Combination of interval set and soft set

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Abstract

Soft set theory and interval set theory are all mathematical tools for dealing with uncertainties. This paper is devoted to the discussion of soft interval set and its application. The notion of soft interval sets is introduced by combining soft set and interval set. Several operations on soft interval sets are presented in a manner parallel to that used in defining operations on soft sets and the lattice structures of soft interval sets are established. In addition, a soft interval set based decision making problem is analyzed.

Keywords: Soft set, Interval set, Soft interval set, Soft equality, Soft quotient algebra.

1. Introduction

To solve complicated problems in economics, engineering, environmental science and social science, methods in classical mathematics are not always successful because of various types of uncertainties present in these problems. While a wide range of theories such as probability theory, fuzzy set theory[1], rough set theory[2], and other mathematical tools are well-known and often serve as useful approaches to describing uncertainty, each of them has its advantages as well as inherent difficulties. In 1999, Molodtsov[3] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory seems to be free from the difficulties affecting the existing methods.

Presently, works on soft set theory are progress-ing rapidly. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov[3]. Maji et al.[4] described the application of soft set theory to a decision making problem. Chen et al.[5] presented a new definition of soft set parameterization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. Kong et al.[6] introduced the notion of normal parameter reduction of soft sets and presented a heuristic algorithm to compute normal parameter reduction. In theoretical aspects, Maji et al.[7] defined several operations on soft sets and made a theoretical study on the theory of soft sets. Irfan et al.[8] introduced some new operations on soft sets and improved the notion of complement of soft set. Based on these operations, Qin et al.[9] established the lattice structures and the soft quotient algebras of soft sets.

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The study of hybrid models combining soft sets with other mathematical structures is emerging as an active research topic of soft set theory. Aktas and Cagman[10] compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups and derived some related properties. Jun[11] introduced the notion of soft BCK/BCI-algebras. Jun and Park[12] discussed the applications of soft sets in ideal theory of BCK/BCI-algebras. Maji et al.[13] initiated the study on hybrid structures involving fuzzy sets and soft sets. They introduced the notion of fuzzy soft sets, which can be seen as a fuzzy generalization of soft sets. Roy et al.[14] presented a novel approach to deal with fuzzy soft set based decision making problems. Yang et al.[15] introduced the notion of interval-valued fuzzy soft sets by combining interval-valued fuzzy set and soft set models. By means of level soft sets and reduct fuzzy soft sets, Feng et al.[16,17] presented some adjustable approaches to fuzzy soft set and interval-valued fuzzy soft set based decision making problems. The combination of soft set and rough set models was also discussed by some researchers[18,19].

The notion of interval sets was proposed by Yao[20] as a mathematical tool for dealing with qualitative information. An interval set is represented by a pair of sets, namely, the lower and upper bounds. It provides a useful and convenient method for representing partially known concepts or for approximating undefinable concepts or complex concepts. A generalized decision logic in interval-set-valued information system is introduced[21]. Interval sets are closely related to, and complementary to, fuzzy sets and rough sets. The relationships among interval sets, rough sets and fuzzy sets were investigated[22,23]. A significant difference between these models lies in the definition and interpretation of their extended set-theoretic operators. More specifically, within a framework of possible-worlds semantics analysis, it has been shown that the rough set model and interval set model correspond to two different extended propositional logics[22].

In many practical soft set based decision making problems, the parameter approximations are extremely individual (dependent on expert's evaluation of alternatives) and thus cannot be lightly confirmed.

Sometimes we can only provide a lower bound and an upper bound of parameter approximations. This leads to the motivation to combine soft set and interval set together. The organization of this paper is as follows. In Section 2 we present the correlative notions of interval sets. Section 3 deals with the algebraic structures of soft interval sets. The concept of soft interval sets is introduced by combining soft set and interval set. Some operations on soft interval sets are proposed and the lattice structures of soft interval sets are established. The soft quotient algebras are presented based the soft equality relations between soft interval sets. A soft interval set based decision making problem is examined in Section 4. We have some conclusions in the concluding Section 5.

2. Interval sets

Let U be a set of objects. We consider the concepts with respect to objects in U. A concept, in the classical view, is defined by a pair of intension and extension. Although the extension of a concept is actually a subset of U, in many practical situations, a lack of knowledge makes us unable to specify this subset. Sometimes we can only provide a lower bound A_l and an upper bound A_u. Any subset A that lies between A_l and A_u, namely, $A_l \subseteq A \subseteq A_u$ can be the actual extension of the concept. Based on these observations, Yao[20] proposed the notion of interval sets.

Definition 1 [20] *Let U be a non-empty set, called the universe, and 2^U be its power set. A subset of 2^U of the form*

$\mathscr{A} = [A_l, A_u] = \{A \in 2^U; A_l \subseteq A \subseteq A_u\}$

is called an interval set on U, where it is assumed $A_l \subseteq A_u$. A_l and A_u are called the lower bound and upper bound of $\mathscr{A}$ respectively.

In other words, an interval set on U is a family of subsets of U. It is just an interval of the power set lattice 2^U. The symbols $\in, \subseteq, \cap, \cup$ and $-$ may be used in their usual set-theoretic sense to represent relationships between elements of 2^U and an interval set, and between different interval sets. Degenerate interval sets of the form $[A, A]$ are equivalent to ordinary sets. Thus, interval sets may be considered

as an extension of elementary sets. In what follows, the set of all interval sets on U is denoted by $I(2^U)$.

Interval set approach provides a tool for modeling and processing partially known concepts and for approximating undefinable or complex concepts. Operations on interval sets are defined by Yao[20] based on the corresponding set-theoretic operations in a manner parallel to that used in defining operations on interval numbers. Thus, interval set algebra may be regarded as a counterpart of the interval number algebra proposed by Moore[24].

