1. Introduction

A key component of medical diagnosis is concerned with the representation and reasoning of medical domain knowledge. Most medical knowledge can be structured in the form of IF-THEN rules. In each rule, we distinguish antecedent attributes (causes) and consequents (effects) e.g. (Ref. 1). For instance, an IF-THEN rule for stroke diagnosis may involve 4 stroke symptoms and 3 consequents. However, stroke symptoms are often described by stoker using linguistic terms. Hence the attributes of initial IF-THEN rules provided by medical expert usually involve linguistic variables. Kong e.g. (Ref. 2) discussed that uncertainty is mostly resulting from subjective domain knowledge or various clinical symptoms. In addition, considering the probability of occurrence of attributes is not always 1, belief degree is taken into account in IF-THEN rules. Lin e.g. (Ref. 3)stated that the result of medical diagnosis is mainly determined by handling various types of uncertainty. In order to complete medical diagnosis, we refer to various frameworks for representing and reasoning for medical domain knowledge under probabilistic and fuzzy uncertainty: multiple attribute decision analysis (MADA), fuzzy set (FS) theory and evidential reasoning (ER). There is a tendency that applying the latest development in these three aspects to medical diagnosis. Evidence theory as a generalization of possibility theory is also known as Dempster-Shafer theory. Jones e.g. (Ref. 4) built a framework for medical diagnosis using the evidence theory. Durbach and Stewart e.g. (Ref. 5) noted that the DS theory of evidence reasoning can deal with probabilistic uncertainty by replacing subjective probabilities with degrees of belief. To model two types of uncertainty, namely fuzziness and probability, DS theory of evidence is extended into fuzzy DS (FDS) theory during the recent years. Xu e.g. (Ref. 6) made a review of the evidential reasoning (ER) approach about its theoretical development and applications. As such, there is a trend that the DS theory of evidence reasoning can be extended to FDS scheme e.g.((Ref. 7), (Ref. 8) and (Ref. 9)). FDS evidence reasoning algorithm can be divided into three steps e.g. (Ref. 10): building a fuzzy evidence structure, combining evidence, and decision making based on ranking consequences.

Uncertainty of evidence is mainly reflected in the following aspects: attribute assessment, attribute weights, rule weights. During the development of the FDS theory, uncertain evidence can be expressed by fuzzy sets, interval numbers, and fuzzy numbers. Yang e.g. (Ref. 11) proposed a rule-based inference methodology using the evidential reasoning (RIMER) approach by adding belief degree of inputs and outputs into the traditional IF-THEN rule base. During the process of building a fuzzy evidence structure, the nonlinear relationship between antecedent attributes and consequents can be established. Max-min operation is adopted to set the matching degree between fuzzy sets in transformation of inputs. Sevastianov e.g. (Ref. 12) discussed that there are two restrictions in the RIMER approach. One of them is that the RIMER approach did not provide the combination method of different evidence. Basically, the ER approach made use of Dempster's rule of combination to aggregate attributes e.g. (Ref. 13).When the evidence structure was expressed by interval evidence, existing combination methods often lead to irrational structure because of improper treatment for the normalization process. Wang e.g. (Ref. 14) analyzed the interval data operations and presented the nonlinear combination method of interval belief degrees. Guo et al. e.g. (Ref. 15) handled the interval beliefs and interval weights to develop an enhanced ER approach. Aminravan et al. e.g. (Ref. 16) developed both fuzzy interval-grade and interval-valued belief degree (IGIB) using one of generalized fuzzy sets (i.e., vague sets). Max-min operation is adopted to set the matching degree between vague sets during the transformation of inputs. Husain e.g. (Ref. 17) concluded that the concept of intuitionistic fuzzy sets (IFSs) can be viewed as another generalized fuzzy set in cases where a conventional fuzzy set is not sufficient for definition of imprecise information. An interpretation of IFSs in terms of evidence theory was presented through converting IFSs into interval fuzzy sets (IVFSs) e.g. (Ref. 18). The operations on IFSs were presented and interval comparison methods were also discussed in the framework of the DS theory of evidence e.g. (Ref. 19). Wang e.g. (Ref. 20) applied triangular intuitionistic fuzzy numbers to fuzzy evidence reasoning. Hence, IFSs can be used to represent those nonspecific attributes. However, in most cases input attributes and antecedent attributes form pieces of linguistic knowledge. Using fuzzy numbers we can represent uncertainty to the highest extent than when dealing with discrete fuzzy sets. Furthermore, max-min operation between fuzzy sets may lead to significant losses of information. In particular, the extreme values representing the dominating attributes are only taken into account in this operation e.g. (Ref. 21). The values in-between are always neglected by decision makers. In order to improve the accuracy of the decision results, it is necessary to ensure the minimal losses of information.

