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α-Minimal Resolution Principle For A Lattice-Valued Logic*

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Abstract

Based on the academic ideas of resolution-based automated reasoning and the previously established research work on binary α-resolution based automated reasoning schemes in the framework of lattice-valued logic with truth-values in a lattice algebraic structure-lattice implication algebra (LIA), this paper is focused on investigating α-n(t)-ary resolution based dynamic automated reasoning system based on lattice-valued logic based in LIA. One of key issues for α-n(t)-ary resolution dynamic automated reasoning is how to choose generalized literals in each resolution. In this paper, the definition of α-minimal resolution principle which determines how to choose generalized literals in LP(X) is introduced firstly, as well as its soundness and completeness being proved. α-minimal resolution principle is then further established in the corresponding lattice-valued first-order logic LF(X) along with its soundness theorem, lifting lemma and completeness theorem. These results lay the theoretical foundation for research of α-n(t)-ary resolution dynamic automated reasoning.

Keywords: Automated reasoning; lattice-valued propositional logic LP(X); lattice-valued first-order logic LF(X); α-minimal resolution principle; α-minimal resolution group.

1. Introduction

As the classical logic can only deal with certain information, to deal with fuzziness and incomparability, Xu et al [1, 2] introduced a lattice-valued logic algebra called lattice implication algebra (LIA) and proposed lattice-valued logic systems based on LIA, which can handle both comparable and incomparable information.

Along with the use of non-classical logics becomes increasingly important in computer science, AI and logic programming, the developing efficient automated theorem proving based on non-classical logic is also an active area of research (e.g., for fuzzy logic and many-valued logic, among others). The essential idea in many of those methods is to transform the resolution algorithm into fuzzy logic and many-valued logic to that of classical logic. To the best of our knowledge, proof theory for lattice-valued logic has so far not been extensively developed. There has also been investigations of resolution-based automated reasoning in lattice-valued logic based on LIA (e.g., among others, [3,8,9,10,12,29,30]). The aim of dealing with

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incomparability leads to the complexity of logical formula in LIA based lattice-valued logic. Correspondingly, the resolution methods in LIA based lattice-valued logic have new features such as (a) resolution is based on generalized literals, which contain constants and implication connectives; (b) resolution is proceeded at a different truth-valued level α chosen from the truth-valued field LIA and the number of resolution generalized literals is fixed at 2 in each resolution in a resolution deduction. So, the α-resolution is also called α-2 ary resolution; (c) it is not easy to judge directly if two generalized literals are α-resolvent or not, because the structure of generalized literal is very complex. Due to these new features, it is not feasible to apply directly the resolution-based automated reasoning theory and methods in classical logic and in many chain-type many-valued logics into that of lattice-valued logic with incomparability. Hence, an α-2 ary resolution principle for a lattice-valued propositional logic LP(X) has been proposed in [10], which can be used to prove whether a lattice-valued logical formula in LP(X) is false at a truth-value level α(i.e., α-false) or not, and the theorems of soundness and completeness for the α-2 ary resolution principle were also proved. In addition, the work in [8] extends the α-2 ary resolution principle for LP(X) to the corresponding lattice-valued first-order logic LF(X).

Xu[4] extended the number of resolution generalized literal from 2 to n, and proposed the general form of α-resolution, and the soundness and completeness are also built. In α-n(t)-ary resolution, the number n(t) of resolution generalized literals is not fixed at some number, but it will be different in the each resolution, where n(t) means the number of resolution generalized literals in the tth resolution.

In each resolution, the conjunction of participated resolution literals should be less or equal to α, in order to achieve this goal, we should make the number of participated resolution clauses the more the better; But from the other hand, considering each clause, except participated resolution literals, all remaining literals are disjunctive, from this point of view, in order to get empty clause, it should make the number of participated resolution clauses the less the better. Based on the Xu and other co-authors' research work [4, 11], the α-minimal resolution principle is proposed in this paper, this resolution principle is efficient for the above problem, it gives how to choose the number of participated resolution clauses in the process of resolution. It reduces the generation of redundant clauses and improves the efficiency of resolution. First-order logic is more expressive and it can better apply and solve more practical problems, so we extend it to the first-order logic LF(X).

This paper is organized as follows: Section 2 reviews some preliminary relevant concepts; In Section 3, α-minimal resolution principle is given in LP(X), as well as its soundness and completeness. In Section 4, α-minimal resolution principle for LP(X) is extended to the corresponding first-order logic LF(X). The paper concludes in Section 5.

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2. Preliminaries

In what follows we provide some elementary concepts and conclusions of lattice-valued propositional logic LP(X) and first-order logic LF(X) with truth-value in lattice implication algebras are introduced. We only provide elementary concepts and conclusions which are closely relevant to this study for the convenience of readers. For further details about the properties and background of LIA, LP(X), and LF(X), see the papers [1-2] and [5, 7-10].

Definition 2.1 [1] Let $(L, \vee, \wedge, O, I)$ be a bounded lattice with an order-reversing involution $'$, I and O the greatest and the smallest element of L respectively, and $\rightarrow: L \times L \rightarrow L$ be a mapping. $(L, \vee, \wedge, ', \rightarrow, O, I)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$:

(I_1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,

(I_2) $x \rightarrow x = I$,

(I_3) $x \rightarrow y = y' \rightarrow x'$,

(I_4) $x \rightarrow y = y \rightarrow x = I$ implies $x = y$,

(I_5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$,

(l_1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$,

(l_2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$.

Example 2.1 [2] **(Łukasiewicz implication algebra on finite chain)** Let $L_n = \{a_i \mid i = 1,2,\ldots,n\}$, $a_1 < a_2 < \ldots < a_n$. For any $1 \le j, k \le n$, define

$$a_j \vee a_k = a_{max\{j, k\}},$$
$$a_j \wedge a_k = a_{min\{j, k\}},$$
$$(a_j)' = a_{n-j+1},$$
$$a_j \rightarrow a_k = a_{min\{n-j+k, n\}}.$$

Then (L_n, ∨, ∧, ′, →, a_1, a_n) is a lattice implication algebra.

Definition 2.2 [5] Let X be the set of propositional variables, (L, ∨, ∧, ′, →, O, I) be a lattice implication algebra, $T = L \cup \{',\rightarrow\}$ be a type with ar(′) = 1, ar(→) = 2 and ar(a) = 0 for any $a \in L$. The proposition algebra of the lattice-valued proposition calculus on the set X of propositional variables is the free T algebra on X and denoted by LP(X).

Definition 2.3 [2] The set $\mathcal{F}$ of formula of LP(X) is the least set Y satisfying the following conditions:

(1) $X \subseteq Y$,

(2) $L \subseteq Y$,

(3) if $p, q \in Y$, then $'(p), \rightarrow(p, q) \in Y$,

where X is the set of propositional variables, L is the set of constants.

In the following, we denote $'(p)$ as p' and $\rightarrow(p, q)$ as $p\rightarrow q$.

Definition 2.4 [2] A mapping v: LP(X) → L is called a valuation of LP(X), if it is a T-homomorphism.