Definition 2 [20] *Let $\mathscr{A} = [A_1, A_2]$ and $\mathscr{B} = [B_1, B_2]$ be two interval sets on U. The interval-set intersection, union, and difference of $\mathscr{A}$ and $\mathscr{B}$ are defined as:*

$$\mathscr{A} \sqcap \mathscr{B} = \{A \cap B; A \in \mathscr{A}, B \in \mathscr{B}\},$$
$$\mathscr{A} \sqcup \mathscr{B} = \{A \cup B; A \in \mathscr{A}, B \in \mathscr{B}\},$$
$$\mathscr{A} \setminus \mathscr{B} = \{A - B; A \in \mathscr{A}, B \in \mathscr{B}\}.$$

Additionally, the interval-set complement of $\mathscr{A}$ is defined as:

$$\neg \mathscr{A} = [U, U] \setminus \mathscr{A}.$$

Interval-set intersection, union, difference and complement are closed operations on $I(2^U)$. Actually, for arbitrary interval sets $\mathscr{A} = [A_1, A_2]$ and $\mathscr{B} = [B_1, B_2]$ on U, we have[20]:

$$\mathscr{A} \sqcap \mathscr{B} = [A_1 \cap A_2, B_1 \cap B_2],$$
$$\mathscr{A} \sqcup \mathscr{B} = [A_1 \cup A_2, B_1 \cup B_2],$$
$$\mathscr{A} \setminus \mathscr{B} = [A_1 - A_2, B_1 - B_2] = \mathscr{A} \sqcap \neg \mathscr{B},$$
$$\neg \mathscr{A} = [A_2^c, A_1^c].$$

The following list summarizes properties of interval sets[25]: for $\mathscr{A}, \mathscr{B}, \mathscr{C} \in I(2^U)$,

(I1) Idempotent: $\mathscr{A} \sqcap \mathscr{A} = \mathscr{A}$, $\mathscr{A} \sqcup \mathscr{A} = \mathscr{A}$.

(I2) Commutativity: $\mathscr{A} \sqcap \mathscr{B} = \mathscr{B} \sqcap \mathscr{A}$, $\mathscr{A} \sqcup \mathscr{B} = \mathscr{B} \sqcup \mathscr{A}$.

(I3) Associativity: $(\mathscr{A} \sqcap \mathscr{B}) \sqcap \mathscr{C} = \mathscr{A} \sqcap (\mathscr{B} \sqcap \mathscr{C})$, $(\mathscr{A} \sqcup \mathscr{B}) \sqcup \mathscr{C} = \mathscr{A} \sqcup (\mathscr{B} \sqcup \mathscr{C})$.

(I4) Distributivity: $\mathscr{A} \sqcap (\mathscr{B} \sqcup \mathscr{C}) = (\mathscr{A} \sqcap \mathscr{B}) \sqcup (\mathscr{A} \sqcap \mathscr{C})$, $\mathscr{A} \sqcup (\mathscr{B} \sqcap \mathscr{C}) = (\mathscr{A} \sqcup \mathscr{B}) \sqcap (\mathscr{A} \sqcup \mathscr{C})$.

(I5) Absorption: $\mathscr{A} \sqcap (\mathscr{A} \sqcup \mathscr{B}) = \mathscr{A}$, $\mathscr{A} \sqcup (\mathscr{A} \sqcap \mathscr{B}) = \mathscr{A}$.

(I6) De Morgan's laws: $\neg(\mathscr{A} \sqcap \mathscr{B}) = \neg \mathscr{A} \sqcup \neg \mathscr{B}$, $\neg(\mathscr{A} \sqcup \mathscr{B}) = \neg \mathscr{A} \sqcap \neg \mathscr{B}$.

(I7) Double negation law: $\neg\neg \mathscr{A} = \mathscr{A}$.

Hence, $(I(2^U), \sqcup, \sqcap)$ is a distributive lattice[25]. The order relation $\sqsubseteq$ on $I(2^U)$ can be defined by:

$[A_1, A_2] \sqsubseteq [B_1, B_2]$ if and only if $A_1 \subseteq B_1$ and $A_2 \subseteq B_2$.

In addition, $[U, U]$ and $[\emptyset, \emptyset]$ are the greatest element and the least element in $I(2^U)$, respectively.

It is worth noticing that, for an interval set $\mathscr{A} = [A_1, A_2]$, $\mathscr{A} \sqcap \neg \mathscr{A}$ is not necessarily equal to $[\emptyset, \emptyset]$, and $\mathscr{A} \sqcup \neg \mathscr{A}$ is not necessarily equal to $[U, U]$. Actually, we have $\mathscr{A} \sqcap \neg \mathscr{A} = [\emptyset, A_2 - A_1]$, $\mathscr{A} \sqcup \neg \mathscr{A} = [(A_2 - A_1)^c, U]$. Hence $\emptyset \in \mathscr{A} \sqcap \neg \mathscr{A}$, $U \in \mathscr{A} \sqcup \neg \mathscr{A}$.

3. Soft interval sets

Soft set theory was initiated by Molodtsov as a mathematical tool for dealing with uncertainty. In this section, by introducing interval set into the theory of soft sets, we propose the concept of soft interval sets and investigate its algebraic structures.

3.1. Concept of and operations on soft interval sets

Let U be an initial universe set and E the set of all possible parameters under consideration with respect to U. Usually, parameters are attributes, characteristics, or properties of objects in U. (U, E) is called a soft universe. Molodtsov defined the notion of a soft set in the following way:

Definition 3 [3] *A pair (F, A) is called a soft set over U, where $A \subseteq E$ and F is a mapping given by $F : A \to 2^U$.*

Example 1 *(1) Suppose that there are five houses in the universe U given by $U = \{h_1, h_2, h_3, h_4, h_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ is the set of parameters. Where e_1, e_2, e_3, e_4, e_5 and e_6 stand for the parameters 'expensive', 'beautiful', 'in the green surroundings', 'in good repair', 'wooden', and 'modern' respectively.*

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. The soft set (F, A) may describe the 'attractiveness of the houses' which Mr.X is going to buy. Suppose that $A = \{e_1, e_2, e_3, e_4\}$ is the set of parameters Mr.X concerned, $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \{h_3, h_4, h_5\}$, and $F(e_4) = \{h_1, h_5\}$. Then the soft set (F, A) is a parameterized family $\{F(e_i); 1 \leqslant i \leqslant 4\}$ of subsets of U and give us a collection of approximate

descriptions of an object. $F(e_1) = \{h_2, h_4\}$ means houses h_2 and h_4 are 'expensive'.

(2) Interval set may be considered as a special case of the soft set. Let $\mathscr{A} = [A_1, A_2]$ be an interval set on the universe U and $C = A_2 - A_1$. It follows that $\mathscr{A} = \{F(B); B \in 2^C\}$ where $F(B) = A_1 \cup B$ for every $B \in 2^C$. Thus, the interval set $\mathscr{A}$ may be considered as the soft set $(F, 2^C)$.

By the definition, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of $e-$approximate elements of the soft set (F, A). We notice that, in some real applications, $F(e)$ cannot be precisely defined because of the complexity of the practical problems. Maybe we can only provide a lower bound and an upper bound of $F(e)$. In this case, interval set is a appropriate substitution for crisp subset of the universe.

