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Some Hybrid Geometric Aggregation Operators with 2-tuple Linguistic Information and Their Applications to Multi-attribute Group Decision Making

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Abstract

A new method is developed to solve multi-attribute group decision making (MAGDM) problem in which the attribute values, attribute weights and expert weights are all in the form of 2-tuple linguistic information. First, the operation laws for 2-tuple linguistic information are defined and the related properties of the operation laws are studied. Then, some new hybrid geometric aggregation operators with 2-tuple linguistic information are developed, involving the 2-tuple hybrid weighted geometric average (THWAG) operator, the 2-tuple hybrid linguistic weighted geometric average (T-HLWG) operator and the extended 2-tuple hybrid linguistic weighted geometric average (ET-HLWG) operator. These hybrid geometric aggregation operators generalize the existing 2-tuple linguistic geometric aggregation operators and reflect the important degrees of both the given 2-tuples and the ordered positions of the 2-tuples. In the proposed decision method, using the ET-HLWG operators the individual overall preference values of the alternatives are integrated into the collective ones of the alternatives, which are used to rank the alternatives. The method can sufficiently consider the importance degrees of different experts and thus relieve the influence of those unfair arguments on the decision results. A real example of evaluating university faculty is given to illustrate the proposed method and the comparison analysis demonstrates the universality and flexibility of the proposed method in this paper.

Keywords: Multi-attribute group decision making; Linguistic preference; 2-tuple linguistic information; hybrid aggregation operator

1. Introduction

Multi-attribute group decision making (MAGDM) problems with linguistic information arise from a wide range of real-world situations (Jiang et al.[1]; Herrera and Herrera-Viedma[2]; Parreiras et al.[3]). In linguistic MAGDM analysis, firstly, experts provide their assessment information from the pre-established linguistic term sets. Then the linguistic information provided by experts is aggregated to form a collective opinion on the alternatives and the most desirable alternative(s) can be selected according to the derived collective opinion (Herrera et al.[4]; Jiang et al.[1]; Xu[5,6]; Wei[7,8]; Merigo et al.[9]).

Herrera et al. proposed 2-tuple linguistic representation model, which composed by a linguistic term and a real number (Herrera and Martínez[10-12]). The 2-tuple linguistic model has exact characteristic in linguistic information processing. It avoided information distortion and losing which occur formerly in the linguistic information processing. In recent years, 2-tuple linguistic model has been widely used in decision making problems (Jiang et al.[1]; Herrera and Martínez[12]; Herrera-Viedma et al.[13]; Liu and Jin[14]; Jiang and Fan[15]; Yang and Chen[16]; García et al.[17]; Wei and Zhao[18]; Wei[19]; Wei and Lin[20]; Wei et al.[21]; Wei[22,23]; Chang and Wen[24]; Zhang and Fan[25]; Wang and Fan[26]; Wang[27]; Herrera et al.[28]; Espinilla et al.[29]; Martínez and Herrera[30]; Rodríguez and Martínez[31]). Wei[8] developed some new geometric aggregation operators: the extended 2-tuple weighted geometric (ET-WG) and the extended 2-tuple ordered weighted geometric (ET-OWG) operator. Then, a MAGDM method is presented based on the ET-WG and ET-OWG operators. Herrera and Martínez[10] developed 2-tuple arithmetic averaging (TAA) operator, 2-tuple weighted averaging (TWA) operator, 2-tuple ordered weighted averaging (TOWA) operator and extended 2-tuple weighted averaging (ET-WA) operator. Herrera and Martinez[12] proposed another method to solve the group decision making problem with multi-granularity linguistic information. They constructed linguistic hierarchy term sets and generalized transformation functions to unify the multi-granularity linguistic information into the linguistic 2-tuples. Jiang and Fan[15] proposed the 2-tuple weighted geometric (TWG) operator and 2-tuple ordered weighted geometric (TOWG) operator. Wei[19] utilized the maximizing deviation method to solve the 2-tuple linguistic MAGDM with incomplete attribute weight information. Wei and Lin[20] and Wei[22] developed grey relational analysis (GRA) MAGDM methods based on 2-tuple linguistic information. Xu et al.[21] adopted the virtual linguistic label to replace 2-tuple linguistic variable and proposed the linguistic power average operators and the uncertain linguistic power average operators. Wei[23] developed three new aggregation operators: generalized 2-tuple weighted average (G-2TWA) operator, generalized 2-tuple ordered weighted average (G-2TOWA) operator and induced generalized 2-tuple ordered weighted average (IG-2TOWA) operator. Chang and Wen[24] proposed a novel technique combining 2-tuple and the Ordered Weighted Averaging (OWA) operator for prioritization of failures in a product design failure mode and effect analysis. Zhang and Fan[25] proposed the extended 2-tuple ordered weighted averaging (ET-OWA) operator. Wang and Fan[26] proposed a TOPSIS method for solving MAGDM problems with 2-tuple linguistic assessment information. Wang[27] presented a 2-tuple fuzzy linguistic evaluation model for selecting appropriate agile manufacturing system in relation to MC production. Herrera et al.[28] developed a fuzzy linguistic methodology to deal with unbalanced linguistic term sets.

Most of the proposals for solving MAGDM problems with 2-tuple linguistic information found in literature did not consider the importance degrees of different experts. However, the experts have their different cultural, educational backgrounds, experience and knowledge, and expertise related with the problem domain. Generally speaking, different experts act as different roles in the decision process. Some experts may assign unduly high or unduly low uncertain preference values to their preferred or repugnant objects. In order to relieve the influence of these unfair arguments on the decision results and reflect the importance degrees of all the experts, it is necessary to pay attention to the different importance degrees of different experts in the real-life MAGDM problems. Therefore, this paper develops some new hybrid geometric aggregation operators with 2-tuple linguistic information and proposes a new method for MAGDM problems with 2-tuple linguistic assessments. The motivation of this paper is based on the following facts:

(i) The existing aggregation operators with 2-tuple linguistic information are mainly focused on the weighted arithmetic (geometric) average and the ordered weighted arithmetic (geometric) average operators. There has no investigation about the hybrid aggregation operators with 2-tuple linguistic information.

(ii) The hybrid aggregation operators can reflect the important degrees of both the given 2-tuples and the ordered positions of the 2-tuples. They are usually used to integrate the individual overall preference values of alternatives into the collective ones of alternatives. To do so, each individual overall preference value should first be weighted by using the corresponding expert weight, which can sufficiently reflect the importance degrees of different experts.

(iii) Wei[8] only considered that the weight information of attributes and experts is in the form of the linguistic variables. The MAGDM method of Wei[8] can not deal with the case that the weight information of attributes and experts takes the form of the 2-tuples. However, this case may appear in some real-life decision problems (see Section 5). These new hybrid geometric aggregation operators with 2-tuple linguistic information proposed in this paper can effectively overcome this drawback.

(iv) The proposed method in this paper is more reasonable and flexible than the existing ones and can be applicable to real-life decision problems in many areas such as risk investment, performance evaluation of military system, engineering management, supply chain, and so on.

The rest of the paper is arranged as follows. Section 2 introduces the notions for 2-tuple linguistic information, gives the operation laws and analyzes the properties of the operation laws. Section 3 presents the existing 2-tuple linguistic geometric aggregation operators and proposes some new hybrid geometric aggregation operators with 2-tuple linguistic information. Section 4 constructs the MAGDM model with 2-tuple linguistic assessments and proposes the corresponding decision method. A real application to evaluating university faculty for tenure and promotion example is given in Section 5. The comparison analysis with other method is conducted in Section 6. Concluding remark is made in Section 7.

2. 2-tuple linguistic information

In this section, some related notions for 2-tuple linguistic information are listed, then the operation laws and properties for 2-tuple linguistic information are investigated

2.1. *2-tuple linguistic information*

Definition 1. Let $S=\{s_0,s_1,s_2,\cdots,s_{t-1}\}$ be a finite and totally ordered discrete linguistic term set with odd cardinality, where s_i represents a possible value for a linguistic variable. $\beta\in[0,t-1]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as:

$$\Delta:[0,t-1]\to S\times[-0.5,0.5)$$
$$\beta\to\Delta(\beta)=(s_i,\alpha) \tag{1}$$

where $i=round(\beta)$, $\alpha=\beta-i$, $\alpha\in[-0.5,0.5)$, $round(\cdot)$ is the usual round operation. s_i has the closest index label to β and α is the value of the symbolic translation (Herrera and Martínez[10-12]; Herrera et al.[4]).