The objective of the intuitionistic fuzzy evidential reasoning (IFER) approach is to offer representation and reasoning on a basis of linguistic knowledge under uncertainty more accurately by rebuilding the evidential structure so as to reduce information losses. In essence, it is to extend the basic RIMER methodology. The main improvement in the proposed IFER approach involves two aspects: on the one hand, the representation of knowledge uses continuous fuzzy numbers (intuitionistic trapezoidal fuzzy numbers, ITFN) instead of conventional discrete fuzzy sets. On the other hand, the matching degree method uses inclusion measure instead of max-min operation for avoiding the effect by extreme values.

This paper is organized as follows: In Section 2, basic concept and existing evidence structure are reviewed. In Section 3, in order to reduce information loss in existing evidence structure, we propose a new intuitionistic fuzzy evidential reasoning (IFER) approach by rebuilding evidence structure. Firstly, intuitionistic fuzzy numbers are converted into interval fuzzy numbers by α-cuts. Secondly, the belief degrees of consequents are updated according to the matching degree between inputs and antecedent attributes. Thirdly, attribute weights are calculated by normalizing the certainty factor. Fourthly, after interval belief degrees and weights are calculated, a nonlinear model is built to combine interval belief degrees. At last, the utility function is adopted to rank the combined results. In Section 4, a stroke diagnosis is presented to illustrate the validity and applicability of the proposed IFER method. In Section 5, conclusions and the further study are summarized.

2. Preliminaries

In this section, basic concepts and former research achievements of intuitionisitc fuzzy set theory and evidential reasoning theory are introduced.

2.1. Basic concept of intuitionistic trapezoidal fuzzy numbers

Definition 1 *Let A be an intuitionistic trapezoidal fuzzy number (ITFN) defined in the space of real numbers R, its membership function is given by e.g. (Ref. 22)*

$$\mu_A(x)=\begin{cases}0, & x<a_1,\\ \dfrac{x-a_1}{a_2-a_1}, & a_1\leqslant x\leqslant a_2,\\ 1, & a_2\leqslant x\leqslant a_3,\\ \dfrac{x-a_3}{a_4-a_3}, & a_3\leqslant x\leqslant a_4,\\ 0, & a_4<x.\end{cases} \tag{1}$$

And its non-membership function is given by

$$\nu_A(x)=\begin{cases}1, & x<b_1,\\ \dfrac{x-b_2}{b_1-b_2}, & b_1\leqslant x\leqslant b_2,\\ 0, & b_2\leqslant x\leqslant b_3,\\ \dfrac{x-b_3}{b_4-b_3}, & b_3\leqslant x\leqslant b_4,\\ 1, & b_4<x.\end{cases} \tag{2}$$

with the parameters $a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4\in R$, $b_1\leqslant a_1\leqslant b_2\leqslant a_2\leqslant a_3\leqslant b_3\leqslant a_4\leqslant b_4$. ITFN is then denoted as $A=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4)\rangle$. A useful tool to deal with the intuitionistic fuzzy numbers is using α-cuts.Every α-cut is a closed interval. They are calculated as follows e.g. (Ref. 23)

$$\begin{aligned} A^+_{Low}(\alpha) &= \inf\{x \in R \mid \mu_A(x) \geqslant \alpha\} \\ &= (a_2 - a_1)\alpha + a_1, \\ A^+_{Up}(\alpha) &= \sup\{x \in R \mid \mu_A(x) \geqslant \alpha\} \\ &= (a_3 - a_4)\alpha + a_4, \\ A^-_{Low}(\alpha) &= \inf\{x \in R \mid \nu_A(x) \leqslant 1-\alpha\} \\ &= (b_1 - b_2)\alpha + b_2, \\ A^-_{Up}(\alpha) &= \sup\{x \in R \mid \nu_A(x) \leqslant 1-\alpha\} \\ &= (b_4 - b_3)\alpha + b_3, \end{aligned} \tag{3}$$

ITFN are converted into interval valued intuitionistic fuzzy sets (IVIFS) e.g. (Ref. 24). $A = \langle x, [A^+_{Low}(\alpha), A^+_{Up}(\alpha)], [A^-_{Low}(\alpha), A^-_{Up}(\alpha)]\rangle$.