Definition 4 *A pair (F, A) is called a soft interval set over U, where $A \subseteq E$ and F is a mapping given by $F : A \to I(2^U)$.*

By this definition, a soft interval set over U is a parameterized family of interval sets on U. Clearly, soft interval set is a generalization of soft set. In what follows, we denote by $SI(2^U)$ the set of all soft interval sets over U.

Example 2 *We consider the soft universe (U, E) given in Example 1(1). (F, A) is a soft interval set, where $A = \{e_1, e_2, e_3, e_4\}$ is the set of parameters we concerned and $F(e_1) = [\{h_2, h_4\}, \{h_1, h_2, h_4\}]$, $F(e_2) = [\{h_1, h_3\}, \{h_1, h_3, h_4\}]$, $F(e_3) = [\{h_3, h_4, h_5\}, \{h_3, h_4, h_5\}]$, and $F(e_4) = [\{h_5\}, \{h_1, h_2, h_5\}]$.*

This soft interval set gives us a collection of approximate descriptions of an object. $F(e_1) = [\{h_2, h_4\}, \{h_1, h_2, h_4\}]$ means houses $h_2, h_4 \in \{h_2, h_4\}$ are definitely expensive, $h_1 \in \{h_1, h_2, h_4\} - \{h_2, h_4\}$ is possibly expensive, and $h_3, h_5 \in U - \{h_1, h_2, h_4\}$ are not expensive.

Maji et al.[7] and Irfan et al.[8] introduced some operations on soft sets. Similarly, we introduce the operations on soft interval sets based on the operations on interval sets.

Definition 5 *Let (F, A) and (G, B) be two soft interval sets over a common universe U.*

(1) The extended union of (F, A) and (G, B), denoted by $(F, A) \cup_e (G, B)$, is the soft interval set (H, C), where $C = A \cup B$, and H is given by:

$$H(a) = \begin{cases} F(a), & if \quad a \in A - B, \\ G(a), & if \quad a \in B - A, \quad (1) \\ F(a) \sqcup G(a), & if \quad a \in A \cap B. \end{cases}$$

(2) The extended intersection of (F, A) and (G, B), denoted by $(F, A) \cap_e (G, B)$, is the soft interval set (H, C), where $C = A \cup B$, and H ia given by:

$$H(a) = \begin{cases} F(a), & if \quad a \in A - B, \\ G(a), & if \quad a \in B - A, \quad (2) \\ F(a) \sqcap G(a), & if \quad a \in A \cap B. \end{cases}$$

(3) The restricted union of (F, A) and (G, B), denoted by $(F, A) \cup_r (G, B)$, is the soft interval set (H, C), where $C = A \cap B$, and $H(a) = F(a) \sqcup G(a)$ for every $a \in C$.

(4) The restricted intersection of (F, A) and (G, B), denoted by $(F, A) \cap_r (G, B)$, is the soft interval set (H, C), where $C = A \cap B$, and $H(a) = F(a) \sqcap G(a)$ for every $a \in C$.

Definition 6 *(1) (F, A) is called a relative null soft interval set(with respect to the parameter set A), denoted by $\emptyset_A$, if $F(e) = [\emptyset, \emptyset]$ for all $e \in A$.*

(2) (G, A) is called a relative whole soft interval set(with respect to the parameter set A), denoted by U_A, if $F(e) = [U, U]$ for all $e \in A$.

Definition 7 *The relative complement of a soft interval set (F, A) is denoted by $(F, A)^r$ and is defined by $(F, A)^r = (F^r, A)$, where $F^r : A \to I(2^U)$ is a mapping given by $F^r(e) = \neg F(e)$ for all $e \in A$.*

Clearly, $((F, A)^r)^r = (F, A)$ holds.

3.2. Algebraic structures of soft interval sets

Algebraic structures play a fundamental role in many fields of mathematics. Borzooei et al.[26] discussed the lattice structures of some fuzzy algebraic systems. We know that an interval set on a universe U is a bounded sublattice of the power set lattice 2^U and the set of all interval sets on U forms a distributive lattice. In addition, Wang et al.[27] proposed $G-$implication and $L-$implication operators on interval sets and constructed $G-$algebra and $MV-$algebra of interval sets. In this subsection, we consider the lattice structure of soft interval sets.

Based on the order relation $\sqsubseteq$ on interval sets, we define the order relation $\subseteq$ on soft interval sets as follows: for arbitrary soft interval sets (F, A) and (G, B), $(F, A) \subseteq (G, B)$ if and only if $A \subseteq B$ and $F(a) \sqsubseteq G(a)$ for every $a \in A$.

We observe that this order relation is compatible with the operation on soft interval sets, i.e.,

$(F, A) \subseteq (G, B)$ if and only if $(F, A) \cup_e (G, B) = (G, B)$.

In fact, let $(F, A) \cup_e (G, B) = (H, A \cup B)$. If $(F, A) \subseteq (G, B)$, then $A \subseteq B$ and hence $A \cup B = B$. For each $a \in B$, if $a \notin A$, then $H(a) = G(a)$ by the definition; if $a \in A$, then $H(a) = F(a) \sqcup G(a) = G(a)$ by $F(a) \sqsubseteq G(a)$. Thus we have $(F, A) \cup_e (G, B) = (G, B)$. Conversely, we assume that $(F, A) \cup_e (G, B) = (G, B)$. It follows that $A \subseteq B$ and for each $a \in A$, by $F(a) \sqcup G(a) = G(a)$ we conclude that $F(a) \sqsubseteq G(a)$ and hence $(F, A) \subseteq (G, B)$. Thus, the order relation $\subseteq$ on soft interval sets is just the order relation induced by $\cup_e$.

It is trivial to verify that $(S I(2^U), \subseteq)$ is a partially ordered sets. Actually, we have:

Theorem 1 *(1) In $(S I(2^U), \subseteq)$, the least upper bound $(F, A) \vee (G, B)$ and greatest lower bound $(F, A) \wedge (G, B)$ of (F, A) and (G, B) can be explicitly computed by using the following formulas:*

$(F, A) \vee (G, B) = (F, A) \cup_e (G, B)$,

$(F, A) \wedge (G, B) = (F, A) \cap_r (G, B)$.

(2) $(S I(2^U), \cup_e, \cap_r)$ is a bounded distributive lattice.

Proof. (1) We prove that $(F, A) \vee (G, B) = (F, A) \cup_e (G, B)$ holds. Suppose that $(F, A) \cup_e (G, B) = (H, A \cup B)$. For every $a \in A$, if $a \notin B$, then $F(a) = H(a)$; if $a \in B$, then $F(a) \sqsubseteq F(a) \sqcup G(a) = H(a)$. We conclude that $(F, A) \subseteq (F, A) \cup_e (G, B)$. Similarly, we have $(G, B) \subseteq (F, A) \cup_e (G, B)$. Hence, $(F, A) \cup_e (G, B)$ is an upper bound of (F, A) and (G, B).