Definition 2. Let $S=\{s_0,s_1,s_2,\cdots,s_{t-1}\}$ be a linguistic term set and (s_i,α) be a linguistic 2-tuple. There is always a function Δ^{-1}, such that, from a 2-tuple it returns its equivalent numerical value $\beta\in[0,t]\subset R$, which is (Herrera and Martínez[10-12]; Herrera et al.[4])

$$\Delta^{-1}:S\times[-0.5,0.5)\mapsto[0,t-1]$$
$$\Delta^{-1}(s_i,\alpha)=i+\alpha=\beta\,. \tag{2}$$

From Definitions 1 and 2, we can conclude that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

$$\Delta(s_i)=(s_i,0)\,. \tag{3}$$

Definition 3. Let (s_k,α_k) and (s_l,α_l) be two 2-tuples, they should have the following properties (Herrera and Martínez[10-12]; Herrera et al.[4]):

1) If $k<l$, then (s_k,α_k) is smaller than (s_l,α_l), denoted by $(s_k,\alpha_k)<(s_l,\alpha_l)$;

2) If $k=l$, then

a) if $\alpha_k=\alpha_l$, then (s_k,α_k) and (s_l,α_l) represent the same information, denoted by $(s_k,\alpha_k)=(s_l,\alpha_l)$;

b) if $\alpha_k<\alpha_l$, then $(s_k,\alpha_k)<(s_l,\alpha_l)$;

c) If $\alpha_k>\alpha_l$, then (s_k,α_k) is bigger than (s_l,α_l), denoted by $(s_k,\alpha_k)>(s_l,\alpha_l)$.

2.2. *Operation laws and properties for 2-tuple linguistic information*

Definition 4. Let (s_k,α_k) and (s_l,α_l) be two 2-tuples and $\lambda\geq 0$. Then the operation laws for 2-tuples are defined as follows:

(i) $(s_k,\alpha_k)\oplus(s_l,\alpha_l)$

$$=\begin{cases}\Delta(\Delta^{-1}(s_k,\alpha_k)+\Delta^{-1}(s_l,\alpha_l)),\ \text{if}\ \Delta^{-1}(s_k,\alpha_k)+\Delta^{-1}(s_l,\alpha_l)\leq t-1\\(s_{t-1},0),\ \text{if}\ \Delta^{-1}(s_k,\alpha_k)+\Delta^{-1}(s_l,\alpha_l)>t-1\end{cases};$$

(ii) $(s_k,\alpha_k)\otimes(s_l,\alpha_l)$

$$=\begin{cases}\Delta(\Delta^{-1}(s_k,\alpha_k)\Delta^{-1}(s_l,\alpha_l)), \text{ if } \Delta^{-1}(s_k,\alpha_k)\Delta^{-1}(s_l,\alpha_l)\le t-1\\(s_{t-1},0), \text{ if } \Delta^{-1}(s_k,\alpha_k)\Delta^{-1}(s_l,\alpha_l)>t-1\end{cases};$$

(iii) $\lambda(s_k,\alpha_k)$

$$=\begin{cases}\Delta(\lambda\Delta^{-1}(s_k,\alpha_k)), \text{ if } \lambda\Delta^{-1}(s_k,\alpha_k)\le t-1\\(s_{t-1},0), \text{ if } \lambda\Delta^{-1}(s_k,\alpha_k)>t-1\end{cases};$$

(iv) $(s_k,\alpha_k)^{\lambda}$

$$=\begin{cases}\Delta((\Delta^{-1}(s_k,\alpha_k))^{\lambda}), \text{ if } (\Delta^{-1}(s_k,\alpha_k))^{\lambda}\le t-1\\(s_{t-1},0), \text{ if } (\Delta^{-1}(s_k,\alpha_k))^{\lambda}>t-1\end{cases};$$

(v) $(s_k,\alpha_k)^{(s_l,\alpha_l)}$

$$=\begin{cases}\Delta((\Delta^{-1}(s_k,\alpha_k))^{\Delta^{-1}(s_l,\alpha_l)}), \text{ if } (\Delta^{-1}(s_k,\alpha_k))^{\Delta^{-1}(s_l,\alpha_l)}\le t-1\\(s_{t-1},0), \text{ if } (\Delta^{-1}(s_k,\alpha_k))^{\Delta^{-1}(s_l,\alpha_l)}>t-1\end{cases}.$$

It should be noted that if the 2-tuple linguistic information comes from different linguistic term sets (i.e. multi-granularity linguistic information), they have to be converted into the fuzzy sets defined in the basic linguistic term set by means of a transformation function (Herrera, et al.[32]), then they can be operated using the above operation laws.

If $\Delta^{-1}(s_k,\alpha_k)+\Delta^{-1}(s_l,\alpha_l)>t-1$, the addition for (s_k,α_k) and (s_l,α_l) may be considered as the maximum 2-tuple $(s_{t-1},0)$ on the linguistic term set *S*. The other operations can be interpreted analogously.

Obviously, Definition 4 can assure that the operation results regarding 2-tuples and linguistic terms must be in $[0,t-1]$, which is accordance with the CWW scheme (Rodríguez and Martínez[31]).

As far as we know, however, there is less investigation on the operation laws of 2-tuples. Definition 4 gives the operation laws of 2-tuples, which can be used to directly compute for 2-tuple linguistic information. Definition 4 is an interesting and valuable work for 2-tuples although there maybe lose little information under some situations. How to define more reasonable operation laws of 2-tuples will be further researched in the future.

In the following, suppose that a given linguistic term set is $S=\{s_0,s_1,s_2,\cdots,s_8\}$, we give some examples to illustrate the above Definition 4.

Example 1. $(s_1,0.1)\oplus(s_3,0.2)=\Delta(\Delta^{-1}(s_1,0.1)+\Delta^{-1}(s_3,0.2))$

$=\Delta(4.3)=(s_4,0.3)$;

$(s_2,0.3)\oplus(s_7,0.2)=(s_8,0)$.

Example 2. $(s_1,0.1)\otimes(s_3,0.2)=\Delta(\Delta^{-1}(s_1,0.1)\cdot\Delta^{-1}(s_3,0.2))$

$=\Delta(3.52)=(s_4,-0.48)$;

$(s_2,0.1)\otimes(s_4,0.3)=(s_8,0)$.

Example 3. $0.6(s_2,0.3)=\Delta(0.6\Delta^{-1}(s_2,0.3))$

$=\Delta(1.38)=(s_1,0.38)$;

$2(s_4,0.2)=(s_8,0))$.

Example 4. $(s_2,0.3)^{0.5}=\Delta((\Delta^{-1}(s_2,0.3))^{0.5})=\Delta(2.3^{0.5})$

$=\Delta(1.5166)=(s_2,-0.4834)$;

$(s_4,0.2)^2=(s_8,0))$.

Example 5. $(s_1,0.1)^{(s_3,0.2)}=\Delta((\Delta^{-1}(s_1,0.1))^{\Delta^{-1}(s_3,0.2)})$

$=\Delta(1.1^{3.2})=\Delta(1.3566)=(s_1,0.3566)$;

$(s_3,0.2)^{(s_4,0.4)}=(s_8,0)$.