Note that L and LP(X) are the algebras with the same type T, where T = L ∪ {′,→}. For example, for any p, q∈$\mathcal{F}$, v is a T-homomorphism, then we have v(p′)=v(p)′ and v(p→q)=v(p)→v(q) hold.

Definition 2.5 [10] Let $G\in\mathcal{F}$ and $\alpha\in L$. For any valuation v of LP(X), if $v(G)\le\alpha$, we say G is always less than α(or G is α-false), denoted by $G \le\alpha$.

Definition 2.6 [10] A lattice-valued propositional logical formula G in lattice-valued propositional logic system LP(X) is called an extremely simple form, in short *ESF*, if a lattice-valued propositional logical formula G^* obtained by deleting any constant or literal or implication term occurring in G is not equivalent to G.

Definition 2.7 [10] A lattice-valued propositional logical formula G in lattice-valued propositional logic system LP(X) is called an indecomposable extremely simple form, in short *IESF*, if the following two conditions hold:

(1) G is an *ESF* containing connective → and ′ at most,

(2) For any $H\in\mathcal{F}$, if $H \in \overline{G}$ in $\overline{\mathrm{LP(X)}}$, then H is an *ESF* containing connectives → and ′ at most, where $\overline{\mathrm{LP(X)}}$= (LP(X)$/_=$,∨,′,→,$\overline{O}$,$\overline{I}$) is the LIA, LP(X)$/_=$ = {$\overline{p}$| p∈LP(X)}, $\overline{p}$ ={ q| q∈LP(X), q=p}, for any $\overline{p},\overline{q} \in \mathrm{LP(X)}/_=$, $\overline{p}\vee\overline{q} = \overline{p\vee q}, \overline{p}\wedge\overline{q} = \overline{p\wedge q}, (\overline{p})' = \overline{p'}$, $\overline{p}\rightarrow\overline{q} = \overline{p\rightarrow q}$.

For example, suppose that x,y, z, p, q are propositional variables in LP(X), b∈L. Then, g_1=(x→y′)∨(z→b) is an *ESF*, g_2=x→y′, g_3=z→b, g_4=x→(y→(p→q)) are three *IESF*s.

Definition 2.8 [3] All the constants, literals and *IESF*s in lattice-valued propositional logic system LP(X) are called generalized literals. Here, the definition of literal is the same as that in classical logic.

For example, the ((x→y) →y) →y is not a generalized literal in LP(X), but the (x→y) is a generalized literal in LP(X), where x, y are propositional variable in LP(X).

Definition 2.9 [3] A lattice-valued propositional logical formula G in lattice-valued propositional logic system LP(X) is called a generalized clause if G is a formula of the form

$$G = g_1\vee\ldots\vee g_i \vee\ldots\vee g_n,$$

where g_i are generalized literals, i = 1,2,…,n. A conjunction(or disjunction) of finite generalized clauses (phrases) is called a generalized conjunctive(or disjunction) normal form.

Definition 2.10 [10] (α-Resolution) Let α∈L, and G_1 and G_2 be two generalized clauses in LP(X) of the forms

$G_1 = g_1\vee\ldots\vee g_i \vee\ldots\vee g_m$, and

$G_2 = h_1\vee\ldots\vee h_j \vee\ldots\vee h_n$,

If $g_i \wedge h_j \le\alpha$, then

$$G= g_1\vee\ldots\vee g_{i-1} \vee g_{i+1} \vee\ldots\vee g_m \vee h_1 \vee\ldots\vee h_{j-1} \vee h_{j+1} \vee\ldots\vee h_n$$

is called an α-resolvent of G_1 and G_2, which is denoted by $G=R_\alpha(G_1,G_2)$, and g_i and h_j form an α-resolution pair, which is denoted by (g_i, h_j)-α. Generation of an α-resolvent from two clauses, which is called α-resolution, is the sole rule of inference of the α-resolution principle.

In the following, we use the symbol α-◎ to represent an α-false generalized clause.

Definition 2.11 [10] In LP(X), suppose that a generalized conjunctive normal form S= $C_1 \wedge C_2 \wedge\ldots\wedge C_n$, α∈L, w={D_1, D_2, …, D_m} is an α-resolution deduction from S to a generalized clause D_m, if

1) $D_i\in\{C_1, C_2, \ldots, C_m\}$; or

2) There exist j, k<i, such that $D_i= R_\alpha(D_j,D_k)$.

If there exists an α-resolution deduction from S to α-◎, then this α-resolution deduction w is called an α-refutation.

Definition 2.12 [8] Suppose V and F are the set of variable symbols and that of functional symbols in LF(X), respectively, the set of terms of LF(X) is defined as the smallest set J satisfying the following conditions:

(1) $V\subseteq$ J,

(2) For any $n \in$ N, if $f^{(n)} \in F$, then for any $t_0, t_1,\ldots,t_n \in$ J , $f^{(n)}(t_0, t_1,\ldots,t_n) \in$ J .

Remark 2.1 $f^{(0)}$ is specified as a constant symbol.

Definition 2.13 [8] Suppose P is the predicate symbol set in LF(X). The set of atoms of LF(X) is defined as the smallest set A_t satisfying the following condition:

For any $n \in$ N, if $P^{(n)} \in P$, then $P^{(n)}(t_0, t_1,\ldots,t_n) \in A_t$ for any $t_0, t_1,\ldots,t_n \in$ J.

Remark 2.2 $P^{(0)}$ is specified as a certain element in L.

Definition 2.14 [8] The set of formulas of LF(X) is defined as the smallest set F satisfying the following conditions:

(1) $A_t \subset$F,

(2) If $p, q \in$ F, then $p \rightarrow q \in$F,

(3) If $p \in$ F, x is a free variable in p, then $(\forall x)\, p$, $(\exists x)\, p \in$F.

Remark 2.3 Note that $p' = p \rightarrow O$, $p \vee q = (p \rightarrow q) \rightarrow q$, $p \wedge q = (p' \vee q')'$, $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$.

Therefore, if $p, q \in$F, then $p', p \vee q, p \wedge q, p \leftrightarrow q \in$F.

Definition 2.15 [8] Suppose $G \in$F , F_G is the set of all functional symbols occurring in G, P_G is the set of all predicate symbols occurring in G, and D ($\neq \phi$) is the domain of interpretation. An interpretation of G over D is a triple $I_D = \langle D, \mu_D, v_D\rangle$, where,

$\mu_D : F_G \rightarrow U_D = \{ f_D^{(n)} : D^n \rightarrow D \mid n \in N \}$

$f^{(0)} \mapsto f_D^{(0)}$, $f_D^{(0)}(D^0) = \{ f_D^{(0)} \} \subseteq D$, $D^{(0)}$ is a non-empty set

$f^{(n)} \mapsto f_D^{(n)}$ $(n \in N^+)$,

$v_D : P_G \rightarrow V_D = \{ P_D^{(n)} : D^n \rightarrow L \mid n \in N \}$

$p^{(0)} \mapsto p_D^{(0)}$, $p_D^{(0)}(D^0) = \{ p_D^{(0)} \} \subseteq L$

$p^{(n)} \mapsto p_D^{(n)}$ $(n \in N^+)$.

