





























$$S1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, S2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$F(S1, S2) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}^T \left( \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} > \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \right)$$

$$-\left( \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} < \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}^T \left( \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}^T \left( \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right) = [1 \ 0 \ 0 \ -1]$$

Where

$$S_1 = [good \ medium \ bad \ bad]$$

$$S_2 = [medium \ medium \ bad \ medium]$$

$$S_1 > S_2 = [good \ 0 \ 0 \ bad]$$

$$S_2 > S_1 = [medium \ 0 \ 0 \ medium]$$

Now we put

$$\begin{bmatrix} good \\ medium \\ bad \end{bmatrix} \approx \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

And

$$(S_1 > S_2) - (S_1 < S_2)$$

$$[good \ 0 \ 0 \ bad] - [medium \ 0 \ 0 \ medium]$$

$$\begin{bmatrix} good & 1 & 0 & 0 & 0 \\ medium & 0 & 0 & 0 & 0 \\ bad & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} good & 0 & 0 & 0 & 0 \\ medium & 1 & 0 & 0 & 1 \\ bad & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} good & 1 & 0 & 0 & 0 \\ medium & -1 & 0 & 0 & -1 \\ bad & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now good as value 3 and medium as value 2 so we have the differences

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = [3 \ 2 \ 1] \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$= (3)(1) + (2)(-1) + (1)(0) = 1$$

good > medium

and

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = -2 + 1 = -1$$

bad < medium

So when we compare S1 with S2 we have a conflicting situation for which when we decrease the value from good to medium, we have as overall or result an increase in the value from bad to medium and this is inconsistent or *non monotonic*.

### 5. Dominance-based rough set approach and agents

Now we study the relation that we have between the active sets theory and dominance with its rule.

#### 5.1 Dominance-based and Agents

Given the map in Figure 15 for the three object<sub>1</sub>, four object<sub>2</sub> and three object<sub>3</sub>

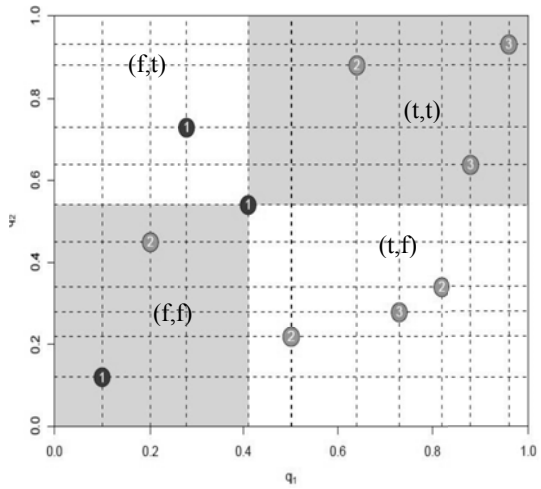


Figure 15. Dominance map.

With active set theory for the three elements of the object<sub>1</sub>

$$Object_1 = \begin{pmatrix} & 1 & 2 & 3 \\ (t & t) & f & t & f \\ (t & f) & f & t & f \\ (f & t) & t & t & f \\ (f & f) & f & t & t \end{pmatrix}$$

The first element of the object<sub>1</sub> is located in place (f,t), the second element of the object<sub>1</sub> is at the frontier of (t,t), (t,f), (f,t), (f,f), the last element of the object<sub>1</sub> is inside the zone (f,f).

In the active set we denote (1, 2, 3) as the elements of the object<sub>1</sub>. We remark that the element two of the object<sub>1</sub> is at the frontier of the four parts of the two dimensional space. So element two belongs to the four classes (parts of the space) at the same time. Figure 18 shows this graphically.

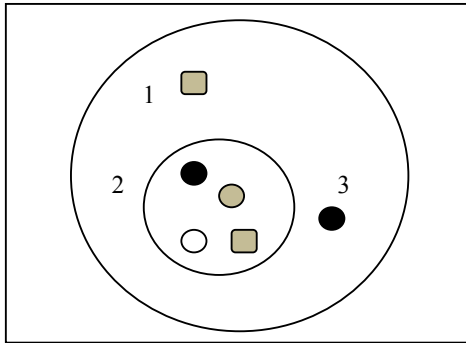


Figure 18. Active sets representation of the dominance in the map of the figure 11 for the object one.

