C_5 \)-Previous period performance, \( C_6 \)-Geographical location, \( C_7 \)-Technology.

Questionnaires are given to the decision makers for the evaluation process. Decision makers’ linguistic evaluation of the criteria and the alternatives with respect to each criterion are transformed to triangular fuzzy numbers (Table 3).

Using equation (5) and (6), opinions and views of the decision makers are integrated (Table 4, Table 5).

Using the fuzzy values in Table 5, gain and loss matrices for each criteria are calculated with Eq. 12-16.
Table 3. Linguistic variables and fuzzy numbers

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low (VL)</td>
<td>(0.00, 0.00, 0.25)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.00, 0.25, 0.50)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.25, 0.50, 0.75)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(0.50, 0.75, 1.00)</td>
</tr>
<tr>
<td>Very High (VH)</td>
<td>(0.75, 1.00, 1.00)</td>
</tr>
</tbody>
</table>

Table 4. Aggregated fuzzy weights of the criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Fuzzy weights (cw)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>l</td>
</tr>
<tr>
<td>C_1</td>
<td>0.60</td>
</tr>
<tr>
<td>C_2</td>
<td>0.60</td>
</tr>
<tr>
<td>C_3</td>
<td>0.65</td>
</tr>
<tr>
<td>C_4</td>
<td>0.60</td>
</tr>
<tr>
<td>C_5</td>
<td>0.30</td>
</tr>
<tr>
<td>C_6</td>
<td>0.45</td>
</tr>
<tr>
<td>C_7</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 5. Fuzzy decision matrix of the alternatives

<table>
<thead>
<tr>
<th></th>
<th>A_1</th>
<th>A_2</th>
<th>A_3</th>
<th>A_4</th>
<th>A_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria</td>
<td>l</td>
<td>m</td>
<td>u</td>
<td>l</td>
<td>m</td>
</tr>
<tr>
<td>C_1</td>
<td>0.45</td>
<td>0.70</td>
<td>0.90</td>
<td>0.50</td>
<td>0.75</td>
</tr>
<tr>
<td>C_2</td>
<td>0.40</td>
<td>0.65</td>
<td>0.90</td>
<td>0.60</td>
<td>0.85</td>
</tr>
<tr>
<td>C_3</td>
<td>0.45</td>
<td>0.70</td>
<td>0.80</td>
<td>0.45</td>
<td>0.70</td>
</tr>
<tr>
<td>C_4</td>
<td>0.50</td>
<td>0.75</td>
<td>0.90</td>
<td>0.50</td>
<td>0.75</td>
</tr>
<tr>
<td>C_5</td>
<td>0.45</td>
<td>0.70</td>
<td>0.95</td>
<td>0.40</td>
<td>0.65</td>
</tr>
<tr>
<td>C_6</td>
<td>0.75</td>
<td>1.00</td>
<td>1.00</td>
<td>0.30</td>
<td>0.55</td>
</tr>
<tr>
<td>C_7</td>
<td>0.45</td>
<td>0.70</td>
<td>0.95</td>
<td>0.60</td>
<td>0.85</td>
</tr>
</tbody>
</table>

G_1 = \[
\begin{bmatrix}
0 & 0 & 0.2843 & 0.3841 & 0.3674 \\
0.0500 & 0 & 0.3342 & 0.4340 & 0.4173 \\
0 & 0 & 0 & 0.2000 & 0.0500 \\
0 & 0 & 0 & 0 & 0.2849 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

G_2 = \[
\begin{bmatrix}
0.1732 & 0.2217 & 0.0408 & 0.1732 \\
0 & 0 & 0 & 0 \\
0.1354 & 0.1848 & 0 & 0.1354 \\
0 & 0 & 0.0500 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

G_3 = \[
\begin{bmatrix}
0 & 0.0577 & 0.2121 & 0.0707 & 0 \\
0.0577 & 0 & 0.2345 & 0.0408 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0957 \\
0.0957 & 0.0500 & 0.2843 & 0.0866 & 0
\end{bmatrix}
\]

G_4 = \[
\begin{bmatrix}
0 & 0.0577 & 0.1258 & 0 & 0.0577 \\
0.0577 & 0 & 0.1500 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0.0707 & 0.0408 & 0.1848 & 0 & 0.0408 \\
0.0577 & 0 & 0.1500 & 0 & 0
\end{bmatrix}
\]