Definition 2.16 [29] A formula G in lattice-valued first-order logic LF(X) is a generalized-literal, if it satisfies the following conditions:

(1) G is a literal, or

(2) G is constructed only by some literals and some implication connectives with the condition that G can not be represented by connectives "$\vee$" or "$\wedge$" and G can not be decomposed into a simpler form (G is called an indecomposable implication form).

The disjunction of a finite number of generalized-literals is a generalized-clause. The conjunction of a finite number of generalized-clauses is a generalized-conjunctive normal form.

Definition 2.17 [8] Let $G \in$F, $\alpha \in L$. G is said to be α-false, if $v_D(G) \leq \alpha$ for any interpretation $I_D = \langle D, \mu_D, v_D\rangle$ of G.

Definition 2.18 [29] Suppose G is a formula of the form $Q_1x_1\ldots Q_nx_nG^*$, where $Q_1,\ldots,Q_n$ are the quantifiers, i.e., $\forall$ or $\exists$, and G^* is a formula without any quantifier. Then G is said to be a generalized-prenex conjunctive normal form, if G^* is a generalized-conjunctive normal form.

Definition 2.19 [29] Suppose a formula $G = Q_1x_1\ldots Q_nx_n M$ is a generalized-prenex conjunctive normal form. The formula G^* obtained by the following steps is called a generalized-Skölem standard form of G:

(1) If Q_r is an existential quantifier and without any universal quantifier occurring ahead it in the prefix $Q_1,\ldots,Q_n$ (from left to right), we choose a new constant c different from other constants occurring in M, replace all x_r occurring in M by c, and then delete Q_r from the prefix $Q_1,\ldots,Q_n$.

(2) If Q_r is an existential quantifier and $Q_{k1},\ldots,Q_{km}$ are all the universal quantifiers occurring ahead Q_r ($m \geq 1$, $1 \leq k_1 <\ldots< k_m < r$), we choose a new m-ary function symbol $f^{(m)}$ different from all other function symbols occurring in M, replace all x_r in M by $f^{(m)}(x_{k1},\ldots,x_{km})$ and then delete Q_r from the prefix $Q_1,\ldots,Q_n$.

(3) Repeating (1) and (2) until there is no existential quantifier occurring in the prefix.

Theorem 2.1 [8] Suppose G^* is a generalized-Skölem standard form of a formula G, and $|L| < \aleph_0$, G^* is α-false if and only if there exists a finite ground instance set G^{*0} of G^* such that G_c^{*0} is α-false, where G_c^{*0} is the conjunction of all ground instances of G^{*0}.

Corollary 2.1[8] Let $G^* = G_1^* \wedge G_2^* \wedge\ldots\wedge G_m^*$ a generalized-Skölem standard form of a formula G, where $G_1^*, G_2^*,\ldots,G_m^*$ are generalized-clauses in LF(X), $\alpha \in L$, and $|L| < \aleph_0$. Then $G^* \leq \alpha$ if and only if there exist $g_1^*, g_2^*,\ldots, g_m^*$ such that $g_1^* \wedge g_2^* \wedge\ldots\wedge g_m^* \leq \alpha$, where g_i^* is an ground instance of G_i^*, $i = 1,2,\ldots,m$.

If generalized literal g is obtained through combining generalized literals $g_1,\ldots, g_m$ with implication connectives, then g is more complex than any element included in $\{g_1,\ldots, g_m\}$. In the following, generalized literals of generalized clause C are the most complex ones occurring in C. For example, if $C = ((x\rightarrow y) \rightarrow (p\rightarrow q)) \vee ((t\rightarrow s) \rightarrow l)$, then generalized literals of C are $(x\rightarrow y) \rightarrow (p\rightarrow q)$ and $(t\rightarrow s) \rightarrow l$, instead of $(x\rightarrow y)$, $(p \rightarrow q)$, $(t\rightarrow s)$ or l.

α occurring in the following is always less than I.

3. α-minimal resolution principle based on lattice-valued propositional logic LP(X)

Definition 3.1 Let $C_i = p_{i1}\vee\ldots\vee p_{in_i}$ be generalized clauses of LP(X), $H_i = \{p_{i1},\ldots, p_{in_i}\}$ the set of all generalized literals occurring in C_i, $x_i \in H_i$, i=1,2,…,n. $\alpha \in L$. If there exist generalized literals such that $x_1 \wedge x_2$

$\wedge\ldots\wedge x_n \le \alpha$, but for any j∈{1, 2,…, n}, $x_1 \wedge\ldots\wedge x_{j-1} \wedge x_{j+1} \wedge\ldots\wedge x_n \not\le \alpha$, then $C_1(x_1=\alpha)\vee C_2(x_2=\alpha)\vee\ldots\vee C_n(x_n=\alpha)$ is called α-minimal resolvent of $C_1,C_2,\ldots,C_n$, which is denoted by

$R_{p(g-\alpha)}^{m}(C_1(x_1),C_2(x_2),\ldots,C_n(x_n))$, here "p" represents the "propositional logic", "m" means "m-ary", and $x_1, \ldots, x_n$ are called an α-minimal resolution group.

Remark 3.1 $C_i(x_i=\alpha)$ in (3.1) means the generalized clause that is obtained by replacing x_i occurring in C_i with α.

Remark 3.2 The α-minimal resolution principle in Definition 3.1 is also hold in classical logic and the binary α-resolution principle based on two generalized literals in LP(X).

Example 3.1 Let $C_1=(x\to y)\vee(s\to t)$, $C_2=(y\to z)\vee(s\to t)'\vee(p\to q)$, $C_3=(x\to z)'\vee(s\to q)$, $C_4=(t\to g)'\vee(z\to g)$, be four generalized clauses in lattice-valued propositional logic $L_9P(X)$ with truth-value in $(L_9, \vee, \wedge, ', \to, a_1, a_9)$, where $(L_9, \vee, \wedge, ', \to, a_1, a_9)$ is the same Łukasiewicz implication algebra with nine elements, and x, y, z, s, t, p, q are propositional variables, $\alpha=a_6$. Then $x\to y$, $y\to z$, $(x\to z)'$, $z\to g$ and $(s\to t)$, $(s\to t)'$, $s\to q$ are α-resolution groups. But $(x\to y)\wedge(y\to z)\wedge(x\to z)'\le\alpha$, and the conjunction of any two of them is not less than or equal to α; $(s\to t)\wedge(s\to t)'\le\alpha$, $s\to t$ and $(s\to t)'$ are all not less than or equal to α. so $x\to y$, $y\to z$, $(x\to z)'$ and $(s\to t)$, $(s\to t)'$ are α-minimal resolution groups.

Theorem 3.1 Every α-resolution group has at least one α-minimal resolution group.