2.2. *The belief rule-base structure*

In a "traditional" IF-THEN rule, decision makers state that the degree of an attribute affecting a consequent is either 100% true or 100% false e.g. (Ref. 2).To take into account belief degrees, attribute weights, and rule weights, the "traditional" rule is extended as e.g. (Ref. 11),

$$\begin{aligned} R_k :& If\ (X_1,\varepsilon_1)\ is\ A^k_1 \wedge (X_2,\varepsilon_2)\ is\ A^k_2 \wedge \cdots \wedge (X_{T_k}, \\ & \varepsilon_{T_k})\ is\ A^k_{T_k},\ then\ \{(D_1,\overline{\beta}_{1k}),(D_2,\overline{\beta}_{2k}),\cdots, \\ & (D_N,\overline{\beta}_{Nk})\},\ with\ a\ rule\ weight\ \theta_k\ and \\ & attribute\ weights\ \delta_{k1},\ \delta_{k2},\ \cdots,\delta_{kT_k}, \\ & and\ with \overline{\beta}_{nk}(n \in \{1,\cdots,N\}),(\sum_{n=1}^{N}\overline{\beta}_{nk} \leqslant 1). \end{aligned} \tag{4}$$

where $X_i \in \{A^*_i, i = 1,\cdots,T_k\}, A^*_i$ is the input set, with the belief degree ε_i to which the input is assessed to the corresponding evaluation grade, and $\varepsilon_i \geqslant 0, \Sigma_{i=1}^{T_k}\varepsilon_i \leqslant 1$. Two kinds of weight are considered. Those are the rule weight θ_k and attribute weightsδ_{ki}, with$\theta_k \geqslant 0, \sum_{k=1}^{L}\theta_k = 1$ and in the kth rule $\delta_{ki} \geqslant, \sum_{i=1}^{T_k}\delta_{ki} = 1$.Given an input, if the rule weightθ_k is greater than zero, the corresponding rule will be activated.

The set $A^k_i \in \{A_{ij}, i = 1,\cdots,T_k, j = 1,\cdots,J_i\}$forms a collection of the referential values of antecedent attributes, where T_k is the number of the antecedent attributes. J_i is the number of the referential values of ith antecedent attributes. Each referential value of antecedent attributes can be treated as an evaluation grade. The input set A^*_i is assessed to the evaluation grade A^k_i with belief degree α_{ij} The input assessment S can be written as follows

$$S(A^*_i) = \{(A_{ij},\alpha_{ij}), i = 1,\cdots,T_k, j = 1,\cdots,J_i\}. \tag{5}$$

$D_n(n = 1,\cdots,N)$ is a set of consequents. $\overline{\beta}_{nk}$ is the degree to which D_n is believed to be the consequent in the kth rule. The belief degree $\overline{\beta}_{nk}$ can be assigned directly based on decision makers' experience, or by using some experimental data. The consequent assessment S can be expressed as follows

$$S(A_{ij}) = \{(D_n,\beta_{nk}), n = 1,\cdots,N, k = 1,\cdots,L\}. \tag{6}$$

with $\beta_{nk} \geqslant 0, \sum_{n=1}^{N}\beta_{nk} \leqslant 1$. From inputs to consequents, information is transformed by belief degrees. Belief transfers from input ε_i to updated consequent β_{nk}, through the intermediate links α_{ij} and $\overline{\beta}_{nk}$. Among these,ε_i and $\overline{\beta}_{nk}$ is given by domain experts before making decision. One important task is to calculate the belief degree α_{ij}.

The two sets A^*_i and A^k_i can be numeric or non-numeric, continuous or discrete. Non-numeric attributes can be labeled by real numbers, traditional fuzzy sets, and generalized fuzzy sets. How to match the degree between two sets is an important step in the process of reasoning. Similarity measure, the t-norm function and t-conorm e.g. (Ref. 25), the area method for fuzzy sets e.g. (Ref. 26), and max-min operation between fuzzy sets (e.g. (Ref. 11) and e.g. (Ref. 16))are used to compute the matching degree as shown below

$$\upsilon(A^*_i, A_{ij}) = \max_x[min(A^*_i(x), A_{ij}(x))] \tag{7}$$

However the max-min operation only considered extreme value. Grzegorzewski e.g. (Ref. 27) presented possible and necessary inclusion of IFSs.

3. Intuitionistic fuzzy evidential reasoning

In this section, an intuitionistic fuzzy evidential reasoning (IFER) method is proposed to perform reasoning. The flowchart of IFER is shown in Fig.1.