Let (M, C) be an upper bound of (F, A) and (G, B), i.e., $(F, A) \subseteq (M, C)$, $(G, B) \subseteq (M, C)$. It follows that $A \subseteq C$, $B \subseteq C$ and hence $A \cup B \subseteq C$. For every $a \in A \cup B$, if $a \in A - B$, then $H(a) = F(a) \sqsubseteq M(a)$; if $a \in B - A$, then $H(a) = G(a) \sqsubseteq M(a)$; if $a \in A \cap B$, then $H(a) = F(a) \sqcup G(a) \sqsubseteq M(a)$ by $F(a) \sqsubseteq M(a)$ and $G(a) \sqsubseteq M(a)$. We conclude that $(F, A) \cup_e (G, B) \subseteq (M, C)$. That is, $(F, A) \cup_e (G, B)$ is the least upper bound of (F, A) and (G, B).

$(F, A) \wedge (G, B) = (F, A) \cap_r (G, B)$ can be proved similarly.

(2) By (1) we know that $(S I(2^U), \cup_e, \cap_r)$ is a lattice. Trivially, $\emptyset_\emptyset$ and U_E are the least and greatest elements in $S I(2^U)$ respectively. We prove that the following distributive law

$(F, A) \cup_e ((G, B) \cap_r (H, C))$

$= ((F, A) \cup_e (G, B)) \cap_r ((F, A) \cup_e (H, C))$

holds for all $(F, A), (G, B), (H, C) \in S I(2^U)$. Suppose that

$(F, A) \cup_e ((G, B) \cap_r (H, C)) = (K, A \cup (B \cap C))$,

$((F, A) \cup_e (G, B)) \cap_r ((F, A) \cup_e (H, C))$

$= (L, (A \cup B) \cap (A \cup C)) = (L, A \cup (B \cap C))$.

For each $a \in A \cup (B \cap C)$,

(a) if $a \notin A$, then $a \in B$ and $a \in C$, it follows that $K(a) = G(a) \sqcap H(a) = L(a)$;

(b) if $a \in A$, $a \notin B$, $a \notin C$, then $K(a) = F(a) = F(a) \sqcap F(a) = L(a)$;

(c) if $a \in A$, $a \in B$, $a \notin C$, then $K(a) = F(a) = (F(a) \sqcup G(a)) \sqcap F(a) = L(a)$;

(d) if $a \in A$, $a \notin B$, $a \in C$, then $K(a) = F(a) = F(a) \sqcap (F(a) \sqcup H(a)) = L(a)$;

(e) if $a \in A$, $a \in B$, $a \in C$, then $K(a) = F(a) \sqcup (G(a) \sqcap H(a)) = (F(a) \sqcup G(a)) \sqcap (F(a) \sqcup H(a)) = L(a)$.

Hence $(F, A) \cup_e ((G, B) \cap_r (H, C)) = ((F, A) \cup_e (G, B)) \cap_r ((F, A) \cup_e (H, C))$. $\square$

We consider soft interval sets over a definite parameter set. Let $A \subseteq E$ and

$$S I_A(2^U) = \{(F, A); F : A \to I(2^U)\}$$

be the set of soft interval sets over the universe U and the parameter set A. It is trivial to verify that $(F, A) \cup_e (G, A), (F, A) \cap_r (G, A) \in S I_A(2^U)$ for all $(F, A), (G, A) \in S I_A(2^U)$. Hence we have the following Corollary.

Corollary 1 *$(S I_A(2^U), \cup_e, \cap_r)$ is a sublattice of $(S I(2^U), \cup_e, \cap_r)$.*

In $(S I_A(2^U), \cup_e, \cap_r), U_A$ and $\emptyset_A$ are the greatest element and the least element respectively.

Now, we consider the algebraic structure of soft interval sets with respect to operations $\cup_r$ and $\cap_e$. We define the order relation $\subseteq'$ on soft interval sets as follows: for arbitrary soft interval sets (F, A) and (G, B), $(F, A) \subseteq' (G, B)$ if and only if $A \supseteq B$ and $F(b) \sqsubseteq G(b)$ for every $b \in B$.

It is easy to see that $(SI(2^U), \subseteq')$ is a partially ordered sets.

Theorem 2 *(1) In $(SI(2^U), \subseteq')$, the least upper bound $(F,A) \vee (G,B)$ and greatest lower bound $(F,A) \wedge (G,B)$ of (F,A) and (G,B) can be explicitly computed by using the following formulas:*

$(F,A) \vee (G,B) = (F,A) \cup_r (G,B)$,

$(F,A) \wedge (G,B) = (F,A) \cap_e (G,B)$.

(2) $(SI(2^U), \cup_r, \cap_e)$ is a bounded distributive lattice.

Proof. (1) We prove $(F,A) \vee (G,B) = (F,A) \cup_r (G,B)$. Suppose that $(F,A) \cup_r (G,B) = (H, A \cap B)$. It follows that $A \cap B \subseteq A$ and $F(a) \sqsubseteq F(a) \sqcup G(a) = H(a)$ for all $a \in A \cap B$. We conclude that $(F,A) \subseteq' (F,A) \cup_r (G,B)$. Similarly, we have $(G,B) \subseteq' (F,A) \cup_r (G,B)$. Hence, $(F,A) \cup_r (G,B)$ is an upper bound of (F,A) and (G,B) with respect to $\subseteq'$.

Let $(F,A) \subseteq' (M,C)$, $(G,B) \subseteq' (M,C)$. It follows that $C \subseteq A$, $C \subseteq B$ and hence $C \subseteq A \cap B$. For every $a \in C$, we have $F(a) \sqsubseteq M(a)$, $G(a) \sqsubseteq M(a)$ and consequently $H(a) = F(a) \sqcup G(a) \sqsubseteq M(a)$. We conclude that $(F,A) \cup_r (G,B) \subseteq' (M,C)$. That is, $(F,A) \cup_r (G,B)$ is the least upper bound of (F,A) and (G,B).

$(F,A) \wedge (G,B) = (F,A) \cap_e (G,B)$ can be proved similarly.

(2) By the definition of $\subseteq'$ we know that $\emptyset_E$ and $\emptyset_\emptyset$ are the least and greatest elements in $SI(2^U)$ with respect to $\subseteq'$, respectively. We prove that the following distributive law

$(F,A) \cup_r ((G,B) \cap_e (H,C))$

$= ((F,A) \cup_r (G,B)) \cap_e ((F,A) \cup_r (H,C))$

holds for all $(F,A), (G,B), (H,C) \in SI(2^U)$.