Proof Known $x_1, x_2, \ldots, x_n$ is an α-resolution group, so we have $x_1\wedge x_2\wedge\ldots\wedge x_n\le\alpha$, if for any j∈{1, 2,…, n}, we have $x_1\wedge\ldots\wedge x_{j-1}\wedge x_{j+1}\wedge\ldots\wedge x_n\not\le\alpha$, then $x_1, x_2, \ldots, x_n$ is an α-minimal resolution group, if there exist i_1∈{1, 2,…, n}, here we assume that i_1 is equal to 1, if not, we can adjust the letter serial number to become 1, such that $x_2\wedge x_3\wedge\ldots\wedge x_n\le\alpha$; if for any j∈{2,…, n}, $x_2\wedge\ldots\wedge x_{j-1}\wedge x_{j+1}\wedge\ldots\wedge x_n\not\le\alpha$, then $x_2, x_3, \ldots, x_n$ is an α-minimal resolution group, Otherwise, there exists i_2 ∈{2,…, n}, here we assume that i_2 is equal to 2, if not, we can adjust the letter serial number to become 2, such that $x_3\wedge\ldots\wedge x_n\le\alpha$, if for any j∈{3,…, n}, $x_3\wedge\ldots\wedge x_{j-1}\wedge x_{j+1}\wedge\ldots\wedge x_n\not\le\alpha$, then $x_3, \ldots, x_n$ is an α-minimal resolution group; Otherwise, take turns to do it according to the above method, we can get an α-minimal resolution group finally. So conclusion holds.

Remark 3.3 By the proof of the theorem 3.1, we know every α-minimal resolution group is an α-resolution group; every α-resolution group is not only has one α-minimal resolution group. But why we still study the special case of multiary α-resolution principle? Multiary α-resolution principle is the extension the binary α-resolution principle, it is the important and meaningful conclusion. But in the process of resolution, it will allow more clauses participate in the resolution and produce more redundant clauses, then reduces the efficiency of resolution. By the Definition 3.1, the α-minimal resolution principle can limit the participated literals, and then determine the number of participated resolution clauses in the process of resolution. This reduces the generation of redundant clauses and improves the efficiency of resolution.

Example 3.2 In the example 3.1, obviously, $x\to y$, $y\to z$, $(x\to z)'$, $(s\to t)$, $(s\to t)'$ is an α-resolution group, $x\to y$, $y\to z$, $(x\to z)'$ and $(s\to t)$, $(s\to t)'$ are α-minimal resolution groups.

Example 3.3 Let $C_1=(x\to y)$, $C_2=(y\to z)\vee(s\to t)'\vee(p\to q)$, $C_3=(x\to g)'\vee(s\to q)$, $C_4=(z\to g)$, be four generalized clauses in lattice-valued propositional logic $L_9P(X)$ with truth-value in $(L_9, \vee, \wedge, ', \to, a_1, a_9)$, where $(L_9, \vee, \wedge, ', \to, a_1, a_9)$ is the same Łukasiewicz implication algebra with nine elements, and x, y, z, s, t, p, q, g are propositional variables, $\alpha=a_6$.

Obviously, $x\to y$, $y\to z$, $z\to g$, $(x\to g)'$ is an α-minimal resolution group. The α-minimal resolvent of C_1, C_2, C_3, C_4, denoted by

$R_{p(g-\alpha)}^{m}(C_1,C_2,C_3,C_4)=(s\to t)'\vee(p\to q)\vee(s\to q)\vee\alpha$.

Theorem 3.2 Let $C_i=p_{i1}\vee\ldots\vee p_{in_i}$ be generalized clauses of LP(X), $H_i=\{p_{i1},\ldots,p_{in_i}\}$ the set of all generalized literals occurring in C_i, i=1,2,…,n, α∈L. If there exist generalized literals $x_i\in H_i$, i = 1,2,…,n, such that $x_1, \ldots, x_n$ is an α-minimal resolution group, then $C_1\wedge C_2\wedge\ldots\wedge C_n\le R_{p(g-\alpha)}^{m}(C_1(x_1),C_2(x_2),\ldots,C_n(x_n))$.

Proof In [4] Theorem 3.1, has proved $C_1\wedge C_2\wedge\ldots\wedge C_n\le R_{p(g-\alpha)}(C_1(x_1),C_2(x_2),\ldots,C_n(x_n))$, Due to α-minimal resolution group is special case of α-resolution group, so we have

$$C_1\wedge C_2\wedge\ldots\wedge C_n\le R_{p(g-\alpha)}^{m}(C_1(x_1),C_2(x_2),\ldots,C_n(x_n)).$$

So the conclusion holds.

Definition 3.2 Suppose $S=C_1\wedge C_2\wedge\ldots\wedge C_n$, where $C_1, C_2,\ldots, C_n$ are generalized clauses in LP(X), α∈L. $\{\Phi_1, \Phi_2,\ldots, \Phi_t\}$ is called an α-minimal resolution deduction from S to generalized clause Φ_t (or S can be α-minimal resolved into Φ_t), H_i is the set of all generalized literals occurring in Φ_i (i =1, 2, …, t), if

(1) $\Phi_i\in S$, or

(2) There exist $r_1, r_2, \ldots, r_{k_i} < i$, and $x_d \in H_{r_d}$ (d =1, 2, …, k_i), such that
$R_{p(g\text{-}\alpha)}{}^{m}(\Phi_{r_1}(x_1), \Phi_{r_2}(x_2), \ldots, \Phi_{r_{k_i}}(x_{k_i})) = \Phi_i$.

Theorem 3.3 (Soundness) Suppose $S = C_1 \wedge C_2 \wedge \ldots \wedge C_n$, where $C_1, C_2, \ldots, C_n$ are generalized clauses in LP(X), α∈L. $\{\Phi_1, \Phi_2, \ldots, \Phi_t\}$ is an α-minimal resolution deduction from S to generalized clause Φ_t. If Φ_t is α-⊚, then $S \le \alpha$, i.e., if $\Phi_t \le \alpha$, then $S \le \alpha$.

Proof We set Φ_i is the α-minimal resolvent of $C_1, \ldots, C_k$, according to Definition 3.2 and Theorem 3.2, we have $C_1 \wedge \ldots \wedge C_k \le \Phi_i$, so $S = C_1 \wedge C_2 \wedge \ldots \wedge C_n = C_1 \wedge C_2 \wedge \ldots \wedge C_n \wedge \Phi_i$, we get the promotion:
$S = C_1 \wedge C_2 \wedge \ldots \wedge C_n = C_1 \wedge C_2 \wedge \ldots \wedge C_n \wedge \Phi_1 \wedge \Phi_2 \wedge \ldots \wedge \Phi_t \le \Phi_1 \wedge \Phi_2 \wedge \ldots \wedge \Phi_t \le \alpha$. The conclusion holds.

Theorem 3.4 (Completeness) Suppose $S = C_1 \wedge C_2 \wedge \ldots \wedge C_n$, where $C_1, C_2, \ldots, C_n$ are generalized clauses in LP(X), α∈L. If $S \le \alpha$, then there exist an α-minimal resolution deduction from S to α-⊚.

Proof (1) S only contains one generalized clause C. By $S \le \alpha$, the conclusion holds.