Where black is the class (t, t), grey round is the class (t,f), grey square is the class (f, t) and white is the class (f, f)

For the second object we have four elements in Figure 18. The active set is

$$Object_2 = \begin{pmatrix} & 1 & 2 & 3 & 4 \\ (t & t) & t & f & f & f \\ (t & f) & f & t & t & f \\ (f & t) & f & f & f & f \\ (f & f) & f & f & f & t \end{pmatrix}$$

The graph is shown in Figure 19.

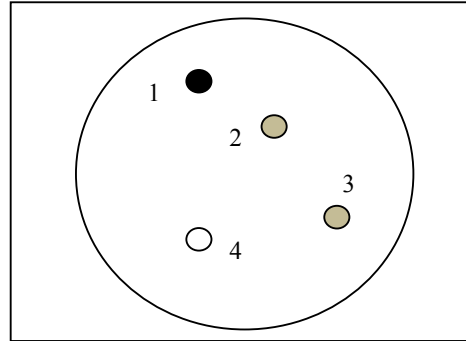


Figure 19. Active sets representation of the dominance in the map of figure 11 for object two.

The third object has three elements so we have the active set

$$Object_3 = \begin{pmatrix} & 1 & 2 & 3 \\ (t & t) & t & t & f \\ (t & f) & f & f & t \\ (f & t) & f & f & f \\ (f & f) & f & f & f \end{pmatrix}$$

The graph of this object is shown in Figure 20

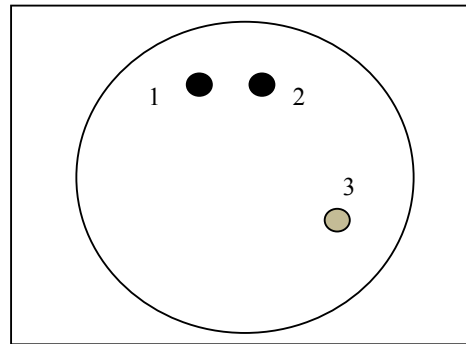


Figure 20. Active sets representation of the dominance in the map of Figure 21 for object three.

By the active sets we have

$$Object_3 = \begin{pmatrix} & 1 & 2 & 3 \\ (t & t) & t & t & f \\ (t & f) & f & t & t \\ (f & f) & f & f & f \end{pmatrix}$$

For the map shown in Figure 21 we have the objects shown in Equations (35-37)

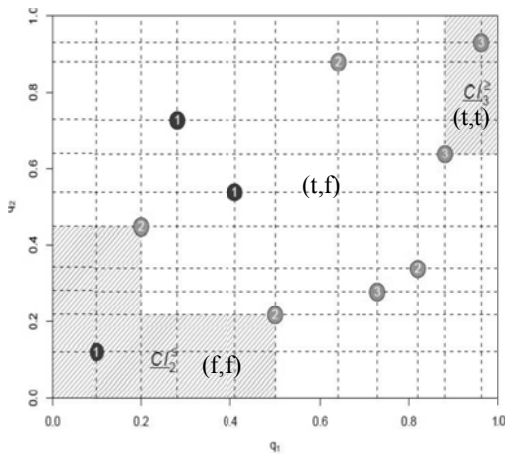


Figure 21. Dominance map.

$$Object_1 = \begin{pmatrix} & 1 & 2 & 3 \\ \begin{pmatrix} t & t \end{pmatrix} & f & f & f \\ \begin{pmatrix} t & f \end{pmatrix} & t & t & f \\ \begin{pmatrix} f & f \end{pmatrix} & f & f & t \end{pmatrix} \quad (35)$$

$$Object_2 = \begin{pmatrix} & 1 & 2 & 3 & 4 \\ \begin{pmatrix} t & t \end{pmatrix} & f & f & f & f \\ \begin{pmatrix} t & f \end{pmatrix} & t & t & t & t \\ \begin{pmatrix} f & f \end{pmatrix} & f & f & t & t \end{pmatrix} \quad (36)$$

$$Object_3 = \begin{pmatrix} & 1 & 2 & 3 \\ \begin{pmatrix} t & t \end{pmatrix} & f & t & t \\ \begin{pmatrix} t & f \end{pmatrix} & t & f & t \\ \begin{pmatrix} f & f \end{pmatrix} & f & f & f \end{pmatrix} \quad (37)$$

At the border of the three regions (classes) we have that the same element include many classes and not only one class or position (t,t),(t,f),(f,t),(f,f).