Suppose that

$(F,A) \cup_r ((G,B) \cap_e (H,C)) = (K, A \cap (B \cup C))$,

$((F,A) \cup_r (G,B)) \cap_e ((F,A) \cup_r (H,C))$

$= (L, (A \cap B) \cup (A \cap C)) = (L, A \cap (B \cup C))$.

For each $a \in A \cap (B \cup C)$, we have $a \in A$ and $a \in B \cup C$.

(a) if $a \in B$ and $a \in C$, then $K(a) = F(a) \sqcup (G(a) \sqcap H(a)) = (F(a) \sqcup G(a)) \sqcap (F(a) \sqcup H(a)) = L(a)$;

(b) if $a \notin B$, $a \in C$, then $K(a) = F(a) \sqcup H(a) = L(a)$;

(c) if $a \in B$, $a \notin C$, then $K(a) = F(a) \sqcup G(a) = L(a)$.

Since K and L are indeed the same interval-set-valued mappings, we conclude that $(F,A) \cup_r$

$((G,B) \cap_e (H,C)) = ((F,A) \cup_r (G,B)) \cap_e ((F,A) \cup_r (H,C))$ as required. $\square$

Corollary 2 $(SI_A(2^U), \cup_r, \cap_e)$ *is a sublattice of* $(SI(2^U), \cup_r, \cap_e)$.

In $(SI_A(2^U), \cup_e, \cap_r)$, $\emptyset_A$ and U_A are the least element and the greatest element respectively. For every $(F,A), (G,A) \in SI_A(2^U)$, we have $(F,A) \subseteq (G,A)$ if and only if $(F,A) \subseteq' (G,A)$. It follows that $(SI_A(2^U), \cup_e, \cap_r)$ and $(SI_A(2^U), \cup_r, \cap_e)$ are the same lattices.

The following types of De Morgan's laws hold in soft interval set theory.

Theorem 3 *Let (F,A) and (G,B) be two soft interval set over the same universe U such that $A \cap B \neq \emptyset$. Then*

(1)$((F,A) \cup_r (G,B))^r = (F,A)^r \cap_r (G,B)^r$,

(2)$((F,A) \cap_r (G,B))^r = (F,A)^r \cup_r (G,B)^r$,

(3)$((F,A) \cup_e (G,B))^r = (F,A)^r \cap_e (G,B)^r$,

(4)$((F,A) \cap_e (G,B))^r = (F,A)^r \cup_e (G,B)^r$.

Proof. (1) Let $(F,A) \cup_r (G,B) = (H, A \cap B)$, $(F,A)^r \cap_r (G,B)^r = (K, A \cap B)$. It follows that $((F,A) \cup_r (G,B))^r = (H^r, A \cap B)$. For all $a \in A \cap B$, we have $H^r(a) = \neg H(a) = \neg(F(a) \sqcup G(a)) = \neg F(a) \sqcap \neg G(a) = F^r(a) \sqcap G^r(a) = K(a)$, and consequently $((F,A) \cup_r (G,B))^r = (F,A)^r \cap_r (G,B)^r$.

(2) By (1) we have $((F,A)^r \cup_r (G,B)^r)^r = ((F,A)^r)^r \cap_r ((G,B)^r)^r = (F,A) \cap_r (G,B)$, and hence $((F,A) \cap_r (G,B))^r = (((F,A)^r \cup_r (G,B)^r)^r)^r = (F,A)^r \cup_r (G,B)^r$.

(3) Let $(F,A) \cup_e (G,B) = (H, A \cup B)$, $(F,A)^r \cap_e (G,B)^r = (K, A \cup B)$. It follows that $((F,A) \cup_e (G,B))^r = (H^r, A \cup B)$. For all $a \in A \cup B$,

(a) if $a \in A - B$, then $H^r(a) = \neg H(a) = \neg F(a) = K(a)$;

(b) if $a \in B - A$, then $H^r(a) = \neg H(a) = \neg G(a) = K(a)$;

(c) if $a \in A \cap B$, then $H^r(a) = \neg H(a) = \neg(F(a) \sqcup G(a)) = (\neg F(a)) \sqcap (\neg G(a)) = K(a)$.

Hence we have $((F,A) \cup_e (G,B))^r = (F,A)^r \cap_e (G,B)^r$.

(4) By (3) we have $((F,A)^r \cup_e (G,B)^r)^r = ((F,A)^r)^r \cap_e ((G,B)^r)^r = (F,A) \cap_e (G,B)$. It follows that $((F,A) \cap_e (G,B))^r = (((F,A)^r \cup_e (G,B)^r)^r)^r = (F,A)^r \cup_e (G,B)^r$. $\square$

3.3. *The soft equality relation on soft interval sets*

Based on the parameter analysis of soft set, Qin et al.[9] introduced the notion of soft equality relation on soft sets. It is used to describe soft sets which are 'almost' equal. With this relation, all the soft sets on a definite universe can be partitioned to different classes in such a way that two soft sets will be partitioned to same class if they are soft equal. Furthermore, the soft equality relation can be used to construct soft quotient algebra. In this subsection, we follow the line of exploration in [9] and intend to generalize this relation to soft interval sets.

Definition 8 *Let (F,A), (G,B) be two soft interval sets over the universe U. (F,A) is called soft equal to (G,B), denoted by $(F,A) \approx_S (G,B)$, if for all $a \in A \cup B$, $a \in A \cap B$ implies $F(a) = G(a)$, $a \in A - B$ implies $F(a) = [\emptyset, \emptyset]$, and $a \in B - A$ implies $G(a) = [\emptyset, \emptyset]$.*

Example 3 *We consider the soft interval set (F,A) given in Example 2. Let (G,B) be a soft interval set, where $B = \{e_1, e_2, e_3, e_4, e_5\}$, $G(e_i) = F(e_i)$ for every $1 \leqslant i \leqslant 4$, and $G(e_5) = [\emptyset, \emptyset]$. Then $(F,A) \approx_S (G,B)$.*

Theorem 4 *Let (F,A) and (G,B) be two soft interval sets over the universe U. Then $(F,A) \approx_S (G,B)$ if and only if $(F,A) \cup_e (G,B) \approx_S (F,A) \cap_r (G,B)$.*

Proof. Let $(F,A) \cup_e (G,B) = (H, A \cup B)$ and $(F,A) \cap_r (G,B) = (T, A \cap B)$.