(2) S contains more than one generalized clause. For any i=1, 2,…, n.

Let H_i be the set of all generalized literals occurring in C_i, denote $|H_i| = \omega_i$.

Let K(S) be disjunction term number and general clauses in the number of difference, i.e.,

$K(S) = \sum_{i=1}^{n} \omega_i - n$. Induction of S, the following conditions exist:

1) If K(S)=0, then S only consist of unit generalized literals, i.e., every generalized clause contains only one generalized literal in S. Because $S \le \alpha$, so all generalized literals form an α-resolution group. By Theorem 3.1, there exist an α-minimal resolution group, the conclusion holds.

2) Assume K(S)<m(m>0) conclusion hold, following we prove K(S)=m conclusion hold.

Let K(S)=m, then S has one non unit generalized clause, let g is a disjunction term of non unit generalized clause in S. Set $C_i = C_i^* \vee g$, and C_i^* is not empty. Let $S_1 = C_1 \wedge \ldots \wedge C_{i-1} \wedge C_i^* \wedge C_{i+1} \wedge \ldots \wedge C_n$, absolutely, $S_1 \le \alpha$ and $K(S_1) < m$. By induction method, there exists an α-minimal resolution deduction D_1^* from S_1 to α-⊚. Change all C_i^* in D_1^* to C_i, get a deduction D_1. From above know, D_1 is an α-minimal resolution deduction from S, and this deduction get α-⊚ or α∨g.

If D_1 is the former, the conclusion holds.

If D_1 is the later, let $S_2 = C_1 \wedge \ldots \wedge C_{i-1} \wedge g \wedge C_{i+1} \wedge \ldots \wedge C_n$, absolutely, $S_2 \le \alpha$ and $K(S_2) < m$. By induction assume, there exist an α-minimal resolution deduction D_2^* from S_2 to α-⊚, and change all g in D_2^* to α∨g, get deduction D_2. Now D_2 is an α-minimal resolution deduction from S, and D_2 deduct to α-⊚ directly. Connect D_1 and D_2, we get an α-minimal resolution deduction from S to α-⊚. This completes the proof.

Example 3.4 Let $C_1 = x \to y$, $C_2 = (x \to z)' \vee (s \to t)$, $C_3 = (y \to z) \vee (y \to a_2) \vee (a_5 \to q)$, $C_4 = (s \to t)'$, $C_5 = (p \to q)'$ be five generalized clauses in lattice-valued propositional logic $L_9P(X)$, where $a_2, a_5 \in L_9$, x, y, z, s, t, p, q are propositional variables, written as $S = C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5$. If $\alpha = a_6$, then $S \le \alpha$ and there exists an α-minimal resolution deduction from S to α-⊚.

In fact, there are four α-minimal resolution groups occurring in S, i.e.,

1) $x \to y, (x \to z)', y \to z$;
2) $x \to y, (x \to z)', y \to a_2$;
3) $s \to t, (s \to t)'$;
4) $a_5 \to q, (p \to q)'$.

Since each α-minimal resolution group satisfies Theorem 3.4, we can obtain an α-minimal resolution deduction from S to α-⊚ as follows:

(1) $x \to y$
(2) $(x \to z)' \vee (s \to t)$
(3) $(y \to z) \vee (y \to a_2) \vee (a_5 \to q)$
(4) $(s \to t)'$
(5) $(p \to q)'$
(6) $(x \to z)' \vee \alpha$ by (2), (4)
(7) $(y \to z) \vee (y \to a_2) \vee \alpha$ by (3), (5)
(8) $(y \to a_2) \vee \alpha$ by (1), (6), (7)
(9) α by (1), (6), (8)

Example 3.5 [4] Let $C_1 = y \to b$, $C_2 = (x \to y) \vee y \vee (p \to q)'$, $C_3 = (x \to z)' \vee (s \to t)$, $C_4 = (s \to t)'$, $C_5 = (q \to w)'$ be five generalized clauses in lattice-valued propositional logic $L_6P(X)$, where $b \in L_6$, x, y, z, s, t, p, q, w are propositional variables, written as $S = C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5$. If $\alpha = b$, then $S \le \alpha$ and there exists an α-minimal resolution deduction from S to α-◎.

In fact, there are four α-minimal resolution groups occurring in S, i.e.,

1) $y \to b, x \to y, (x \to z)'$; 2) $y \to b, y$;
3) $(p \to q)', (q \to w)'$; 4) $s \to t, (s \to t)'$.

Since each α-minimal resolution group satisfies Theorem 3.4, so we can obtain an α-minimal resolution deduction from S to α-⊚ as follows:

(1) $y \to b$

(2) $(x \to y) \vee y \vee (p \to q)'$
(3) $(x \to z)' \vee (s \to t)$
(4) $(s \to t)'$
(5) $(q \to w)'$
(6) $(x \to y) \vee (p \to q)' \vee \alpha$ by (1), (2)
(7) $(x \to z)' \vee \alpha$ by (3), (4)
(8) $(x \to y) \vee \alpha$ by (5), (6)
(9) α by (1), (7), (8)

From the example 3.5, there exists an α-minimal resolution deduction from S to α-◎. But according to the binary α-resolution principle, the generalized clause (8) occurring in deduction does not have a binary α-resolution pair, we will stop at (8). Because this example is simple, so we can find there does not exist a binary α-resolution deduction from S to α-◎ easily. Binary α-resolution principle does not have the completeness and inefficiency. Therefore, we need to break through the limitations of binary α-resolution automated reasoning and research α-n(t) resolution dynamic automated reasoning based on lattice-valued logic. This paper is the theoretical guidance of research of α-n(t)-ary resolution dynamic automated reasoning, it not only breaks through the limitations of binary α-resolution, but reduces the generation of redundant clauses also.

The determination of α-minimal resolution of generalized literals is very important in α-minimal resolution automated reasoning, so corresponding α-minimal resolution method research is important and meaning.

4. α-minimal resolution principle based on lattice-valued first-order logic LF(X)

Generalized-clauses and generalized-literals occurring in this section always belong to a generalized-Skölem standard form, i.e., for any generalized-clause C and generalized-literal g, all variables of C and g are bound variables with the quantifier ∀. For any generalized-clauses $G_1,G_2,\ldots,G_n$ ($n\geq 3$), there always exists a renamed substitution such that $G_1,G_2,\ldots,G_n$ have no common variables. Therefore, generalized-clauses $C_1,C_2,\ldots,C_n$($n\geq 3$) occurring in the following have no common variables. In addition, the definitions of substitution, the most general unifier, ground substitution, instance, ground instance occurring in the following are the same as those in classical logic.