L_1 = \[
\begin{bmatrix}
0 & 0 & 0.0500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.2843 & -0.3342 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.3841 & -0.4340 & -0.1000 & 0 & 0 & -0.2849 & 0 & 0 & 0 & 0 \\
-0.3674 & -0.4173 & -0.0866 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

L_2 = \[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.1732 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

L_3 = \[
\begin{bmatrix}
0 & 0.0577 & 0.2121 & 0.0707 & 0 \\
0.0577 & 0 & 0.2345 & 0.0408 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0957 \\
-0.0707 & -0.0408 & 0 & 0 & -0.0866 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

L_4 = \[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & -0.0707 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.0408 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.1258 & -0.1500 \\
0 & 0 & 0 & 0 & 0 & -0.1848 & -0.1500 \\
0 & 0 & 0 & 0 & 0 & -0.0408 & 0
\end{bmatrix}
\]
Criteria weights \((w_j)\) given in Table 4 are defuzzicated using Equations (7) - (11) and standardized. Relative criteria weights \((w_{jr})\) are calculated with Equation (17), in which \(C_j\) is selected as the reference criterion.

Table 6. Relative weights of the criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>(w_j)</th>
<th>(w_{jr})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1)</td>
<td>0.177</td>
<td>0.879</td>
</tr>
<tr>
<td>(C_2)</td>
<td>0.177</td>
<td>0.879</td>
</tr>
<tr>
<td>(C_3)</td>
<td>0.201</td>
<td>1.000</td>
</tr>
<tr>
<td>(C_4)</td>
<td>0.177</td>
<td>0.879</td>
</tr>
<tr>
<td>(C_5)</td>
<td>0.062</td>
<td>0.310</td>
</tr>
<tr>
<td>(C_6)</td>
<td>0.111</td>
<td>0.551</td>
</tr>
<tr>
<td>(C_7)</td>
<td>0.096</td>
<td>0.479</td>
</tr>
</tbody>
</table>

Dominance degree matrices concerning attributes are given below:

\[
G_5 = \begin{bmatrix}
0 & 0.0707 & 0.2000 & 0.1190 & 0.0500 \\
0 & 0 & 0.1354 & 0.0500 & 0.0289 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0866 & 0 & 0 \\
0 & 0.0289 & 0.1500 & 0.0707 & 0 \\
\end{bmatrix}
\]

\[
L_5 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
-0.0707 & 0 & 0 & 0 & 0 \\
-0.2000 & -0.1354 & 0 & -0.0866 & -0.1500 \\
-0.1190 & -0.0500 & 0 & 0 & -0.0707 \\
\end{bmatrix}
\]

\[
G_6 = \begin{bmatrix}
0 & 0.3851 & 0.1658 & 0.0408 & 0.0408 \\
0 & 0.2217 & 0 & 0 & 0 \\
0 & 0.3464 & 0.1258 & 0 & 0 \\
0 & 0.3464 & 0.1258 & 0 & 0 \\
\end{bmatrix}
\]

\[
L_6 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
-0.3851 & 0 & -0.2217 & -0.3464 & -0.3464 \\
-0.1658 & 0 & 0 & -0.1258 & -0.1258 \\
-0.0408 & 0 & 0 & 0 & 0 \\
-0.0408 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
G_7 = \begin{bmatrix}
0 & 0 & 0.0289 & 0.0500 & 0 \\
0 & 0 & 0.1258 & 0.1354 & 0.1732 & 0.0816 \\
0.0289 & 0 & 0 & 0.0408 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0.0500 & 0 & 0.0707 & 0.1000 & 0 \\
\end{bmatrix}
\]

\[
L_7 = \begin{bmatrix}
0 & 0 & -0.1258 & 0 & 0 & -0.0500 \\
0 & 0 & 0 & -0.1354 & 0 & 0 \\
0 & 0 & -0.1354 & 0 & 0 & -0.0707 \\
0 & 0 & -0.1732 & -0.0408 & 0 & -0.1000 \\
0 & 0 & -0.0816 & 0 & 0 & 0 \\
\end{bmatrix}
\]