From Definition 4, the following properties are proven:

1) $(s_k,\alpha_k)\oplus(s_l,\alpha_l)=(s_l,\alpha_l)\oplus(s_k,\alpha_k)$;

2) $(s_k,\alpha_k)\otimes(s_l,\alpha_l)=(s_l,\alpha_l)\otimes(s_k,\alpha_k)$;

3) $\lambda((s_k,\alpha_k)\oplus(s_l,\alpha_l))=\lambda(s_k,\alpha_k)\oplus\lambda(s_l,\alpha_l)$;

4) $((s_k,\alpha_k)^{\lambda})^k=(s_k,\alpha_k)^{\lambda k}$,

$(s_k,\alpha_k)^{\lambda}\otimes(s_k,\alpha_k)^k=(s_k,\alpha_k)^{\lambda+k}$, $k\ge 0$;

5) For any (s_i,α_i), there have

$$[(s_k,\alpha_k)\oplus(s_l,\alpha_l)]\otimes(s_i,\alpha_i)$$
$$=[(s_k,\alpha_k)\otimes(s_i,\alpha_i)]\oplus[(s_l,\alpha_l)\otimes(s_i,\alpha_i)]$$

and

$$[(s_k,\alpha_k)\otimes(s_l,\alpha_l)]\otimes(s_i,\alpha_i)$$
$$=(s_k,\alpha_k)\otimes[(s_l,\alpha_l)\otimes(s_i,\alpha_i)].$$

3. Some geometric aggregation operators with 2-tuple linguistic information

In this section, we first present the existing 2-tuple linguistic geometric aggregation operators, and then propose some new hybrid geometric aggregation operators with 2-tuple linguistic information.

3.1. *The existing 2-tuple linguistic geometric aggregation operators*

Based on Definitions 2 and 3, the existing 2-tuple linguistic geometric aggregation operators are presented in this subsection. For convenience, let *T* be the set composed of all 2-tuples.

Definition 5. Let $x=\{(r_1,a_1),(r_2,a_2),\cdots,(r_n,a_n)\}$ be a set of 2-tuples, and $\boldsymbol{w}=(w_1,w_2,\cdots,w_n)^{\mathrm{T}}$ be the weighting vector of 2-tuples (r_j,a_j) $(j=1,2,\cdots,n)$, satisfying that $0\le w_j\le 1$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n}w_j=1$.

The 2-tuple weighted geometric (TWG) average operator is defined as (Jiang and Fan[15])

$$\mathrm{TWG}_{\boldsymbol{w}}((r_1,a_1),(r_2,a_2),\cdots,(r_n,a_n))=\Delta(\prod_{j=1}^{n}\Delta^{-1}(r_j,a_j)^{w_j}) \quad .(4)$$

Lemma 1. Let $a_j > 0$, $w_j > 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$, then

$$\prod_{j=1}^{n} (a_j)^{w_j} \le \sum_{j=1}^{n} w_j a_j ,$$

with equality if and only if $a_1 = a_2 = \cdots = a_n$ (Torra and Narukawa[33]).

Theorem 1. In Definition 5, the argument of delta in Eq. (4), $\prod_{j=1}^{n} \Delta^{-1}(r_j, a_j)^{w_j}$, is defined in $[0, t-1]$.

Proof. By Definition 2, we know that $\Delta^{-1}: S \times [-0.5, 0.5) \mapsto [0, t-1]$. Thus, $0 \le \Delta^{-1}(r_j, a_j) \le t-1$ $(j = 1, 2, \cdots, n)$.

Since the weighted vector $\boldsymbol{w} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ satisfies that $0 \le w_j \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$, it yields by Lemma 1 that

$$0 \le \prod_{j=1}^{n} \Delta^{-1}(r_j, a_j)^{w_j} \le \sum_{j=1}^{n} w_j \Delta^{-1}(r_j, a_j) \le \sum_{j=1}^{n} w_j (t-1) = (t-1) .$$

Therefore, $\prod_{j=1}^{n} \Delta^{-1}(r_j, a_j)^{w_j} \in [0, t-1]$ and the proof of Theorem 1 is completed.

Definition 6. Let $x = \{(r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)\}$ be a set of 2-tuples. The 2-tuple ordered weighted geometric (TOWG) average operator of dimension n is a mapping $\mathrm{TOWG}: T^n \to T$ so that

$$\mathrm{TOWG}_{\boldsymbol{w}}((r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)) = \Delta(\prod_{j=1}^{n} \Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)})^{w_j}) . \quad (5)$$

where $\boldsymbol{w} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ is the weighted vector correlating with TOWG, satisfying that $0 \le w_j \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$. $(\sigma_{(1)}, \sigma_{(2)}, \cdots, \sigma_{(n)})$ is a permutation of $(1, 2, \cdots, n)$ such that $(r_{\sigma(j-1)}, a_{\sigma(j-1)}) \ge (r_{\sigma(j)}, a_{\sigma(j)})$ for any j (Jiang and Fan[15]).

Remark 1. In Definition 6, since the weighted vector $\boldsymbol{w} = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$ satisfies that $0 \le w_j \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$, the argument of delta in Eq. (5), $\prod_{j=1}^{n} \Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)})^{w_j}$, is also defined in $[0, t-1]$, which can be proven by the similar way to Theorem 1.

Definition 7. Let $x = \{(r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)\}$ be a set of 2-tuples, and $\boldsymbol{C} = ((c_1, b_1), (c_2, b_2), \cdots, (c_n, b_n))^{\mathrm{T}}$ be the linguistic weighting vector of 2-tuples (r_j, a_j) $(j = 1, 2 \cdots, n)$. The extended 2-tuple weighted geometric (ET-WG) average operator is defined as (Wei[8])

$$\text{ET-WG}_{\boldsymbol{C}}((r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)) = \Delta(\prod_{j=1}^{n} (\Delta^{-1}(r_j, a_j))^{\frac{\Delta^{-1}(c_j, b_j)}{\sum_{j=1}^{n} \Delta^{-1}(c_j, b_j)}}) . \quad (6)$$

Remark 2. In Definition 7, since the power index $\frac{\Delta^{-1}(c_j, b_j)}{\sum_{j=1}^{n} \Delta^{-1}(c_j, b_j)}$ satisfies that $0 \le \frac{\Delta^{-1}(c_j, b_j)}{\sum_{j=1}^{n} \Delta^{-1}(c_j, b_j)} \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} \frac{\Delta^{-1}(c_j, b_j)}{\sum_{j=1}^{n} \Delta^{-1}(c_j, b_j)} = 1$, the argument of delta in Eq (6), $\prod_{j=1}^{n} (\Delta^{-1}(r_j, a_j))^{\frac{\Delta^{-1}(c_j, b_j)}{\sum_{j=1}^{n} \Delta^{-1}(c_j, b_j)}}$, is defined in $[0, t-1]$, which can be easily proven by the similar way to Theorem 1.

Definition 8. Let $x = \{(r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)\}$ be a set of 2-tuples. The extended 2-tuple ordered weighted geometric (ET-OWG) average operator of dimension n is a mapping $\text{ET-OWG}: T^n \to T$ so that

$$\text{ET-OWG}_{\boldsymbol{L}}((r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)) = \Delta(\prod_{j=1}^{n} (\Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)}}) , \quad (7)$$

where $\boldsymbol{L} = ((l_1, \eta_1), (l_2, \eta_2), \cdots, (l_n, \eta_n))^{\mathrm{T}}$ is the linguistic weighted vector correlating with $ET-OWG$, $(\sigma_{(1)}, \sigma_{(2)}, \cdots, \sigma_{(n)})$ is a permutation of $(1, 2, \cdots, n)$ such that $(r_{\sigma(j-1)}, a_{\sigma(j-1)}) \ge (r_{\sigma(j)}, a_{\sigma(j)})$ for any j (Wei[8]).

Remark 3. In Definition 8, since the power index $\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)}$ satisfies that $0 \le \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)} \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)} = 1$, the argument of delta in Eq (7), $\prod_{j=1}^{n} (\Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)}}$, is also defined in $[0, t-1]$, which can be easily proven by the similar way to Theorem 1.

3.2. *The new hybrid geometric aggregation operators with 2-tuple linguistic information*

It can be seen from Definitions 7 and 8 that the ET-WG operator weights the 2-tuple linguistic arguments while the ET-OWG operator weights the ordered positions of the 2-tuple linguistic arguments instead of weighting the arguments themselves. Therefore, weights represent different aspects in both the ET-WG and ET-OWG operators. However, both the operators consider only one of them. To solve this drawback, based on Definitions 2, 3 and 4, some new hybrid geometric aggregation operators with 2-tuple linguistic information are developed in the following.