Definition 4.1 Let $C_i = p_{i1} \vee \ldots \vee p_{in_i}$ (i=1,2,…,n) be generalized-clauses without common variables in LF(X), $H_i = \{p_{i1},\ldots, p_{in_i}\}$ is the set of all generalized-literals occurring in C_i, $i=1,2,\ldots,n$, $\alpha\in L$. If there exist generalized-literals $x_i \in H_i$ and a substitution σ such that $x_1^\sigma \wedge x_2^\sigma \wedge\ldots\wedge x_n^\sigma \leq\alpha$, but for any $j\in\{1, 2,\ldots, n\}$, $x_1^\sigma\wedge\ldots\wedge x_{j-1}^\sigma\wedge x_{j+1}^\sigma\wedge\ldots\wedge x_n^\sigma \not\leq \alpha$, then

$$C_1^\sigma(x_1^\sigma=\alpha) \vee C_2^\sigma(x_2^\sigma=\alpha) \vee\ldots\vee C_n^\sigma(x_n^\sigma=\alpha) \quad (4.1)$$

is called an α-minimal resolvent of $C_1, C_2,\ldots,C_n$, which is denoted by $R_{f(g\text{-}\alpha)}{}^{m}(C_1(x_1), C_2(x_2),\ldots, C_n(x_n))$. Where "f" means "first-order logic", "m" represents "m-ary", and $x_1, x_2,\ldots, x_n$ are called an α-minimal resolution group.

Let $C_i = C_i^* \vee x_i \vee p_{i1}\vee\ldots\vee p_{in_i}$, $i = 1,2,\ldots, n$, satisfy $\{x_i, p_{i1},\ldots, p_{in_i}\} = \{q_i \mid q_i$ is a generalized-literal in C_i, $q_i^\sigma = x_i^\sigma\}$.

and $x_1, x_2,\ldots, x_n$ is an α-minimal resolution group. So

$$R_{f(g\text{-}\alpha)}{}^{m}(C_1(x_1), C_2(x_2),\ldots,C_n(x_n)) = C_1^{*\sigma}\vee C_2^{*\sigma}\vee\ldots\vee C_n^{*\sigma}\vee\alpha$$

Theorem 4.1 Let $C_i = p_{i1}\vee\ldots\vee p_{in_i}$ be generalized-clauses without common variables in LF(X), $H_i = \{p_{i1},\ldots, p_{in_i}\}$ the set of all generalized-literals occurring in C_i, $i = 1,2,\ldots,n$, $\alpha\in L$. If there exist a substitution σ and generalized-literals $x_i\in H_i$, $i= 1,2,\ldots,n$, such that $x_1, \ldots, x_n$ is an α-minimal resolution group, then

$$C_1 \wedge C_2 \wedge\ldots\wedge C_n \leq R_{f(g\text{-}\alpha)}{}^{m}(C_1(x_1), C_2(x_2),\ldots,C_n(x_n)),$$

i.e., $C_1 \wedge C_2 \wedge\ldots\wedge C_n \leq C_1^\sigma (x_1^\sigma=\alpha) \vee C_2^\sigma (x_2^\sigma=\alpha) \vee\ldots\vee C_n^\sigma (x_n^\sigma=\alpha)$.

Proof Similar to Theorem 3.2, we can get the following conclusion:

$$C_1^\sigma \wedge C_2^\sigma \wedge\ldots\wedge C_n^\sigma \leq C_1^\sigma (x_1^\sigma=\alpha) \vee C_2^\sigma (x_2^\sigma=\alpha) \vee\ldots\vee C_n^\sigma (x_n^\sigma=\alpha).$$

Since σ is a substitution, so we can obtain

$$C_1 \wedge C_2 \wedge\ldots\wedge C_n \leq C_1^\sigma\wedge C_2^\sigma\wedge\ldots\wedge C_n^\sigma.$$

Hence the conclusion holds.

Definition 4.2 Suppose $S=C_1\wedge C_2\wedge\ldots\wedge C_n$, where $C_1, C_2,\ldots,C_n$ are generalized-clauses in LF(X), $\alpha\in L$. $\{\Phi_1, \Phi_2,\ldots,\Phi_t\}$ is called an α-minimal resolution deduction from S to generalized-clause Φ_t, H_i is the set of all generalized-literals occurring in Φ_i ($i = 1, 2, \ldots, t$), if

(1) $\Phi_i \in S$, or

(2) There exist $r_1, r_2,\ldots,r_{k_i} < i$, and $x_d \in H_{r_d}$ ($d = 1, 2, \ldots, k_i$), such that

$$R_{f(g\text{-}\alpha)}{}^{m}(\Phi^0_{r_1}(x_1), \Phi^0_{r_2}(x_2), \ldots, \Phi^0_{r_{k_i}}(x_{k_i})) = \Phi_i,$$ where $\Phi^0_{r_d}$ is Φ_{r_d} or an instance of Φ_{r_d}.

Theorem 4.2 (**Soundness**) Suppose $S = C_1 \wedge C_2 \wedge\ldots\wedge C_n$, where $C_1, C_2,\ldots,C_n$ are generalized-clauses in LF(X), $\alpha\in L$. $\{\Phi_1, \Phi_2,\ldots,\Phi_t\}$ is an α-minimal resolution deduction from S to generalized-clause Φ_t. If Φ_t is α-◎, then $S \leq \alpha$, i.e., if $\Phi_t \leq \alpha$, then $S \leq \alpha$.

Proof According to Definition 4.2 and Theorem 4.1, we can obtain

$$S \leq \Phi_1 \wedge \Phi_2 \wedge \ldots \wedge \Phi_t \leq \alpha \text{ easily.}$$

Theorem 4.3 (Lifting Lemma) Let $C_1, C_2, \ldots, C_n$ be generalized-clauses without common variables in LF(X), C_i^0 an instance of C_i, $i = 1,2,\ldots,n$. If Ω_0 is an α-minimal resolvent of $C_1^0, C_2^0, \ldots, C_n^0$, then there exists an α-minimal resolvent Ω of $C_1, C_2, \ldots, C_n$ such that Ω_0 is an instance of Ω, i.e., Fig.4.1 holds.

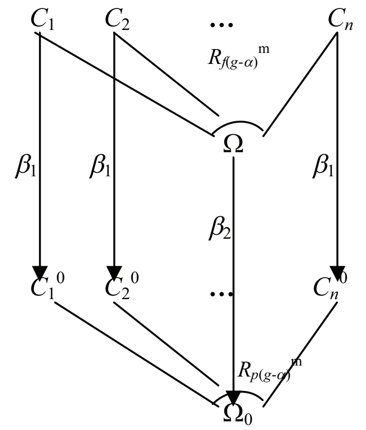

Fig.4.1

Proof Since C_i^0 is an instance of generalized-clause C_i, $i = 1,2,\ldots,n$, so there exists a substitution ε_i such that $C_i^0 = C_i^{\varepsilon_i}$. Let H_i^0 be the set of all generalized-literals occurring in C_i^0, $i = 1,2,\ldots,n$. Since Ω_0 is an α-minimal resolvent of $C_1^0, C_2^0, \ldots, C_m^0$, so there exist a substitution σ and generalized-literals $x_i^0 \in H_i^0$ such that $x_1^{0\sigma} \wedge x_2^{0\sigma} \wedge \ldots \wedge x_n^{0\sigma} \leq \alpha$ and $\Omega_0 = C_1^{0\sigma}(x_1^{0\sigma} = \alpha) \vee C_2^{0\sigma}(x_2^{0\sigma} = \alpha) \vee \ldots \vee C_n^{0\sigma}(x_n^{0\sigma} = \alpha)$.