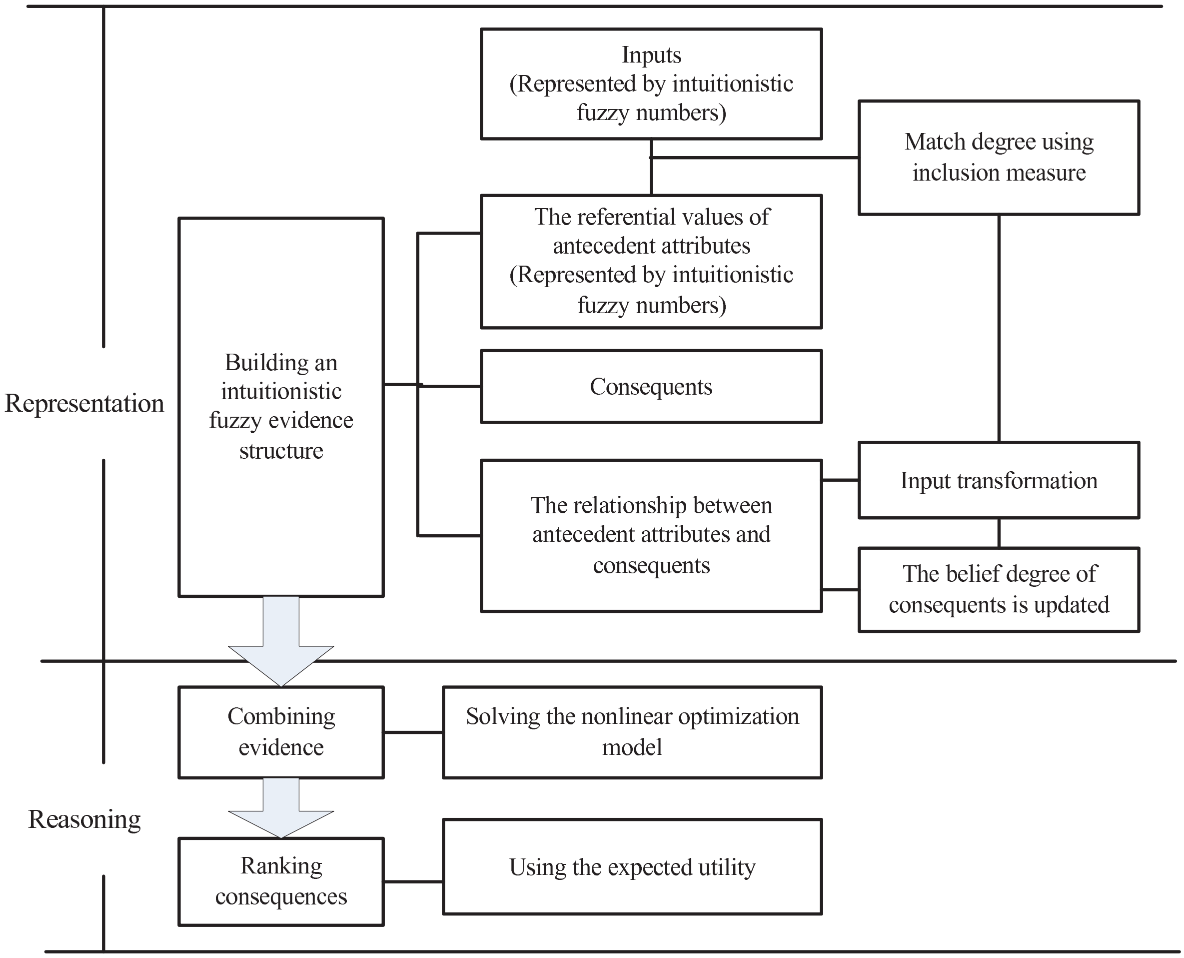

Fig. 1. The flowchart of intuitionistic fuzzy evidential reasoning (IFER).

3.1. *Intuitionistic fuzzy structure*

3.1.1. *The framework of evidence structure*

In evidential reasoning, the result depends on the suitable modeling of the domain knowledge and uncertainties. Dymova (e.g. (Ref. 18) and e.g. (Ref. 19)) proposed three hypotheses as $Yes(x \in A)$, $No(x \notin A)$ and $(Yes,No)(hesitasion)$. $m(A)(Yes)$, $m(A)(No)$, $m(A)(Yes,No)$ represent the focal element of the basic assignment function respectively. The sum of them is equal to 1.

Let $\Phi = \{\phi_1, \phi_2, \cdots, \phi_T\}$ be a set of collectively exhaustive and mutually exclusive hypotheses. It is called the frame of discernment. The T assessment grades for all attributes, the referential values of antecedent attributes A_{ij} is any subset of Φ. The intuitionistic fuzzy structure is expressed as

$$\begin{aligned}\Re = &\{(A_{ij}, [m(A_{ij})(Yes), m(A_{ij})(Yes) + m(A_{ij})(Yes,\\ &No)], \mu_{A_{ij}}(x), \nu_{A_{ij}}(x)) \mid A_{ij} \in \Phi, x \in U\}\\ &with \quad i = 1, \cdots, T_k, j = 1, \cdots, J_i.\end{aligned} \tag{8}$$

where A_{ij} denotes the fuzzy propositions in the frame of discernment Φ. Among these,$m(A_{ij})(Yes)$ indicates the membership of belief mass assigned to the set A_{ij} .U represents the universe of discourse.

The belief interval BI_A is a possibility interval $BI_A = [Bel_A, Pl_A]$ .The belief and plausibility of the