Suppose that $(F,A) \approx_S (G,B)$. For all $a \in A \cap B$, by Definition 8, we have $F(a) = G(a)$, and hence $H(a) = F(a) \sqcup G(a) = F(a) \sqcap G(a) = T(a)$. For all $a \in A \cup B - A \cap B$, (a) if $a \in A - B$, then $F(a) = [\emptyset, \emptyset]$ and hence $H(a) = F(a) = [\emptyset, \emptyset]$; (b) if $a \in B - A$, then $G(a) = [\emptyset, \emptyset]$ and hence $H(a) = G(a) = [\emptyset, \emptyset]$. Consequently, $(F,A) \cup_e (G,B) \approx_S (F,A) \cap_r (G,B)$.

Conversely, suppose that $(F,A) \cup_e (G,B) \approx_S (F,A) \cap_r (G,B)$. For all $a \in A \cap B$, we have $F(a) \sqcup G(a) = F(a) \sqcap G(a)$ and hence $F(a) = G(a)$. For all $a \in A - B$, it follows that $a \in A \cup B$, $a \notin A \cap B$, and hence $F(a) = H(a) = [\emptyset, \emptyset]$. For all $a \in B - A$, $G(a) = [\emptyset, \emptyset]$ can be proved similarly. Hence $(F,A) \approx_S (G,B)$. $\square$

Theorem 5 *Let (F,A), (G,B) be two soft sets over the universe U.*

(1) $(F,A) \approx_S (G,B)$ if and only if $(F,A) \cup_r (G,B) \approx_S (F,A) \cap_e (G,B)$.

(2) $(F,A) \approx_S (G,B)$ implies $(F,A) \cup_e (G,B) = (F,A) \cap_e (G,B)$, $(F,A) \cup_r (G,B) = (F,A) \cap_r (G,B)$.

The proof is similar to that of Theorem 4 and we omit it.

Theorem 6 *$\approx_S$ is a congruence relation on $SI(2^U)$ with respect to operations $\cap_r$ and $\cup_e$, i.e.,*

(1) $\approx_S$ is an equivalence relation;

(2) $(F,A) \approx_S (G,B)$ and $(H,C) \approx_S (L,D)$ imply $(F,A) \cap_r (H,C) \approx_S (G,B) \cap_r (L,D)$ and $(F,A) \cup_e (H,C) \approx_S (G,B) \cup_e (L,D)$.

Proof. (1) It is trivial to verify that $\approx_S$ is reflexive and symmetric.

Suppose that $(F,A) \approx_S (G,B)$ and $(G,B) \approx_S (H,C)$. For all $e \in A \cap C$, if $e \in B$, then $e \in A \cap B$ and $e \in B \cap C$, it follows that $F(e) = G(e) = H(e)$; if $e \notin B$, then $e \in A - B$ and $e \in C - B$, it follows that $F(e) = [\emptyset, \emptyset] = H(e)$.

For all $e \in A - C$, it follows that $e \in A$ and $e \notin C$. If $e \in B$, then $e \in A \cap B$ and $e \in B - C$. Consequently, $F(e) = G(e) = [\emptyset, \emptyset]$; if $e \notin B$, then $e \in A - B$ and $F(e) = [\emptyset, \emptyset]$.

For all $e \in C - A$, $H(e) = [\emptyset, \emptyset]$ can be proved similarly. Hence $(F,A) \approx_S (H,C)$. We conclude that $\approx_S$ is a transitive relation as required.

(2) Suppose that $(F,A) \approx_S (G,B)$ and $(H,C) \approx_S (L,D)$. Let $(F,A) \cap_r (H,C) = (M_1, A \cap C)$, $(G,B) \cap_r (L,D) = (M_2, B \cap D)$.

(a) If $e \in (A \cap C) \cap (B \cap D)$, then $e \in (A \cap B)$ and $e \in (C \cap D)$. It follows that $F(e) = G(e)$, $H(e) = L(e)$ and hence

$M_1(e) = F(e) \sqcap H(e) = G(e) \sqcap L(e) = M_2(e)$.

(b) If $e \in (A \cap C) - (B \cap D)$, then $e \in A$, $e \in C$ and $e \notin B \cap D$. It follows that $e \notin B$ or $e \notin D$. If $e \notin B$, then $e \in A - B$ and hence $F(e) = [\emptyset, \emptyset]$. Consequently, $M_1(e) = F(e) \sqcap H(e) = [\emptyset, \emptyset]$. If $e \notin D$, then $e \in C - D$ and hence $H(e) = [\emptyset, \emptyset]$. It follows that $M_1(e) = F(e) \sqcap H(e) = [\emptyset, \emptyset]$.

(c) If $e \in (B \cap D) - (A \cap C)$, then $M_2(e) = [\emptyset, \emptyset]$ can be proved similarly.

We conclude that $(F,A) \cap_r (H,C) \approx_S (G,B) \cap_r (L,D)$.

Let $(F,A) \cup_e (H,C) = (T_1, A \cup C)$, $(G,B) \cup_e (L,D) = (T_2, B \cup D)$.

For all $e \in (A \cup C) \cap (B \cup D)$, we have $e \in A \cup C$ and $e \in B \cup D$. Without losing of generality, we suppose that $e \in A$ and $e \in D$.

(a) If $e \in B$ and $e \in C$, then $e \in A \cap B$ and $e \in C \cap D$. It follows that $F(e) = G(e)$, $H(e) = L(e)$ and hence $T_1(e) = F(e) \sqcup H(e) = G(e) \sqcup L(e) = T_2(e)$.

(b) If $e \notin B$ and $e \in C$, then $e \in A - B$ and $e \in C \cap D$. It follows that $F(e) = [\emptyset, \emptyset]$, $H(e) = L(e)$ and hence $T_1(e) = F(e) \sqcup H(e) = H(e) = L(e) = T_2(e)$.

(c) If $e \in B$ and $e \notin C$, then $e \in A \cap B$ and $e \in D - C$. It follows that $F(e) = G(e)$, $L(e) = [\emptyset, \emptyset]$ and hence $T_1(e) = F(e) = G(e) = G(e) \sqcup L(e) = T_2(e)$.

(d) If $e \notin B$ and $e \notin C$, then $e \in A - B$ and $e \in D - C$. It follows that $F(e) = [\emptyset, \emptyset]$, $L(e) = [\emptyset, \emptyset]$ and hence $T_1(e) = F(e) = [\emptyset, \emptyset] = L(e) = T_2(e)$.

For all $e \in A \cup C - B \cup D$, we have $e \in A \cup C$, $e \notin B$ and $e \notin D$.