Let $C_i = C_i^* \vee x_i \vee g_{i1} \vee \ldots \vee g_{ik_i}$, $i = 1,2,\ldots,n$, satisfy

① $x_i^0 = x_i^{\varepsilon_i}$

② $\{x_i, g_{i1}, \ldots, g_{ik_i}\} = \{g_i \mid g_i$ is a generalized-literal occurring in C_i, $g_i^{\varepsilon_i\sigma} = x_i^{\varepsilon_i\sigma}\}$. Set $\varepsilon = \varepsilon_1 \cup \varepsilon_2 \cup \ldots \cup \varepsilon_n$.

So

$$\Omega_0 = C_1^{*\varepsilon_1\sigma} \vee C_2^{*\varepsilon_2\sigma} \vee \ldots \vee C_n^{*\varepsilon_n\sigma} \vee \alpha$$
$$= C_1^{*\varepsilon\sigma} \vee C_2^{*\varepsilon\sigma} \vee \ldots \vee C_n^{*\varepsilon\sigma} \vee \alpha.$$

Let λ_i be the most general unifier of $x_i, g_{i1}, \ldots, g_{ik_i}$, $V_i = \{y_{i1}, y_{i2}, \ldots, y_{is_i}\}$ the set of all variables occurring in C_i, $i \in \{1,2,\ldots,n\}$. Since $C_1, C_2, \ldots, C_n$ are generalized-clauses without common variables, so $V_1 \cap V_2 \cap \ldots \cap V_n = \varnothing$. In the following, $v_1, v_2, \ldots, v_h$ occurring in the substitution $\{t_1/v_1, t_2/v_2, \ldots, t_h/v_h\}$ are called the denominator part of substitution $\{t_1/v_1, t_2/v_2, \ldots, t_h/v_h\}$.

Suppose the denominator part of substitution $\varepsilon\sigma$ only have variables $y_{11}, \ldots, y_{1s_1}, \ldots, y_{i1}, \ldots, y_{is_i}, \ldots, y_{n1}, \ldots, y_{ns_n}$. Let

$(\varepsilon\sigma)_i = \{u \mid u \in \varepsilon\sigma$, the denominator of u occurs in $\{y_{i1}, y_{i2}, \ldots, y_{is_i}\}$, $i = 1,2,\ldots,n$.

Hence we can obtain $(\varepsilon\sigma)_i = \lambda_i \cdot \delta_i$, where δ_i is a substitution. Since the denominator of $(\varepsilon\sigma)_i$ only have variables $y_{i1}, y_{i2}, \ldots, y_{is_i}$, so the denominator of δ_i also only have variables $y_{i1}, y_{i2}, \ldots, y_{is_i}$. We set $\lambda_i = \{t_{i1}/y_{i1}, \ldots, t_{is_i}/y_{is_i}\}$. Hence we have

$$(\lambda_1 \cup \lambda_2 \cup \ldots \cup \lambda_n) \cdot (\delta_1 \cup \delta_2 \cup \ldots \cup \delta_n)$$
$$= \{t_{11}^{(\delta 1 \cup \delta 2 \cup \ldots \cup \delta n)}/y_{11}, t_{12}^{(\delta 1 \cup \delta 2 \cup \ldots \cup \delta n)}/y_{12}, \ldots, t_{s_1}^{(\delta_1 \cup \delta_2 \cup \ldots \cup \delta_n)}/y_{1s_1}$$
$$\ldots$$
$$t_{i1}^{(\delta 1 \cup \delta 2 \cup \ldots \cup \delta n)}/y_{i1}, t_{i2}^{(\delta 1 \cup \delta 2 \cup \ldots \cup \delta n)}/y_{i2}, \ldots, t_{is_i}^{(\delta_1 \cup \delta_2 \cup \ldots \cup \delta_n)}/y_{is_i}$$
$$\ldots$$
$$t_{n1}^{(\delta 1 \cup \delta 2 \cup \ldots \cup \delta n)}/y_{n1}, t_{n2}^{(\delta 1 \cup \delta 2 \cup \ldots \cup \delta n)}/y_{n2}, \ldots, t_{ns_n}^{(\delta_1 \cup \delta_2 \cup \ldots \cup \delta_n)}/y_{ns_n}, \delta_1 \cup \delta_2 \cup \ldots \cup \delta_n\}$$
$$= \{t_{11}^{\delta 1}/y_{11}, t_{12}^{\delta 1}/y_{12}, \ldots, t_{1s_1}^{\delta_1}/y_{1s_1}, \ldots, t_{i1}^{\delta i}/y_{i1}, t_{i2}^{\delta i}/y_{i2}, \ldots, t_{is_i}^{\delta_i}/y_{is_i}, \ldots, t_{n1}^{\delta n}/y_{n1}, t_{n2}^{\delta n}/y_{n2}, \ldots, t_{ns_n}^{\delta_n}/y_{ns_n}, \delta_1 \cup \delta_2 \cup \ldots \cup \delta_n\}$$
$$= \{t_{11}^{\delta 1}/y_{11}, t_{12}^{\delta 1}/y_{12}, \ldots, t_{1s_1}^{\delta_1}/y_{1s_1}, \delta_1, \ldots, t_{i1}^{\delta i}/y_{i1}, t_{i2}^{\delta i}/y_{i2}, \ldots, t_{is_i}^{\delta_i}/y_{is_i}, \delta_i, \ldots, t_{n1}^{\delta n}/y_{n1}, t_{n2}^{\delta n}/y_{n2}, \ldots, t_{ns_n}^{\delta_n}/y_{ns_n}, \delta_n\}$$
$$= (\lambda_1 \cdot \delta_1) \cup (\lambda_2 \cdot \delta_2) \cup \ldots \cup (\lambda_n \cdot \delta_n).$$

Denote $\lambda = \lambda_1 \cup \lambda_2 \cup \ldots \cup \lambda_n$, $\delta = \delta_1 \cup \delta_2 \cup \ldots \cup \delta_n$, so we have

$$\varepsilon\sigma = (\lambda_1 \cdot \delta_1) \cup (\lambda_2 \cdot \delta_2) \cup \ldots \cup (\lambda_n \cdot \delta_n) = \lambda \cdot \delta$$

Since $x_1^{0\sigma} \wedge x_2^{0\sigma} \wedge \ldots \wedge x_n^{0\sigma} \leq \alpha$, i.e., $x_1^{\varepsilon\sigma} \wedge x_2^{\varepsilon\sigma} \wedge \ldots \wedge x_n^{\varepsilon\sigma} \leq \alpha$, so

$x_1^{\lambda\delta} \wedge x_2^{\lambda\delta} \wedge \ldots \wedge x_n^{\lambda\delta} \leq \alpha$. Hence