Definition 9. Let $x=\{(r_1,a_1),(r_2,a_2),\cdots,(r_n,a_n)\}$ be a set of 2-tuples. If $\text{THWG}:T^n\to T$ so that

$$\text{THWG}_{w,\omega}((r_1,a_1),(r_2,a_2),\cdots,(r_n,a_n))$$
$$=\Delta(\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{w_j}), \quad (8)$$

where $w=(w_1,w_2,\cdots,w_n)^{\mathrm{T}}$ is the weighted vector correlating with THWG, satisfying that $0\le w_j\le 1$ $(j=1,2\cdots,n)$ and $\sum_{j=1}^{n}w_j=1$. $(r'_{\sigma(j)},a'_{\sigma(j)})$ is the jth largest 2-tuple of 2-tuples (r'_i,a'_i) $(i=1,2,\cdots,n)$ with $(r'_i,a'_i)=(r_i,a_i)^{n\omega_i}$, $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^{\mathrm{T}}$ is the weight vector of 2-tuples (r_j,a_j) $(j=1,2\cdots,n)$, satisfying that $0\le\omega_j\le 1$ $(j=1,2\cdots,n)$ and $\sum_{j=1}^{n}\omega_j=1$, n is the balancing coefficient (in this case, if $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^{\mathrm{T}}$ goes to $(\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n})^{\mathrm{T}}$, then (r'_i,a'_i) goes to (r_i,a_i) $(i=1,2,\cdots,n)$). Then, the function THWG is called the 2-tuple hybrid weighted geometric average operator of dimension n.

Theorem 2. In Definition 9, the argument of delta in Eq. (8), $\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{w_j}$, is defined in $[0,t-1]$.

Proof. According to (iv) of Definition 4, we have

$$(r'_i,a'_i)=(r_i,a_i)^{n\omega_i}$$
$$=\begin{cases}\Delta((\Delta^{-1}(r_i,a_i))^{n\omega_i}), \text{ if } (\Delta^{-1}(r_i,a_i))^{n\omega_i}\le t-1\\ (s_{t-1},0), \text{ if } (\Delta^{-1}(r_i,a_i))^{n\omega_i}>t-1\end{cases}.$$

Hence, in Definition 9, $\Delta^{-1}(r'_i,a')\in[0,t-1]$ and thus $\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)})\in[0,t-1]$.

In addition, since the weighted vector $w=(w_1,w_2,\cdots,w_n)^{\mathrm{T}}$ satisfies that $0\le w_j\le 1$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n}w_j=1$, it follows from Lemma 1 that

$$0\le\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{w_j}\le\sum_{j=1}^{n}w_j\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)})$$
$$\le\sum_{j=1}^{n}w_j(t-1)=t-1.$$

Namely, $\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{w_j}$ is defined in $[0,t-1]$.

Example 6. Assume that $(r_1,a_1)=(s_1,0.1)$, $(r_2,a_2)=(s_3,0.3)$, $(r_3,a_3)=(s_2,0.2)$, $(r_4,a_4)=(s_4,0.3)$, w=(0.2, 0.3, 0.3, 0.2)$^{\mathrm{T}}$ and ω=(0.1, 0.4, 0.3, 0.2)$^{\mathrm{T}}$, then,

$$(r'_1,a'_1)=(s_1,0.1)^{4\times0.1}=(s_1,0.0389),$$
$$(r'_2,a'_2)=(s_3,0.3)^{4\times0.4}=(s_7,-0.245),$$
$$(r'_3,a'_3)=(s_2,0.2)^{4\times0.3}=(s_3,-0.4242)$$

and

$$(r'_4,a'_4)=(s_4,0.3)^{4\times0.2}=(s_3,0.212).$$

Therefore,

$$(r'_{\sigma(1)},a'_{\sigma(1)})=(s_7,-0.245),$$
$$(r'_{\sigma(2)},a'_{\sigma(2)})=(s_3,0.212),$$
$$(r'_{\sigma(3)},a'_{\sigma(3)})=(s_3,-0.4242)$$

and

$$(r'_{\sigma(4)},a'_{\sigma(4)})=(s_1,0.0389).$$

Thus,

$$\text{THWG}_{w,\omega}((r_1,a_1),(r_2,a_2),(r_3,a_3),(r_4,a_4))$$
$$=\Delta(\prod_{j=1}^{4}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{w_j})=\Delta(2.9980)=(s_3,-0.002).$$

Theorem 3. The TOWG operator is a special case of the THWG operator.

Proof. Let $\omega_j=\frac{1}{n}$ $(j=1,2,\cdots,n)$, then $(r'_i,a'_i)=(r_i,a_i)^{n\omega_i}=(r_i,a_i)$ $(i=1,2,\cdots,n)$. This completes the proof of Theorem 3.

Theorem 4. The TWG operator is a special case of the THWG operator.

Proof. Let $w_j=\frac{1}{n}$ $(j=1,2,\cdots,n)$, then

$$\text{THWG}_{w,\omega}((r_1,a_1),(r_2,a_2),\cdots,(r_n,a_n))$$
$$=\Delta(\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{w_j})$$
$$=\Delta(\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)},a'_{\sigma(j)}))^{\frac{1}{n}})$$
$$=\Delta(\prod_{j=1}^{n}(\Delta^{-1}((r_i,a_i)^{n\omega_i}))^{\frac{1}{n}})$$

$$= \Delta(\prod_{j=1}^{n}(\Delta^{-1}(r_i, a_i))^{n\omega_i \frac{1}{n}})$$
$$= \Delta(\prod_{j=1}^{n}(\Delta^{-1}(r_i, a_i))^{\omega_i})$$
$$= \text{TWG}_{\omega}((r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)),$$

which completes the proof of Theorem 4.

From Theorems 3 and 4, we know that, the THWG operator first weights the given arguments, and then reorders the weighted arguments in descending order and weights these ordered arguments, and finally aggregates all the weighted arguments into a collective one. The THWG operator generalizes both the TWG and TOWG operators. The THWG operator reflects the important degrees of both the given 2-tuples and the ordered positions of the 2-tuples.

Definition 10. Let $x = \{(r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)\}$ be a set of 2-tuples. If $\text{T-HLWG}: T^n \to T$ so that

$$\text{T-HLWG}_{L,\omega}((r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n))$$
$$= \Delta(\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(l_j, \eta_j)}}), \quad (9)$$

where $\boldsymbol{L} = ((l_1, \eta_1), (l_2, \eta_2), \cdots, (l_n, \eta_n))^{\text{T}}$ is the linguistic weighted vector correlating with T-HLWG, $(r'_{\sigma(j)}, a'_{\sigma(j)})$ is the jth largest 2-tuple of 2-tuples (r'_i, a'_i) $(i = 1, 2, \cdots, n)$ with $(r'_i, a'_i) = (r_i, a_i)^{n\omega_i}$, $\boldsymbol{\omega}=(\omega_1, \omega_2, \cdots, \omega_n)^{\text{T}}$ is the weight vector of 2-tuples (r_j, a_j) $(j = 1, 2, \cdots, n)$, satisfying that $0 \le \omega_j \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n}\omega_j = 1$, n is the balancing coefficient (in this case, if $\boldsymbol{\omega}=(\omega_1, \omega_2, \cdots, \omega_n)^{\text{T}}$ goes to $(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})^{\text{T}}$, then (r'_i, a'_i) goes to (r_i, a_i) $(i = 1, 2, \cdots, n)$). Then, the function T-HLWG is called the 2-tuple hybrid linguistic weighted geometric average operator of dimension n.

Remark 4. According to (iv) of Definition 4,

$$(r'_i, a'_i) = (r_i, a_i)^{n\omega_i}$$
$$= \begin{cases} \Delta((\Delta^{-1}(r_i, a_i))^{n\omega_i}), \text{ if } (\Delta^{-1}(r_i, a_i))^{n\omega_i} \le t-1 \\ (s_{t-1}, 0), \text{ if } (\Delta^{-1}(r_i, a_i))^{n\omega_i} > t-1 \end{cases}.$$

Hence, in Definition 10, $\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}) \in [0, t-1]$. Meanwhile, since the power index $\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(l_j, \eta_j)}$ satisfies that $0 \le \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(l_j, \eta_j)} \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n}\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(l_j, \eta_j)} = 1$, the argument of delta in Eq. (9), $\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(l_j, \eta_j)}}$, is defined in $[0, t-1]$, which can be easily proven by the similar way to Theorem 2.