(a) If $e \in A$ and $e \in C$, then $e \in A - B$ and $e \in C - D$. It follows that $F(e) = H(e) = [\emptyset, \emptyset]$, and hence $T_1(e) = F(e) \sqcup H(e) = [\emptyset, \emptyset]$.

(b) If $e \in A$ and $e \notin C$, then $e \in A - B$. It follows that $F(e) = [\emptyset, \emptyset]$, and hence $T_1(e) = F(e) = [\emptyset, \emptyset]$.

(c) If $e \notin A$ and $e \in C$, then $e \in C - D$. It follows that $H(e) = [\emptyset, \emptyset]$, and hence $T_1(e) = H(e) = [\emptyset, \emptyset]$.

For all $e \in B \cup D - A \cup C$, $T_2(e) = [\emptyset, \emptyset]$ can be proved similarly.

Hence $(F, A) \cup_e (H, C) \approx_S (G, B) \cup_e (L, D)$. □

Let $(F, A)_{\approx_S} = \{(G, B); (G, B) \approx_S (F, A)\}$ be the congruence class including (F, A) and $SI(2^U)/\approx_S = \{(F, A)_{\approx_S}; (F, A) \in SI(2^U)\}$ be the set of all congruence classes. We define operations $\cup_S$ and $\cap_S$ on $SI(2^U)/\approx_S$ as follows:

$$(F, A)_{\approx_S} \cup_S (G, B)_{\approx_S} = ((F, A) \cup_e (G, B))_{\approx_S}, \quad (3)$$

$$(F, A)_{\approx_S} \cap_S (G, B)_{\approx_S} = ((F, A) \cap_r (G, B))_{\approx_S}. \quad (4)$$

These two operations are well defined by Theorem 6. We call $(SI(2^U)/\approx_S, \cup_S, \cap_S)$ the soft quotient algebra with respect to $\approx_S$. It is routine to verify that $(SI(2^U)/\approx_S, \cup_S, \cap_S)$ is a distributive lattice.

Theorem 7 $(SI(2^U)/\approx_S, \cup_S, \cap_S)$ and $(SI_E(2^U), \cup_e, \cap_r)$ are isomorphic.

Proof. Let $\varphi : SI_E(2^U) \to SI(2^U)/\approx_S$ be defined as $\varphi(F, E) = (F, E)_{\approx_S}$ for every $(F, E) \in SI_E(2^U)$.

(1) If $(F_1, E), (F_2, E) \in SI_E(2^U)$ and $(F_1, E) \neq (F_2, E)$, then there exists $a \in E$ such that $F_1(a) \neq F_2(a)$, and consequently $(F_1, E)_{\approx_S} \neq (F_2, E)_{\approx_S}$.

(2) Let $(F, A)_{\approx_S} \in SI(2^U)/\approx_S$. We define $(F', E) \in SI_E(2^U)$ as follows: $F'(a) = [\emptyset, \emptyset]$ for each $a \in E - A$ and $F'(a) = F(a)$ for each $a \in A$. Hence $(F', E) \approx_S (F, A)$ and

$\varphi(F', E) = (F', E)_{\approx_S} = (F, A)_{\approx_S}$.

By (1) and (2), we conclude that φ is a one-to-one mapping.

(3) For every $(F_1, E), (F_2, E) \in SI_E(2^U)$, we have:
$\varphi((F_1, E) \cup_e (F_2, E)) = ((F_1, E) \cup_e (F_2, E))_{\approx_S}$
$= (F_1, E)_{\approx_S} \cup_S (F_2, E)_{\approx_S} = \varphi(F_1, E) \cup_S \varphi(F_2, E)$,
$\varphi((F_1, E) \cap_r (F_2, E)) = ((F_1, E) \cap_r (F_2, E))_{\approx_S}$
$= (F_1, E)_{\approx_S} \cap_S (F_2, E)_{\approx_S} = \varphi(F_1, E) \cap_S \varphi(F_2, E)$.

We conclude that φ is an isomorphism, $(SI(2^U)/\approx_S, \cup_S, \cap_S)$ and $(SI_E(2^U), \cup_e, \cap_r)$ are isomorphic. □

In order to establish the soft quotient algebra with respect to operations $\cup_r$ and $\cap_e$, we discuss another kind of soft equality relation $\approx^S$.

Definition 9 Let $(F, A), (G, B) \in SI(2^U)$. $(F, A) \approx^S (G, B)$ if for all $e \in A \cup B$, $e \in A \cap B$ implies $F(e) = G(e)$, $e \in A - B$ implies $F(e) = [U, U]$, and $e \in B - A$ implies $G(e) = [U, U]$.

Theorem 8 Let (F, A) and (G, B) be two soft interval sets over the universe U. Then $(F, A) \approx^S (G, B)$ if and only if $(F, A)^r \approx_S (G, B)^r$.

Proof. Suppose that $(F, A) \approx^S (G, B)$. For all $e \in A \cap B$, by definition, we have $F(e) = G(e)$, and hence $F^r(e) = \neg F(e) = G^r(e)$. For all $e \in A - B$, we have $F(e) = [U, U]$ and hence $F^r(e) = [\emptyset, \emptyset]$. Similarly, $G^r(e) = [\emptyset, \emptyset]$ for all $e \in B - A$. We conclude that $(F, A)^r \approx_S (G, B)^r$ as required.

Conversely, suppose that $(F, A)^r \approx_S (G, B)^r$. For all $e \in A \cap B$, we have $F^r(e) = G^r(e)$, and hence $F(e) = G(e)$. For all $e \in A - B$, we have $F^r(e) = [\emptyset, \emptyset]$ and hence $F(e) = [U, U]$. Similarly, $G(e) = [U, U]$ for all $e \in B - A$. Hence $(F, A) \approx^S (G, B)$. □

Theorem 9 $\approx^S$ is a congruence relation with respect to operations $\cup_r$ and $\cap_e$.
The proof of this Theorem is similar to that of Theorem 6.

Let $(F, A)_{\approx^S} = \{(G, B); (G, B) \approx^S (F, A)\}$ be the congruence class (with respect to $\approx^S$) including

(F,A) and $SI(2^U)/\approx^S = \{(F,A)_{\approx^S}; (F,A) \in SI(2^U)\}$. We define operations $\cup^S$ and $\cap^S$ on $SI(2^U)/\approx^S$ as follows:

$$(F,A)_{\approx^S} \cup^S (G,B)_{\approx^S} = ((F,A) \cup_r (G,B))_{\approx^S}, \quad (5)$$

$$(F,A)_{\approx^S} \cap^S (G,B)_{\approx^S} = ((F,A) \cap_e (G,B))_{\approx^S}. \quad (6)$$

We call $(SI(2^U)/\approx^S, \cup^S, \cap^S)$ the soft quotient algebra with respect to $\approx^S$. It is routine to verify that $(SI(2^U)/\approx^S, \cup^S, \cap^S)$ is a distributive lattice.