$C_1^{\lambda}(x_1^{\lambda} = \alpha) \vee C_2^{\lambda}(x_2^{\lambda} = \alpha) \vee \ldots \vee C_n^{\lambda}(x_n^{\lambda} = \alpha)$ is an α-minimal resolvent of $C_1, C_2, \ldots, C_n$. Because only generalized-literals $g_{i1}, \ldots, g_{ik_i}$ are equal to x_i under substitution $\varepsilon\sigma$ and $g_{i1}, \ldots, g_{ik_i}$ are also equal to x_i under substitution λ_i, so all the generalized-literals that are equal to x_i under substitution λ are $g_{i1}, \ldots, g_{ik_i}$. Therefore, we have

$$\Omega_0 = C_1^{*\varepsilon\sigma} \vee C_2^{*\varepsilon\sigma} \vee \ldots \vee C_n^{*\varepsilon\sigma} \vee \alpha$$
$$= (C_1^* \vee C_2^* \vee \ldots \vee C_n^* \vee \alpha)^{\varepsilon\sigma}$$
$$= (C_1^* \vee C_2^* \vee \ldots \vee C_n^* \vee \alpha)^{\lambda\delta}$$
$$= (C_1^{*\lambda} \vee C_2^{*\lambda} \vee \ldots \vee C_n^{*\lambda} \vee \alpha)^{\delta}$$
$$= \Omega^{\delta}.$$

This completes the proof.

Theorem 4.4 (Completeness) Suppose $S = C_1 \wedge C_2 \wedge\ldots\wedge C_n$, where $C_1, C_2,\ldots,C_n$ are generalized-clauses in LF(X), $\alpha\in L$, and $|L| < \aleph_0$. If $S\leq\alpha$. then there exists an α-minimal resolution deduction from S to α-⊚.

Proof In fact, if $S\leq\alpha$, then there at least exists a ground instance S^σ of S such that S^σ is α-false by Corollary 2.1. According to Theorem 3.3, there exists an α-minimal resolution deduction D_0 from S^σ to α-⊚. Moreover, we can lift D_0 to a deduction D from S to α-⊚ by Theorem 4.3. So the conclusion holds.

Example 4.1 Let $C_1 = P(f(x)) \to Q(x)$, $C_2 = (P(y_1) \to R(y_2))' \vee (S(u) \to T(g(u)))$, $C_3 = (Q(a) \to R(z)) \vee (Q(x) \to N(b)) \vee (Q(x)\to M(z))'$, $C_4 = (S(w_1) \to T(g(b)))' \vee (S(w_1) \to T(w_2))'$, $C_5 = (M(c) \to N(v))'$ be five generalized-clauses in lattice-valued first-order logic L_9F(X), where $x, y_1, y_2, z, u, v, w_1, w_2$ are variables and a, b, c are constants, written as $S = C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5$. If $\alpha=a_6$, then $S\leq\alpha$ and there exists an α-minimal resolution deduction from S to α-⊚.

In fact, there exists a ground substitution $\sigma = \{a/x, f(a)/y_1, c/y_2, c/z, b/u, a/v, b/w_1, g(b)/w_2\}$ such that $C_1^\sigma = P(f(a)) \to Q(a)$, $C_2^\sigma = (P(f(a)) \to R(c))' \vee (S(b) \to T(g(b)))$, $C_3^\sigma = (Q(a) \to R(c)) \vee (Q(a) \to N(b))\vee (Q(a) \to M(c))'$, $C_4^\sigma = (S(b) \to T(g(b)))'$, $C_5^\sigma = (M(c) \to N(a))'$ and $S^\sigma = C_1^\sigma \wedge C_2^\sigma \wedge C_3^\sigma \wedge C_4^\sigma \wedge C_5^\sigma \leq\alpha$.

Furthermore, there are four α-minimal resolution groups occurring in S^σ, i.e.,

1) $P(f(a)) \to Q(a)$, $(P(f(a)) \to R(c))'$, $Q(a) \to R(c)$;
2) $P(f(a)) \to Q(a)$, $(P(f(a)) \to R(c))'$, $Q(a) \to N(b)$;
3) $S(b) \to T(g(b))$, $(S(b) \to T(g(b)))'$;
4) $(Q(a)\to M(c))'$, $(M(c) \to N(a))'$.

According to Theorem 4.2 and 4.4, we only need to prove there exists an α-minimal resolution deduction from S^σ to α-⊚. We have the following α-minimal resolution deduction ω*:

(1) $P(f(a)) \to Q(a)$
(2) $(P(f(a)) \to R(c))' \vee (S(b) \to T(g(b)))$
(3) $(Q(a) \to R(c)) \vee (Q(a) \to N(b))\vee (Q(a) \to M(c))'$
(4) $(S(b) \to T(g(b)))'$
(5) $(M(c) \to N(a))'$
(6) $(P(f(a)) \to R(c))' \vee\alpha$ by (2), (4)
(7) $(Q(a) \to R(c)) \vee (Q(a) \to N(b))\vee \alpha$ by (3), (5)
(8) $(Q(a) \to N(b)) \vee\alpha$ by (1), (6), (7)
(9) α by (1), (6), (8)

Since ω* is an α-minimal resolution deduction from S^σ to α-⊚. We have an α-minimal resolution deduction ω from S to α-⊚ as follows:

(1) $P(f(x)) \to Q(x)$
(2) $(P(y_1) \to R(y_2))' \vee (S(u) \to T(g(u)))$
(3) $(Q(a) \to R(z)) \vee (Q(x) \to N(b)) \vee (Q(x)\to M(z))'$
(4) $(S(w_1) \to T(g(b)))' \vee (S(w_1) \to T(w_2))'$
(5) $(M(c) \to N(v))'$
(6) $(P(y_1) \to R(y_2))' \vee\alpha$ by (2), (4)
(7) $(Q(a) \to R(c)) \vee (Q(a) \to N(b)) \vee\alpha$ by (3), (5)
(8) $(Q(a) \to N(b)) \vee\alpha$ by (1), (6), (7)
(9) α by (1), (6), (8)

In fact, according to the binary α-resolution principle, from the above example, the generalized clauses (6) and (7) occurring in the deduction do not have any α-resolution pair. So there does not exist a binary α-resolution deduction from S to α-⊚. So we select the number of generalized literals based on the Definition 4.1 in each resolution. This not only avoids the limitations of binary α-resolution principle, but also reduces the generation of redundant clauses. Thus it improves the resolution efficiency.

5. Conclusions

In this paper, α-minimal resolution principle based on lattice-valued propositional logic system LP(X) was established firstly, as well as its soundness and completeness being proved. α-minimal resolution principle is then further established in the corresponding lattice-valued first-order logic LF(X) along with its soundness theorem, lifting lemma and completeness theorem. Based on this α-minimal resolution principle, we know how to choose the generalized literals in each resolution, it is the theoretical guidance for α-n(t)-ary resolution dynamic automated reasoning. It not only jumps out of the limitation of binary α-resolution principle, but reduces the generation of redundant clauses also. This can reduce many unnecessary resolution, thus improves resolution efficiency. All these works will place a theoretical support for establishing α-n(t)-ary resolution-based dynamic automated reasoning method, algorithm and its implementation with further applications.

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