In this study \(\theta\) is taken as 1, which means that losses will contribute with their real values to the global value. Using Eq.21, overall dominance matrix can be found:

\[
\Delta = \begin{bmatrix}
0 & -2.1839 & 0.9734 & -0.9060 & -0.9322 \\
-2.3433 & 0 & -0.3882 & -1.6119 & -1.6898 \\
-6.6367 & -7.0026 & 0 & -5.9941 & -5.9954 \\
-4.5126 & -4.4561 & -0.6503 & 0 & -2.9067 \\
-2.6370 & -3.1098 & 0.0926 & -0.9886 & 0 \\
\end{bmatrix}
\]
Finally overall value of each alternative can be obtained as $\xi(A_1) = 1$, $\xi(A_2) = 0.8623$, $\xi(A_3) = 0$, $\xi(A_4) = 0.5629$ and $\xi(A_5) = 0.8342$. According to these values ranks of the alternatives are $A_1 > A_2 > A_5 > A_4 > A_3$.

In order to study the influence of the parameter $\theta$, sensitivity analysis for different $\theta$ values is also given. Order of the alternatives for the different $\theta$ values are given in Table 7. The order of the alternatives are not changed for $\theta = 1, 2, 3$ and 4. But when $\theta$ is equal to 5, alternative 1 and 2 obtain the same priority.

As we can notice, the alternative $A_1$ represents the best alternative for all $\theta$ values, also the order of the alternatives do not change. This means that the alternative $A_1$ even though affected by uncertainty remains the best alternative. The results for $\theta = 1$ and $\theta = 4$ are almost the same, which indicates the robustness of the method.

Table 7. Ranking orders of alternatives with different values of $\theta$

<table>
<thead>
<tr>
<th>Different values of $\theta$</th>
<th>Ranking orders of alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 1$</td>
<td>$A_1 &gt; A_2 &gt; A_5 &gt; A_4 &gt; A_3$</td>
</tr>
<tr>
<td>$\theta = 2$</td>
<td>$A_1 &gt; A_2 &gt; A_5 &gt; A_4 &gt; A_3$</td>
</tr>
<tr>
<td>$\theta = 3$</td>
<td>$A_1 &gt; A_2 &gt; A_5 &gt; A_4 &gt; A_3$</td>
</tr>
<tr>
<td>$\theta = 4$</td>
<td>$A_1 &gt; A_2 &gt; A_5 &gt; A_4 &gt; A_3$</td>
</tr>
<tr>
<td>$\theta = 5$</td>
<td>$A_1 = A_2 &gt; A_5 &gt; A_4 &gt; A_3$</td>
</tr>
</tbody>
</table>

Table 8. Fuzzy VIKOR, TOPSIS and TODIM results

<table>
<thead>
<tr>
<th>Index</th>
<th>Fuzzy VIKOR ($v=0.5$)</th>
<th>Fuzzy TOPSIS</th>
<th>Fuzzy TODIM</th>
<th>Closeness coefficient (Ci)</th>
<th>Total value</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.112</td>
<td>0.407</td>
<td>2</td>
<td>1.000</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.107</td>
<td>0.429</td>
<td>1</td>
<td>0.862</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$A_3$</td>
<td>1.000</td>
<td>0.329</td>
<td>5</td>
<td>0.000</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.578</td>
<td>0.380</td>
<td>3</td>
<td>0.562</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.485</td>
<td>0.379</td>
<td>4</td>
<td>0.834</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusions

In this study, fuzzy TODIM method is proposed for the supplier selection problem which is a multi criteria decision making problem including both qualitative and quantitative criteria. Existence of the uncertainty affects the efficiency of the decision process, to lessen this disadvantage fuzzy logic is applied. Fuzzy TODIM is a relatively new MCDM method based on the Prospect Theory. This study is one of the first applications of the proposed methodology to supplier selection problem.

For each of the criterion, each alternative is compared with other alternatives one by one and this is the most important point where TODIM differentiates from the other well-known MCDM methods. According to these comparing, gain and losses matrices are determined and used for the ranking of the alternatives.

A real life case study is solved with fuzzy TODIM and compared against fuzzy VIKOR and fuzzy TOPSIS. Proposed methodology can further be developed with integrating different methods. For example AHP can be used to determine the criteria weights. Also it can be used to solve different MCDM problems like facility location selection, project selection and supplier performance measuring. Visual Basic based macros in Excel are prepared to solve the above mentioned three methods. Decision makers can do sensitivity analysis to see effects of change in parameters. These macros can be developed to used as a decision support system software.
Acknowledgement

This paper was supported by The Scientific Research Projects Coordination Unit of the Akdeniz University.

References