4. Applications to decision making problems

Maji et al.[4] proposed tabular representation of soft set to deal with soft set based decision making problems. We can represent soft interval set similarly in a tabular form. Let (F,A) be a soft interval set, $a \in A$ and $F(a) = [F(a)_-, F(a)_+]$. By intuitionistic semantics of soft interval set, we know that elements in $F(a)_-$ belong definitely to $F(a)$, elements in $U - F(a)_+$ does not belong to $F(a)$, while elements in $F(a)_+ - F(a)_-$ are possibly belong to $F(a)$. Following the idea of incomplete information system, we represent soft interval set (F,A) in a binary table and its entries h_{ij} are defined as: $h_i \in F(a)_-$ implies $h_{ij} = 1$; $h_i \in F(a)_+ - F(a)_-$ implies $h_{ij} = *$, and $h_i \in U - F(a)_+$ implies $h_{ij} = 0$. $h_{ij} = *$ means we are not sure whether $h_i \in F(a)$ or not.

The tabular representation of the soft interval set (F,A) given in Example 2 is as in Table 1.

Table 1. The tabular representation of (F,A)

	e_1	e_2	e_3	e_4
h_1	*	1	0	*
h_2	1	0	0	*
h_3	0	1	1	0
h_4	1	*	1	0
h_5	0	0	1	1

Zou and Xiao[28] proposed a data analysis approach to soft set under incomplete information. The incomplete data will be predicted based on the method of average probability. Suppose p_a stands for probability that an object in $F(a)_+ - F(a)_-$ belongs to $F(a)$, it can be defined by [28]

$$p_a = \frac{|F(a)_-|}{|F(a)_-| + |U - F(a)_+|}, a \in A.$$

We may take p_a as the value of elements in $F(a)_+ - F(a)_-$ with respect to parameter a. By this definition, for the soft interval set in Table 1, we have $p_{e_1} = \frac{1}{2}$, $p_{e_2} = \frac{1}{2}$, $p_{e_4} = \frac{1}{3}$. The extended tabular representation of (F,A) is as in Table 2.

Table 2. The extended tabular representation of (F,A)

	e_1	e_2	e_3	e_4
h_1	$\frac{1}{2}$	1	0	$\frac{1}{3}$
h_2	1	0	0	$\frac{1}{3}$
h_3	0	1	1	0
h_4	0	$\frac{1}{2}$	1	0
h_5	0	0	1	1

In real decision making problems, the choice parameters may not be of equal importance. To cope with such problems, we can impose different weights to different decision parameters. Additionally, there may be some parameters which is 'negative' to optimal choice. So the weights can range from -1 to 1, that is, the values domain of weights is the interval $[-1, 1]$.

Maji et al.[4] proposed an approach to weighted soft set based decision making problems. The choice value c_i of object h_i will be computed by:

$$c_i = \sum_j w_j h_{ij},$$

where w_j is the weight of e_j. The object with the maximum choice value will be selected as the optimal alternative. In this decision making problem, we know that e_1 stands for 'expensive' and it is negative to the optimal choice. We assume that the weights of e_1, e_2, e_3, e_4 are $-0.4, 0.6, 0.4, 0.9$, respectively. The choice values c_i of objects h_i can be computed as follows:

$$c_1 = (-0.4) \times \frac{1}{2} + 0.6 \times 1 + 0.4 \times 0 + 0.9 \times \frac{1}{3} = 0.7,$$

$$c_2 = (-0.4) \times 1 + 0.6 \times 0 + 0.4 \times 0 + 0.9 \times \frac{1}{3} = -0.1,$$

$$c_3 = (-0.4) \times 0 + 0.6 \times 1 + 0.4 \times 1 + 0.9 \times 0 = 1,$$

$$c_4 = (-0.4) \times 1 + 0.6 \times \frac{1}{2} + 0.4 \times 1 + 0.9 \times 0 = 0.3,$$

$$c_5 = (-0.4) \times 0 + 0.6 \times 0 + 0.4 \times 1 + 0.9 \times 1 = 1.3.$$

Hence, the optimal decision is to select h_5. The extended tabular representation of (F,A) with weights and choice values is given by Table 3.

Table 3. The extended tabular representation of (F,A) with weights and choice values

	$e_{1,-0.4}$	$e_{2,0.6}$	$e_{3,0.4}$	$e_{4,0.9}$	choice-value
h_1	$\frac{1}{2}$	1	0	$\frac{1}{3}$	0.7
h_2	1	0	0	$\frac{1}{3}$	-0.1
h_3	0	1	1	0	1
h_4	1	$\frac{1}{2}$	1	0	0.3
h_5	0	0	1	1	1.3

This is only a simple example to show the possibility of using this method for decision making problems which could be improved based on domain knowledge and other competing data analysis methods. We summarize the soft interval set based decision making approach presented in this section as the following algorithm:

Step 1: Input the soft interval set (F,A) and place it in tabular form.

Step 2: Compute average probability p_a for each parameter $a \in A$ and construct the extended tabular representation of (F,A), where $p_a = \frac{|F(a)_-|}{|F(a)_-|+|U-F(a)_+|}$.

Step 3: Input the weights w_j of parameters in A.

Step 4: Compute the choice values c_i of h_i, where $c_i = \sum_j w_j h_{ij}$ and h_{ij} are entries in extended tabular representation of (F,A).

Step 5: The optimal decision is h_k if $c_k = max_i c_i$.

Step 6: If k has more that one value then any one of h_k may be chosen.

5. Conclusions

Soft set theory and interval set theory are all mathematical tools for dealing with uncertainty. They are closely related. This paper deals with the combination of soft set and interval set models and its application. The notion of soft interval sets is introduced and the algebraic structure of soft interval sets is investigated. Additionally, a soft interval set based decision making problem is examined.

This paper focuses on the theoretical study of soft interval sets. Because of limitation of spaces, only a numerical example is provided to illustrate the application method of soft interval sets. We think that soft interval set theory may be applied to the knowledge acquisition for interval-valued information systems, qualitative information representation and qualitative information reasoning. Based on this paper, we can further probe the applications of soft interval sets in practical fields. The combination of soft interval set and other competing data analysis models is another important and interesting issue to be addressed.

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