Example 7. Assume that

$$(l_1, \eta_1) = (s_3, 0.4),\ (l_2, \eta_2) = (s_2, 0.2),$$
$$(l_3, \eta_3) = (s_1, 0.1),\ (l_4, \eta_4) = (s_5, 0.2),$$
$$(r_1, a_1) = (s_1, 0.1),\ (r_2, a_2) = (s_3, 0.3),$$
$$(r_3, a_3) = (s_2, 0.2),\ (r_4, a_4) = (s_4, 0.3)$$

and

$$\boldsymbol{\omega}=(0.1, 0.4, 0.3, 0.2)^{\text{T}},$$

then,

$$(r'_1, a'_1) = (s_1, 0.1)^{4\times 0.1} = (s_1, 0.0389),$$
$$(r'_2, a'_2) = (s_3, 0.3)^{4\times 0.4} = (s_7, -0.245),$$
$$(r'_3, a'_3) = (s_2, 0.2)^{4\times 0.3} = (s_3, -0.4242),$$
$$(r'_4, a'_4) = (s_4, 0.3)^{4\times 0.2} = (s_3, 0.212).$$

Therefore,

$$(r'_{\sigma(1)}, a'_{\sigma(1)}) = (s_7, -0.245),$$
$$(r'_{\sigma(2)}, a'_{\sigma(2)}) = (s_3, 0.212),$$
$$(r'_{\sigma(3)}, a'_{\sigma(3)}) = (s_3, -0.4242)$$

and

$$(r'_{\sigma(4)}, a'_{\sigma(4)}) = (s_1, 0.0389).$$

Thus,

$$\text{T-HLWG}_{L,\omega}((r_1, a_1), (r_2, a_2), (r_3, a_3), (r_4, a_4))$$
$$= \Delta(\prod_{j=1}^{4}(\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{4}\Delta^{-1}(l_j, \eta_j)}})$$
$$= \Delta(2.3765) = (s_2, 0.3765).$$

Theorem 5. The ET-OWG operator is a special case of the T-HLWG operator.

Proof. Let $\omega_j = \frac{1}{n}$ $(j = 1, 2, \cdots, n)$, then $(r'_i, a'_i) = (r_i, a_i)^{n\omega_i} = (r_i, a_i)$, $(i = 1, 2, \cdots, n)$. This completes the proof of Theorem 5.

Definition 11. Let $x = \{(r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n)\}$ be a set of 2-tuples. If $\text{ET-HLWG}: T^n \to T$ so that

$$\text{ET-HLWG}_{L,C}((r_1, a_1), (r_2, a_2), \cdots, (r_n, a_n))$$
$$= \Delta(\prod_{j=1}^{n}(\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(l_j, \eta_j)}}), \quad (10)$$

where $\boldsymbol{L} = ((l_1, \eta_1), (l_2, \eta_2), \cdots, (l_n, \eta_n))^{\text{T}}$ is the linguistic weighted vector correlating with ET-HLWG. $(r'_{\sigma(j)}, a'_{\sigma(j)})$ is the jth largest 2-tuple of 2-tuples

(r_i', a_i') $(i = 1, 2, \cdots, n)$ with $(r_i', a_i') = (r_i, a_i)^{n(c_i, b_i)}$, $\boldsymbol{C} = ((c_1, b_1), (c_2, b_2), \cdots, (c_n, b_n))^{\mathrm{T}}$ is the linguistic weight vector of 2-tuples (r_j, a_j) $(j = 1, 2, \cdots, n)$, n is the balancing coefficient. Then, the function ET-HLWG is called the extended 2-tuple hybrid linguistic weighted geometric average operator of dimension n.

Theorem 6. In Definition 11, the argument of delta in Eq. (10), $\prod_{j=1}^{n} (\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)}}$, is defined in $[0, t-1]$.

Proof. According to (iii) of Definition 4, we get that

$$n(c_i, b_i) = \begin{cases} \Delta(n\Delta^{-1}(c_i, b_i)), \text{ if } n\Delta^{-1}(c_i, b_i) \le t-1 \\ (s_{t-1}, 0), \text{ if } n\Delta^{-1}(c_i, b_i) > t-1 \end{cases}.$$

According to (v) of Definition 4,

$$(r_i', a_i') = (r_i, a_i)^{n(c_i, b_i)}$$

$$= \begin{cases} \Delta((\Delta^{-1}(r_i, a_i))^{n\Delta^{-1}(c_i, b_i)}), \text{ if } (\Delta^{-1}(r_i, a_i))^{n\Delta^{-1}(c_i, b_i)} \le t-1 \\ (s_{t-1}, 0), \text{ if } (\Delta^{-1}(r_i, a_i))^{n\Delta^{-1}(c_i, b_i)} > t-1 \end{cases}.$$

Hence, in Definition 11, $\Delta^{-1}(r_i', a_i') \in [0, t-1]$ and thus $\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}) \in [0, t-1]$.

Moreover, since the power index $\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)}$ satisfies that $0 \le \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)} \le 1$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)} = 1$, it yields by Lemma 1 that

$$0 \le \prod_{j=1}^{n} (\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)}}$$

$$\le \sum_{j=1}^{n} \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)} \Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)})$$

$$\le \sum_{j=1}^{n} \frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{n} \Delta^{-1}(l_j, \eta_j)} (t-1) = t-1.$$

Therefore, the argument of delta in Eq. (10) is also defined in $[0, t-1]$.

Example 8. Assume that

$$(l_1, \eta_1) = (s_3, 0.4),\ (l_2, \eta_2) = (s_2, 0.2),$$
$$(l_3, \eta_3) = (s_1, 0.1),\ (l_4, \eta_4) = (s_5, 0.2),$$
$$(r_1, a_1) = (s_1, 0.1),\ (r_2, a_2) = (s_1, 0.3),$$
$$(r_3, a_3) = (s_1, 0.002),\ (r_4, a_4) = (s_1, 0.03),$$
$$(c_1, b_1) = (s_2, 0.3),\ (c_2, b_2) = (s_1, 0.1),$$
$$(c_3, b_3) = (s_1, 0.2),\ (c_4, b_4) = (s_1, 0.3),$$

then,

$$(r_1', a_1') = (s_1, 0.1)^{4(s_2, 0.3)} = (s_2, 0.4033),$$
$$(r_2', a_2') = (s_1, 0.3)^{4(s_1, 0.1)} = (s_3, 0.1721),$$
$$(r_3', a_3') = (s_1, 0.002)^{4(s_1, 0.2)} = (s_1, 0.0080)$$

and

$$(r_4', a_4') = (s_1, 0.03)^{4(s_1, 0.3)} = (s_1, 0.0157).$$

Therefore,

$$(r'_{\sigma(1)}, a'_{\sigma(1)}) = (s_3, 0.1721),$$
$$(r'_{\sigma(2)}, a'_{\sigma(2)}) = (s_2, 0.4033),$$
$$(r'_{\sigma(3)}, a'_{\sigma(3)}) = (s_1, 0.0157)$$

and

$$(r'_{\sigma(4)}, a'_{\sigma(4)}) = (s_1, 0.0080).$$

Thus,

$$\text{ET-HLWG}_{L,C}((r_1, a_1), (r_2, a_2), (r_3, a_3), (r_4, a_4))$$
$$= \Delta(\prod_{j=1}^{4} (\Delta^{-1}(r'_{\sigma(j)}, a'_{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{4} \Delta^{-1}(l_j, \eta_j)}})$$
$$= \Delta(1.6436) = (s_2, -0.3564).$$

4. MAGDM model and method with 2-tuple Linguistic assessments

In the following, we apply the 2-tuple hybrid geometric aggregation operators to solve the MAGDM problems with 2-tuple linguistic assessments.