### 5.2 Dominance-based rules matrix logic computation

Given the eight students evaluation by the matrices

$$S1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, S2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$S3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$S5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

$$S7 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, S8 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Where 1 is the true value and 0 is the false value in the chapter 2 of rough set definition.

In the previous sections we define the active set for the student S1 and for good, medium and bad

$$V(Student_1 \in (professor_i, class_j)) = K_{i,j}^1 = \begin{pmatrix} \text{active sets} & \text{Math} & \text{Phys} & \text{Lit} & \text{Overall} \\ \text{good} & \text{true} & \text{false} & \text{false} & \text{false} \\ \text{medium} & \text{false} & \text{true} & \text{false} & \text{false} \\ \text{bad} & \text{false} & \text{false} & \text{true} & \text{true} \end{pmatrix}$$

where the four columns are math, phys, lit and overall or conclusion. So we put constrains on the math, phys and lit variables and we consider the Overall as the conclusion of the decision rule. We show the active set interpretation examples of decision rules.

Now given the premise

$$\text{mathematics} \succ = \text{good}, \text{Literature} \succ = \text{medium}$$

We can select the students



$$S4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$S6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

And with the union operation we have the conclusion

$$S4 \vee S5 \vee S6 =$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

That formally can write in this way

Then student = good.

Now given the premise

*mathematics*  $\succ=$  *medium*,  
*Literature*  $\succ=$  *medium*

We can select the students

$$S3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$S5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And with the union operation we have the conclusion

$$S3 \vee S4 \vee S5 \vee S6 =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

That formally can write in this way

Then student  $\succ=$  medium for overall column

Now given the premise

*mathematics*  $\succ=$  *good*, *Literature* = *bad*

We can select the students

$$S4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, S5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$S6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

And with the union operation we have the conclusion

$$S4 \vee S5 \vee S6 =$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

That formally can write in this way

Then student = good. for overall column

Now given the premise

*mathematics*  $\succ=$  *medium*,  
*Literature*  $\succ=$  *medium*

We can select the students

$$S1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, S2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

And with the union we have the conclusion

$$S1 \vee S2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

That formally can write in this way

Then student is bad or medium for overall column

### 6. Dominance-based rules by matrix numerical computation

Given the numerical analogy

$$\begin{bmatrix} \text{good} \\ \text{medium} \\ \text{bad} \end{bmatrix} \approx \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

The table of the student for math, phys, Lit can be written by this matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \\ 3 & 2 & 3 \\ 3 & 3 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

and the overall as

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

Now we want to solve the equation

$$A x = b$$

This equation cannot be solved because we do not have the inverse matrix of A. Now we can change the previous equation in a way to have the pseudo inverse in this way

$$A^T A x = A^T b$$

So we have

$$x = (A^T A)^{-1} A^T b$$

Where

$$A^T A = \begin{bmatrix} 40 & 34 & 33 \\ 34 & 30 & 29 \\ 33 & 29 & 32 \end{bmatrix}$$

Is the self and cross relations among the three professors math (M), phys (P) and Lit (L) and the relations between professor and overall.

The diagonal part is

$$\begin{bmatrix} & M & Ph & L \\ M & 40 & 0 & 0 \\ Ph & 0 & 30 & 0 \\ L & 0 & 0 & 38 \end{bmatrix}$$

Where we have the math has the max variation in the marks (good, medium, bad) and Ph has the minimum variation. Now for the correlation or entanglement or synchronisation of the math professor with the others and with the conclusion is

$$\begin{bmatrix} & M & Ph & L \\ M & 0 & 34 & 33 \\ Ph & 34 & 0 & 27 \\ L & 33 & 27 & 0 \end{bmatrix}$$

We remark that M the minimum relation is between Ph and Literature only 27. The max correlation as is intuitive is between math and Physics professor for which we have 34. We are not surprised to see that we have a relative good relation between mathematic professor and the Literature professor for which we have 33.

now if

$$b = A x$$

we have

$$(A^T A)^{-1} A^T A x = x$$

Now from the table of the student we have

$$x = \begin{bmatrix} \text{math} & -0.23 \\ \text{phy} & 0.816 \\ \text{Lit} & 0.529 \end{bmatrix}$$

For math we have a negative value and this means that math generate a lot of inconsistency because has a negative value on the conclusion or overall. Physics is the most consistent element and is the more important professor that give the final conclusion or overall. The Literature professor is not so important as physics in the definition of the result but does not generate inconsistency because has a weight that is positive. Now by the x we can compute the vector b' that is

$$A x = b1$$

Where

$$b1 = \begin{bmatrix} 1.471 \\ 1.701 \\ 2.23 \\ 2.23 \\ 2.529 \\ 3.345 \\ 1.644 \end{bmatrix}$$

The lapse between student S1 and S2 is reduced the minimum inconsistency. For S2, S3 we have no inconsistency so the lapse is near to one. Now for the three professors the student S3 and S4 are equal so is irrational to have as overall the values medium and good. The marks for S5, S6, S7 are in agreement with the conclusion overall. Now given a new student to be evaluated.