4.1. *MAGDM model description with 2-tuple linguistic assessments*

This subsection describes the MAGDM problem with 2-tuple linguistic assessments.

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of m possible alternatives and $F = \{a_1, a_2, \cdots, a_n\}$ be a finite set of n attributes, where A_i denotes the ith alternative and a_j denotes the jth attribute. Let $D = \{D_1, D_2, \cdots, D_t\}$ be a finite set of t experts, where D_k denotes the kth expert.

The expert D_k provides his/her assessment information of an alternative A_i on an attribute a_j as a 2-tuple $r_{ij}^k = (s_{ij}^k, \alpha_{ij}^k)$ according to a predefined linguistic term set S, where $s_{ij}^k \in S$ and $\alpha_{ij}^k \in [-0.5, 0.5)$ $(i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n;\ k = 1, 2, \cdots, t)$. Thus, the experts' assessment information can be represented by the 2-tuple linguistic decision matrixes $\boldsymbol{R}^k = (r_{ij}^k)_{m\times n}$ $(k = 1, 2, \cdots, t)$.

Suppose that the weight information of attributes and experts also can be represented by the 2-tuple linguistic information. Let $\boldsymbol{W} = ((w_1, \theta_1), (w_2, \theta_2), \cdots, (w_n, \theta_n))^{\mathrm{T}}$ be the 2-tuple linguistic weight vector of the attributes a_j $(j = 1, 2, \cdots, n)$ and $\boldsymbol{C} = ((c_1, b_1), (c_2, b_2), \cdots, (c_t, b_t))^{\mathrm{T}}$ be the 2-tuple linguistic weight vector of the experts D_k $(k = 1, 2, \cdots, t)$, where $w_j \in S$, $c_k \in S$, $\theta_j \in [-0.5, 0.5)$ and $b_k \in [-0.5, 0.5)$ $(j = 1, 2, \cdots, n;\ k = 1, 2, \cdots, t)$.

The problem concerned in this paper is how to rank alternatives or select the most desirable alternative(s) among the finite set A based on the 2-tuple linguistic assessment information given by the experts and the 2-tuple linguistic weight information of attributes and experts.

4.2. *The MAGDM method with 2-tuple Linguistic assessment Information*

In this subsection, we propose a new method based on the ET-WG and ET-HLWG operators to solve the MAGDM problems with 2-tuple linguistic assessments. The process and algorithm may be summarized as follows.

Step 1. Utilized the ET-WG operator to integrate the ith line elements of the decision matrix $\boldsymbol{R}^k$, the individual overall preference value of the alternative A_i given by the expert D_k is derived as follows:

$$z_i^k = (s_i^k, \alpha_i^k) = \text{ET-WG}_{\boldsymbol{W}}((s_{i1}^k, \alpha_{i1}^k), (s_{i2}^k, \alpha_{i2}^k), \cdots, (s_{in}^k, \alpha_{in}^k))$$
$$= \Delta\Big(\prod_{j=1}^{n} (\Delta^{-1}(s_{ij}^k, \alpha_{ij}^k))^{\frac{\Delta^{-1}(w_j, \theta_j)}{\sum_{j=1}^{n}\Delta^{-1}(w_j, \theta_j)}}\Big), \qquad (11)$$

where $\boldsymbol{W} = ((w_1, \theta_1), (w_2, \theta_2), \cdots, (w_n, \theta_n))^{\mathrm{T}}$ be the 2-tuple linguistic weight vector of the attributes, $s_i^k \in S$ and $\alpha_i^k \in [-0.5, 0.5)$.

Step 2. Used the ET-HLWG operator to integrate all the individual overall preference values $z_i^k = (s_i^k, \alpha_i^k)$ $(k = 1, 2, \cdots, t)$ of alternative A_i, the collective overall preference value of alternative A_i is obtained as follows:

$$z_i = (s_i, \alpha_i) = \text{ET-HLWG}_{\boldsymbol{L},\boldsymbol{C}}((s_i^1, \alpha_i^1), (s_i^2, \alpha_i^2), \cdots, (s_i^t, \alpha_i^t))$$
$$= \Delta\Big(\prod_{j=1}^{t} (\Delta^{-1}(s_i'^{\sigma(j)}, \alpha_i'^{\sigma(j)}))^{\frac{\Delta^{-1}(l_j, \eta_j)}{\sum_{j=1}^{t}\Delta^{-1}(l_j, \eta_j)}}\Big), \qquad (12)$$

where $\boldsymbol{L} = ((l_1, \eta_1), (l_2, \eta_2), \cdots, (l_t, \eta_t))^{\mathrm{T}}$ is the linguistic weighted vector correlating with ET-HLWG, $(s_i'^{\sigma(j)}, \alpha_i'^{\sigma(j)})$ is the jth largest 2-tuple of 2-tuples $(s_i'^k, \alpha_i'^k)$ $(k = 1, 2, \cdots, t)$ with $(s_i'^k, \alpha_i'^k) = (s_i^k, \alpha_i^k)^{t(c_k, b_k)}$, $\boldsymbol{C} = ((c_1, b_1), (c_2, b_2), \cdots, (c_t, b_t))^{\mathrm{T}}$ is the 2-tuple linguistic weight vector of experts.

Step 3. Rank all the alternatives and select the best one(s) in accordance with $z_i = (s_i, \alpha_i)$ $(i = 1, 2, \cdots, m)$. If any alternative has the highest z_i value, then it is the best alternative.

5. A real application to evaluating university faculty for tenure and promotion

In this subsection, a real case study of evaluating university faculty for tenure and promotion is examined to illustrate the proposed method in this paper.

Nanchang University of China intends to evaluate five faculties for tenure and promotion. The five faculty candidates (alternatives) are Information technology faculty A_1, Software faculty A_2, Humanities faculty A_3, Mathematics faculty A_4 and Chemistry faculty A_5, respectively. The university committee invites four experts D_k $(k = 1, 2, 3, 4)$ from the other famous universities to evaluate these faculties. Since the expert D_3 has engaged in university evaluation for many years and accumulated rich experience, the university committee names the expert D_3 as the group leader which is responsible for the whole evaluating work.

Generally, many attributes should be used to evaluate these faculties. To improve the efficiency and rapidly make decision, three attributes are chosen by the four experts after preliminary screening. These attributes are teaching a_1, research a_2 and service a_3, respectively. These attributes are all qualitative attributes, it is reasonable for the experts to use linguistic variables or 2-tuples to represent the evaluation information of the faculties with respective to the attributes. Consequently, the five faculty candidates are to be evaluated using the 2-tuple linguistic information according to the linguistic term set:

S ={ s_0 = extremely poor (bad); s_1 = very poor (bad); s_2 = poor (bad); s_3 = slightly poor (bad); s_4 = fair (important); s_5 = slightly good (important); s_6 = good (important); s_7 = very good (important); s_8 = extremely good (important)} by the four experts under these three attributes. The 2-tuple linguistic decision matrixes provided by each expert are respectively as follows:

$$\boldsymbol{R}^1=\begin{pmatrix}(s_0,0.4)&(s_3,0.2)&(s_8,0.1)\\(s_4,0.3)&(s_1,0.4)&(s_7,-0.2)\\(s_2,0.2)&(s_4,0.3)&(s_6,0.3)\\(s_1,0.3)&(s_5,-0.4)&(s_7,0.2)\\(s_7,-0.2)&(s_8,0.1)&(s_0,0.1)\end{pmatrix},$$

$$\boldsymbol{R}^2=\begin{pmatrix}(s_2,0.1)&(s_4,0.2)&(s_6,0.1)\\(s_5,-0.3)&(s_3,0.1)&(s_6,0.2)\\(s_2,0.2)&(s_7,-0.3)&(s_6,0.3)\\(s_2,0.3)&(s_1,0.4)&(s_7,0.2)\\(s_6,0.2)&(s_7,-0.1)&(s_8,0.1)\end{pmatrix},$$

$$\boldsymbol{R}^3=\begin{pmatrix}(s_4,0.3)&(s_2,0.4)&(s_7,0.3)\\(s_3,0.4)&(s_2,0.1)&(s_5,-0.2)\\(s_1,0.3)&(s_4,0.3)&(s_6,0.3)\\(s_5,0.1)&(s_8,-0.3)&(s_7,0.2)\\(s_7,-0.2)&(s_7,0.4)&(s_2,0.4)\end{pmatrix}$$

and

$$\boldsymbol{R}^4=\begin{pmatrix}(s_1,0.3)&(s_0,0.4)&(s_7,0.1)\\(s_3,0.3)&(s_5,0.4)&(s_8,-0.2)\\(s_1,0.2)&(s_6,0.2)&(s_8,0.3)\\(s_1,0.4)&(s_5,0.3)&(s_8,-0.2)\\(s_6,0.3)&(s_3,0.1)&(s_1,0.3)\end{pmatrix}.$$