Student	Mathematics	Physics	Literature
S9	medium	medium	good

In this case the matrix A is increased by a new student so we have a new matrix B given by the explicit form

$$B = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \\ 3 & 2 & 3 \\ 3 & 3 & 3 \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

With the value of x we can compute the result b by the expression

$$B x = b2$$

So we have the new b = b2

$$b2 = \begin{bmatrix} 1.471 \\ 1.701 \\ 2.23 \\ 2.23 \\ 2.529 \\ 3.345 \\ 1.644 \\ 2.759 \end{bmatrix}$$

In agreement with the other students the valuation for the new student is 2.759 or with approximation we have "good". In conclusion for the new student we have

student	math	Physics	Literature	Overall
S9	medium	medium	good	good

### 7. Conclusion

With the suggestion of the paper Dominance-based Rough Set Approach to Reasoning about Vague Data and with the introduction of the agents in the active set theory, we give a new image of the rough set with a formal logic description of the vague or approximate data. Connection with evidence theory and many valued logic by lattice evaluation gives us a more general image of the rough sets and reasoning. Compensation of the inconsistency in rough set approximation is used to give reasoning for new data in agreement with previous vague data. Because active set was used also for fuzzy set model we suggest a bridge between fuzzy set, rough set and active set.

## 8. References

1. Resconi G. and Jain, L. (2004) Intelligent agents, Springer Verlag
2. Resconi, G., Klir G.J., and U. St. Clair, (1992) Hierarchical uncertainty metatheory based upon modal logic. *International Journal of General Systems* 21 (23-50).
3. Resconi, G., Klir G.J., U. St. Clair, and D. Harmanec. (1993). The integration of uncertainty theories. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 1, 1-18.
4. Resconi, G., Klir G.J. Harmanec, D. and U. St. Clair. (1996). Interpretation of various uncertainty theories using models of modal logic: a summary, *Fuzzy Sets and Systems* 80, 7-14.
5. Harmanec, D., Resconi, G., Klir, G. J. and Pan, Y.. (1995). On the computation of uncertainty measure in Dempster-Shafer theory, *International Journal General Systems*, vol.25 (2). 153-163.
6. Resconi, G., Murai, T. and Shimbo, M. (2000). Field Theory and Modal Logic by Semantic field to make Uncertainty Emerge from Information, *International Journal of General Systems*, vol 29 (5), 737-782.
7. Resconi G. and Turksen I.B. (2001). Canonical Forms of Fuzzy Truthhoods by Meta-Theory Based Upon Modal Logic, *Information Sciences* 131, 157-194. Elsevier
8. Resconi G. and Kovalerchuk, B. (2006). The Logic of Uncertainty with Irrational Agents In: Proc. of JCIS-2006 Advances in Intelligent Systems Research, Taiwan, Atlantis Press
9. Kahneman, D. (2003). Maps of Bounded Rationality: Psychology for Behavioral Economics. *The American Economic Review*. 93(5), 1449-1475.
10. Kovalerchuk B., (1990). Analysis of Gaines' logic of uncertainty, In: Proceeding of NAFIPS '90 vol.2 edited by I.B. Turksen, Toronto, Canada, pp. 293-295.
11. Kovalerchuk B. (1996). Context spaces as necessary frames for correct approximate reasoning. *International Journal of General Systems*, v.25, n 1, 61-80.
12. Kovalerchuk B. and Vityaev E. (2000). Data mining in finance: advances in relational and hybrid methods, Kluwer
13. Baldwin, J. (1991). A theory of mass assignments for artificial intelligence. In: *Fuzzy Logic and Fuzzy Control*. pp. 22-34. IJCAI'91 Workshops on fuzzy logic and fuzzy control, Springer-Verlag, Sydney, Australia (Jan 1991)
14. Baldwin, J. (1994). A calculus for mass assignments in evidential reasoning. In: Yager, R., Fedrizzi, M., Kacprzyk, J. (eds.) *Advances in the Dempster-Shafer Theory of Evidence*, pp. 513-531. John Wiley & Sons, Inc., New York (1994)