With ever increasing complexity in real-life university evaluation management, it is very difficult to give precisely the linguistic assessment information on the expert weights and attribute weights according to the given linguistic term set in advance. For example, the experts think that the attribute a_3 is important and the weight may be s_6 but less than s_6, thus the weight of attribute a_3 can be represented using the linguistic 2-tuple $(w_3,\theta_3)=(s_6,-0.2)$. After the negotiation and investigation of the experts, they determine the 2-tuple linguistic weight vector $\boldsymbol{W}=((w_1,\theta_1),(w_2,\theta_2),(w_3,\theta_3))^{\mathrm{T}}$ of the attributes, where $(w_1,\theta_1)=(s_8,-0.4)$, $(w_2,\theta_2)=(s_1,0.3)$ and $(w_3,\theta_3)=(s_6,-0.2)$.

As the stated earlier, the expert D_3, named as the group leader, has rich experience, knowledge and speciality in university evaluation. Obviously, his importance degree is extremely high and may be s_8 but less than s_8, therefore, the weight of expert D_3 can be represented using the linguistic 2-tuple $(c_3,\beta_3)=(s_8,-0.1)$. Analogously, the 2-tuple linguistic weight vector $\boldsymbol{C}=((c_1,b_1),(c_2,b_2),(c_3,b_3),(c_4,b_4))^{\mathrm{T}}$ of the experts can be obtained, where $(c_1,b_1)=(s_5,0.1)$, $(c_2,b_2)=(s_1,0.2)$, $(c_3,b_3)=(s_8,-0.1)$ and $(c_4,b_4)=(s_3,0.4)$.

Next, we adopt the proposed method in this paper to solve this faculty evaluation problem.

Step 1. Combined the decision matrix $\boldsymbol{R}^1$ and $\boldsymbol{W}=((w_1,\theta_1),(w_2,\theta_2),(w_3,\theta_3))^{\mathrm{T}}$ with the ET-WG operator , the individual overall preference value of the faculty A_1 given by expert D_1 is generated as follows:

$$z_1^1=(s_1^1,\alpha_1^1)=\text{ET-WG}_{\boldsymbol{W}}((s_{11}^1,\alpha_{11}^1),(s_{12}^1,\alpha_{12}^1),(s_{13}^1,\alpha_{13}^1))$$

$$=\Delta\Big(\prod_{j=1}^{3}(\Delta^{-1}(s_{1j}^1,\alpha_{1j}^1))^{\frac{\Delta^{-1}(w_j,\theta_j)}{\sum_{j=1}^{3}\Delta^{-1}(w_j,\theta_j)}}\Big)=(s_2,-0.4245).$$

Similarly, we have

$$z_2^1=(s_2^1,\alpha_2^1)=(s_5,-0.3446),$$
$$z_3^1=(s_3^1,\alpha_3^1)=(s_4,-0.4646),$$
$$z_4^1=(s_4^1,\alpha_4^1)=(s_3,-0.1438),$$
$$z_5^1=(s_5^1,\alpha_5^1)=(s_1,0.3068).$$
$$z_1^2=(s_1^2,\alpha_1^2)=(s_3,0.4007),$$
$$z_2^2=(s_2^2,\alpha_2^2)=(s_5,0.5003),$$
$$z_3^2=(s_3^2,\alpha_3^2)=(s_4,-0.3232),$$
$$z_4^2=(s_4^2,\alpha_4^2)=(s_3,0.4531),$$
$$z_5^2=(s_5^2,\alpha_5^2)=(s_7,0.0449).$$
$$z_1^3=(s_1^3,\alpha_1^3)=(s_5,0.0323),$$
$$z_2^3=(s_2^3,\alpha_2^3)=(s_1,-0.2670),$$
$$z_3^3=(s_3^3,\alpha_3^3)=(s_3,-0.3065),$$
$$z_4^3=(s_4^3,\alpha_4^3)=(s_6,0.0601),$$
$$z_5^3=(s_5^3,\alpha_5^3)=(s_5,0.4575).$$
$$z_1^4=(s_1^4,\alpha_1^4)=(s_2,0.2887),$$
$$z_2^4=(s_2^4,\alpha_2^4)=(s_5,-0.1602),$$
$$z_3^4=(s_3^4,\alpha_3^4)=(s_3,-0.0239),$$
$$z_4^4=(s_4^4,\alpha_4^4)=(s_3,0.1016)$$

and

$$z_5^4=(s_5^4,\alpha_5^4)=(s_3,0.1745).$$

Step 2. For the ET-HLWG operator, assume that the correlated 2-tuple weighted vector is $\boldsymbol{L}=((l_1,\eta_1),(l_2,\eta_2),(l_3,\eta_3),(l_4,\eta_4))^{\mathrm{T}}$, where $(l_1,\eta_1)=(s_2,0.2)$, $(l_2,\eta_2)=(s_5,0.1)$, $(l_3,\eta_3)=(s_7,-0.2)$ and $(l_4,\eta_4)=(s_6,0.3)$. Then, used $\boldsymbol{C}=((c_1,b_1),(c_2,b_2),(c_3,b_3),(c_4,b_4))^{\mathrm{T}}$ and the ET-HLWG operator to integrate all the individual overall preference values $z_1^k=(s_1^k,\alpha_1^k)$ $(k=1,2,3,4)$ of faculty A_1, the collective overall preference value of faculty A_1 is thus derived as follows:

$$z_1 = \text{ET-HLWG}_{L,C}((s_1^1,\alpha_1^1),(s_1^2,\alpha_1^2),(s_1^3,\alpha_1^3),(s_1^4,\alpha_1^4))$$

$$=\Delta\left(\prod_{j=1}^{4}\left(\Delta^{-1}(s_1'^{\sigma(j)},\alpha_1'^{\sigma(j)})\right)^{\frac{\Delta^{-1}(l_j,\eta_j)}{\sum_{j=1}^{4}\Delta^{-1}(l_j,\eta_j)}}\right)=(s_3,0.1561).$$

In the same way, we have

$$z_2 = \text{ET-HLWG}_{L,C}((s_2^1,\alpha_2^1),(s_2^2,\alpha_2^2),(s_2^3,\alpha_2^3),(s_2^4,\alpha_2^4))$$

$$=\Delta\left(\prod_{j=1}^{4}\left(\Delta^{-1}(s_2'^{\sigma(j)},\alpha_2'^{\sigma(j)})\right)^{\frac{\Delta^{-1}(l_j,\eta_j)}{\sum_{j=1}^{4}\Delta^{-1}(l_j,\eta_j)}}\right)=(s_4,0.4690),$$

$$z_3 = \text{ET-HLWG}_{L,C}((s_3^1,\alpha_3^1),(s_3^2,\alpha_3^2),(s_3^3,\alpha_3^3),(s_3^4,\alpha_3^4))$$

$$=\Delta\left(\prod_{j=1}^{4}\left(\Delta^{-1}(s_3'^{\sigma(j)},\alpha_3'^{\sigma(j)})\right)^{\frac{\Delta^{-1}(l_j,\eta_j)}{\sum_{j=1}^{4}\Delta^{-1}(l_j,\eta_j)}}\right)=(s_3,0.0919),$$

$$z_4 = \text{ET-HLWG}_{L,C}((s_4^1,\alpha_4^1),(s_4^2,\alpha_4^2),(s_4^3,\alpha_4^3),(s_4^4,\alpha_4^4))$$

$$=\Delta\left(\prod_{j=1}^{4}\left(\Delta^{-1}(s_4'^{\sigma(j)},\alpha_4'^{\sigma(j)})\right)^{\frac{\Delta^{-1}(l_j,\eta_j)}{\sum_{j=1}^{4}\Delta^{-1}(l_j,\eta_j)}}\right)=(s_4,-0.0523)$$

and

$$z_5 = \text{ET-HLWG}_{L,C}((s_5^1,\alpha_5^1),(s_5^2,\alpha_5^2),(s_5^3,\alpha_5^3),(s_5^4,\alpha_5^4))$$

$$=\Delta\left(\prod_{j=1}^{4}\left(\Delta^{-1}(s_5'^{\sigma(j)},\alpha_5'^{\sigma(j)})\right)^{\frac{\Delta^{-1}(l_j,\eta_j)}{\sum_{j=1}^{4}\Delta^{-1}(l_j,\eta_j)}}\right)=(s_4,-0.0451).$$

Step 3. Since $z_2 > z_5 > z_4 > z_1 > z_3$, the ranking order of the faculties is $A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$, and therefore the best faculty is Software faculty A_2.

6. Comparison analysis with the similar method

To illustrate the superiority of the proposed method, we use the proposed method in this paper to solve the investment selection problem of Wei[8], and then conduct a comparison analysis.

An investment company wants to invest a sum of money in the best option. There is a panel with five possible alternatives to invest the money: a car company A_1, a food company A_2, a computer company A_3, an arms company A_4 and a TV company A_5. The investment company must take a decision according to the four attributes: the risk analysis a_1, the growth analysis a_2, the social-political impact analysis a_3 and the environmental impact analysis a_4. The five possible alternatives A_i $(i=1,2,3,4,5)$ are to be evaluated using the linguistic term set S ={ s_0 =extremely poor (EP); s_1 =very poor (VP); s_2 =poor (P); s_3 =medium (M); s_4 =good (G); s_5 = very good (VG); s_6 =extremely good (EG)} by three decision makers D_k (k=1,2,3) under the above four attributes. They respectively construct the decision matrices $\boldsymbol{R}_k = (\tilde{r}_{ij}^k)_{5\times 4}$ (k=1,2,3) as follows:

$$\boldsymbol{R}_1 = \begin{pmatrix} M & G & P & P \\ P & VP & M & P \\ G & M & G & EP \\ VG & P & P & G \\ EG & EP & VP & M \end{pmatrix},$$

$$\boldsymbol{R}_2 = \begin{pmatrix} P & M & VP & VP \\ VP & EP & G & G \\ M & G & P & EG \\ EG & VP & VP & M \\ P & VP & M & VP \end{pmatrix},$$

and

$$\boldsymbol{R}_3 = \begin{pmatrix} G & P & VP & VG \\ VP & G & P & G \\ VG & VP & G & P \\ G & VG & EG & VP \\ M & VP & M & G \end{pmatrix}.$$

In Wei[8], the linguistic weight vector of the attributes is $\boldsymbol{H}=(s_2',s_0',s_1',s_3')^{\mathrm{T}}$ using the linguistic term set S' ={ s_0' =extremely important; s_1' =very important; s_2' = important; s_3' =medium; s_4' =bad; s_5' = very bad; s_6' =extremely bad}. For the ET-OWG operator of Wei[8], the correlated linguistic weighted vector is taken as $\boldsymbol{V}=(s_5',s_3',s_1')^{\mathrm{T}}$.(Note that all the subscripts in the linguistic term sets S and S' are minus 1 in order to be unified with Definition 1)

We suppose that the weight vector of decision makers is $\boldsymbol{\omega}=(s_3',s_3',s_4')^{\mathrm{T}}$ according to the linguistic term set S'. In addition, for the ET-HLWG operator of this paper, we also take the correlated linguistic weighted vector as $\boldsymbol{V}=(s_5',s_3',s_1')^{\mathrm{T}}$.

Applying the proposed method in this paper, the above linguistic decision matrices, the linguistic weight vectors of the attributes and experts, and the correlated linguistic weighted vector should be firstly transformed into 2-tuple linguistic forms by using Eq. (3). Then, the collective overall preference values of alternatives can be obtained. Table 1 lists the collective overall preference values of alternatives obtained by the method Wei[8] and method of this paper.

Table 1 The collective overall preference values of alternatives obtained by both methods

Alternatives	A_1	A_2	A_3	A_4	A_5	Ranking result
Wei[8]	$(s_3, -0.25)$	$(s_3, 0.43)$	$(s_3, 0.15)$	$(s_3, 0.33)$	$(s_3, -0.32)$	$A_4 \succ A_3 \succ A_1 \succ A_5 \succ A_2$
This paper	$(s_3, 0.0040)$	$(s_3, 0.4685)$	$(s_3, 0.2450)$	$(s_3, 0.2140)$	$(s_3, -0.1392)$	$A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_2$

It is easily seen from Table 1 that the ranking results obtained by the method Wei[8] and the method of this paper are slightly different. The difference is the ranking order of A_4 and A_3, i.e., $A_4 \succ A_3$ by the former while $A_3 \succ A_4$ by the latter. The worst alternative is A_2 by both methods, but the best alternative by the former is A_4, while the best alternative by the latter is A_3. Compared with the former, the main advantages of the latter mainly lie in the following:

(i) The latter sufficiently takes the importance degrees of different experts into consideration. Before utilizing the ET-HLWG operator, the individual overall preference values of alternatives should be firstly weighted by the expert weights and then the collective overall preference values of alternatives can be obtained. However, the former is based on the ET-WG and ET-OWG operators, which doesn't consider the importance degrees of different experts at all.

In fact, different experts act as different roles in the decision process (such as the expert D_1 in Section 5). Some experts may assign unduly high or unduly low uncertain preference values to their preferred or repugnant objects. To relieve the influence of these unfair arguments on the decision results and reflect the importance degrees of all the experts, the latter first weights each individual overall preference value by using the corresponding expert weight, and then utilizes the ET-HLWG operator to aggregate all the individual weighted overall preference values of each alternative into the collective ones of alternatives. Therefore, the ET-HLWG or T-HLWG operator can make the decision results more reasonable through assigning low weights to those "false" or "biased" arguments. These advantages can not be reflected in the former.

(ii) The former is only suitable for the case where the weight information of attributes is in the form of the linguistic variables, whereas the latter can deal with the three cases: the linguistic variables, the 2-tuples and numerical values for the weight information of attributes and experts.

If the weight information of experts is given by linguistic variables or 2-tuples, the ET-HLWG operator can be used to integrate the individual overall preference values of alternatives into the collective ones; If the weight information of experts is given by the numerical values, we can use the T-HLWG operator to replace the ET-HLWG operator to derive the collective overall preference values of alternatives, which demonstrates that the latter is of universality and flexibility.

7. Conclusions

The traditional aggregation operators are generally suitable for aggregating the information taking the form of numerical values, and yet they will fail in dealing with linguistic information. A new decision method is proposed for the MAGDM problem with 2-tuple linguistic assessments. Firstly, the operation laws for 2-tuple linguistic information are defined. After reviewing the existing 2-tuple linguistic geometric aggregation operators, some new hybrid geometric aggregation operators with 2-tuple linguistic information are developed including THWG, T-HLWG and ET-HLWG operators. The THWG operator generalizes both the TWG and TOWG operators. The ET-OWG operator is a special case of the T-HLWG operator.

The decision method proposed in this paper is based on hybrid geometric aggregation operators which can sufficiently consider the importance degrees of different experts and thus relieve the influence of those unfair arguments on the decision results. The proposed hybrid geometric aggregation operators with 2-tuple linguistic information enlarge the research content on 2-tuple linguistic information and enrich the ideas for solving the MAGDM problems with linguistic information.

However, how to reasonably determine the linguistic (or 2-tuple linguistic) weighted vector correlating with these hybrid geometric aggregation operators is a critical problem, which will be investigated in the near future.

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