15. Baldwin, J.F., Martin, T. and Pilsworth, B. (1995). *Frial - Fuzzy and Evidential Reasoning in Artificial Intelligence*. Research Studies Press Ltd, Taunton UK (1995)
16. Hinde, C.J. (1993) Inference of Fuzzy Relational Tableaux from Fuzzy Exemplifications. *Fuzzy Sets and Systems* 11, 91-101.
17. Hinde, C.J., (2009) Intuitionistic Fuzzy Set, Interval Valued Fuzzy Sets and Mass Assignment: a Unifying Treatment including Inconsistency and Contradiction. *International Journal of Computational Intelligence Research* 4(4), 372-391
18. Hinde, C.J., (2009) Knowledge from Contradiction and Inconsistency. In: Atanassov, K., Baczynski, M., Drewniak, J., Kacprzyk, J., Krawczak, M., Szmidt, E., Wygralak, M., Zadrozny, S. (eds.) *Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics*. Volume 1: Foundations., pp. 95-107. Academic Publishing House EXIT, Warsaw
19. Hinde, C.J., (2009) The statement is contradictory. In: *Proceedings of 2009 International Conference on Artificial Intelligence and Soft Computing*. pp. 235-240. IASTED
20. Hinde, C.J. and Atanassov, K., (2008) On Intuitionistic Fuzzy Negations and Intuitionistic Fuzzy Modal operators with Contradictory Evidence. In: Dimitrov, D., Mladenov, V., Jordanova, S., Mastorakis, N. (eds.) *Proceedings of 9th WSEAS International Conference on Fuzzy Systems (FS '08)*
21. Hinde, C. and Patching, R., (2007) Inconsistent Intuitionistic Fuzzy Sets. In: Atanassov, K., Bustince, H., Hryniewicz, O., Kacprzyk, J., Krawczak, M., Riecan, B., Szmidt, E. (eds.) *Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics*. Foundations., pp. 155-174. Academic Publishing House EXIT, Warsaw
22. Hinde, C.J., Patching, R. and McCoy, S., (2007), Inconsistent Intuitionistic Fuzzy Sets and Mass Assignment. In: Atanassov, K., Bustince, H., Hryniewicz, O., Kacprzyk, J., Krawczak, M., Riecan, B., Szmidt, E. (eds.) *Developments in Fuzzy Sets, Intuition-istic Fuzzy Sets, Generalized Nets and Related Topics*. Foundations., pp. 133-153. Academic Publishing House EXIT, Warsaw
23. Hinde, C.J., Patching, R., Stone, R., Xhemali, D. and McCoy, S. (2007) Reasoning Consistently about Inconsistency. In: Garibaldi, J., Angelov, P. (eds.) *Proceedings of 2007 IEEE International Conference on Fuzzy Systems*. pp. 769-775
24. Patching, R., Hinde, C.J. and McCoy, S. (2006), Inconsistency and semantic unification. *Fuzzy Sets and Systems* 157, 2513-2539 (2006)
25. Resconi, G. and Hinde, C.J., (2010) Active sets, fuzzy sets and inconsistency. In: Aranda, J., Xambo, S. (eds.) *Proceedings of FUZZIEEE 2010*. pp. 354-357. IEEE
26. Resconi, G. and Kovalerchuk, B., (2011), Agents in Quantum and Neural Uncertainty. *Medical Information Science Reference* (Hershey, New York)
27. Scott, D., (1982), Some ordered sets in computer science. In: Rival, I. (ed.) *Ordered Sets*, pp. 677-718. Reidel Publishing Company, Boston
28. Xu, Y., Ruan, D., Qin, K., Liu, J., (2003) *lattice-valued Logic*, Springer
29. Figueira, J. Greco, S., Mousseau, V. and Slowinski, R., (2008) Interactive Multiobjective Optimization using a Set of Additive Value Functions. In J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, *Multiobjective*

- Optimization: Interactive and Evolutionary Approaches, 99--122
30. Greco, S., Mousseau, V. and Slowinski, R. (2008), Ordinal regression revisited: multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, 191(2):415-435, December 2008.
  31. Blaszczynski, J. Greco, S. and Slowinski, R. (2007) Multi-criteria classification - A new scheme for application of dominance-based decision rules. *European Journal of Operational Research*, 181(3):1030 - 1044