An Interactive Approach Based on Alternative Achievement Scale and Alternative Comprehensive Scale for Multiple Attribute Decision Making under Linguistic Environment

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Abstract

The aim of this paper is to develop an interactive approach for multiple attribute decision making with incomplete weight information under linguistic environment. Some of the concepts are defined, such as the distance between two 2-tuple linguistic variables, the expectation level of alternative, the achievement scale, the alternative comprehensive scale under linguistic environment. Based on these concepts, we establish some linear programming models, through which the decision maker interacts with the analyst. Furthermore, we establish a practical interactive approach for selecting the most desirable alternative(s). The interactive process can be realized by giving and revising the achievement scale and comprehensive scale of alternatives till the achievement scale and the comprehensive scale are achieved to the decision maker's request. Finally, an illustrative example is also given.

Keywords: Multiple attribute decision making; Interactive approach; Achievement scale; Comprehensive scale; 2-tuple linguistic.

1. Introduction

Multiple attribute decision making (MADM) consists of selecting the most desirable alternative from a given alternative set. Usually, it consists the following two phases[3,6,20]: (1) Aggregation phase: it consists of establishing an appropriate aggregation operator for aggregating and combining the individual preferences values provided from decision maker to obtain a collective preference value for each alternative; (2) Exploitation phase: it consists of establishing a rank ordering among the alternatives according to the collective preference value for choosing the best alternative(s). For the above processes, the selection process is only made in a single step, that means each phase is only implemented once. But in the real world, the increasing complexity of the socio-economic environment makes it less and less impossible for the decision maker (DM) to make a decision in an order step, perhaps in some phase, the DM does not satisfy with the result, it needs to revise some value of the phase, thus, interactive process is important for the DM to determine the best alternative. Interactive approaches have been received more and more attention in recent years[2,11-14,19,31,32]. As a result, the above methods can only be suitable for dealing with the MADM problems with numerical information on attributes values. In the real-life word, there are many

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decision situations in which the information cannot be assessed by precisely in a quantitative form but may be in a qualitative one, for example, when evaluating the "comfort" or "design" of a car, terms like "good", "medium", "bad" are usually used, and evaluating a car's speed, terms like "very fast", "fast", "slow" can be used instead of numeric values[1,10,15,24,25,27-29], and thus, in such situations, the use of linguistic approach is necessary. Xu[30] developed an interactive approach to multiple attribute group decision making with multigranular uncertain linguistic information. Xu[32] presented an interactive procedure for linguistic multiple attribute decision making with incomplete weight information. The method establishes a practical interactive procedure for selecting the most desirable alternative(s). The interactive process can be realized by giving and revising the satisfactory degrees of alternatives till an optimum satisfactory solution is achieved. But in the paper, the satisfactory degree of alternative is so simple, and far away from the real situations. And in fact, in the process of the interactive, the satisfactory degree of alternative, in our sense, is a subjective index, and only expressed by a numerical value, it is not only difficult, but also deprives the DM's leading role to some extent. All of the papers are failed to deal with such situations. Therefore, it is important to pay attention to this issue. And also Xu[32] used the virtual linguistic variable to compute. However, Martínez et al.[17] said that the appearance of virtual terms without syntax either semantics limits the interpretability of the results. The 2-tuple linguistic model is a symbolic computational one introduced by Herrera and Martínez[8] in order to improve the accuracy and facilitate the processes of CW[18,35,36] by treating the linguistic domain as continuous but keeping the linguistic basis (syntax and semantics). The 2-tuple has been discussed extensively[5-7,9,16,26] in the last years and also has a new version[4,21]. Therefore, we use 2-tuple linguistic model to deal with linguistic information in this paper.

In this paper, we focus our attention on developing a practical interactive procedure for selecting the most desired alternative(s) by 2-tuple linguistic model under linguistic environment. We investigate the MADM problems, in which the information about attribute weights is partly known and the attribute values are expressed by means of linguistic variables. We propose the objective index of achievement scale. It depends on the DM to judge the satisfactory degree of alternative

according to the achievement scale. It not only avoids expressing the satisfactory degree by a simple index, but also maintains the DM's leading role. In order to measure the overall value of all the alternatives, we introduce the comprehensive scale. We don't need the index of comprehensive scale to reflect the overall value precisely, but it provides the DM to know the overall value in the interactive decision process. The interactive process can be realized by giving and revising the achievement scale and comprehensive scale till an optimum satisfactory solution is achieved.

In order to do this, this paper is structured as follows. Section 2 gives the concept of the 2-tuple linguistic representation, the distance between two 2-tuple linguistic variables. Section 3 defines the expectation level of alternative, the 2-tuple linguistic negative point, the achievement scale, the alternative comprehensive scale under linguistic environment. Based on these concepts, an interactive approach is developed. Section 4, a practical application of the developed approach to evaluate the university faculty for tenure and promotion has also been given to show the effectiveness of the proposed approach. Section 5, concludes the paper and points out the future work.

2. Preliminaries

The linguistic variables are used in processes of computing with words that imply their fusion, aggregation, comparison, etc. The most often used models dealing with linguistic information are: (ⅰ) the semantic model that uses the linguistic terms just as labels for fuzzy numbers, while the computations over them are done directly over those fuzzy numbers; (ⅱ) the second one, is the symbolic model that uses the order index of the linguistic terms to make direct computation on labels; (ⅲ) the third mode is based on the linguistic 2-tuple.

The 2-tuple linguistic representation model presented in Ref. 8,9, is based on the symbolic one and in a concept called *Symbolic Translation*. In this section, we shall make a brief review of some concepts of the 2-tuple.

Definition 1. *Let $S=\{s_0,s_1,...,s_g\}$ be a linguistic term set. For example, a set of nine terms S could be given as follows:*

$S = \{s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor},$

$s_3 = \text{slightly poor}, s_4 = \text{fair}, s_5 = \text{slightly good},$

$s_6 = \text{good}, s_7 = \text{very good}, s_8 = \text{extremely good}\}$

Usually, in these cases, it is required that the linguistic term set satisfies the following additional characteristics.

1) There is a negation operator: $Neg(s_i)=s_j$ such that $j=g-i$ ($g+1$ is the cardinality).

2) $s_i \le s_j \Leftrightarrow i \le j$. Therefore, there exists a minimization and a maximization operator.

Definition 2[8]. *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation.* $\beta \in [0,g]$*, being g the cardinality of S. Let i= round(β) and α= β-i be two values such that* $i \in [0,g]$ *and* $\alpha \in [-0.5,0.5]$*, then α is called a symbolic translation.*

From this concept it is developed a linguistic representation model which represents the linguistic information by means of 2-tuple (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-0.5,0.5]$:

• s_i represents the linguistic label center of the information;

• α_i is a numerical value expressing the value of the translation from the original result β to the closest index label i in the linguistic term set (s_i), i.e., the symbolic translation.

Definition 3[8]. *Let* $S=\{s_0,s_1,...,s_g\}$ *be a linguistic term set and* $\beta \in [0,g]$ *be a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0,g] \to S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha_i), with \begin{cases} s_i, i = round(\beta) \\ \alpha_i = \beta - i, \alpha_i \in [-0.5, 0.5) \end{cases}$$

where round(·)is the usual round operation, s_i *has the closest index label to β and* α_i *is the value of the symbolic translation.*

Contrarily, let $S=\{s_0,s_1,...,s_g\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple. There is always a Δ^{-1} function:

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0,g]$$

$$\Delta^{-1}(s_i, \alpha_i) = i + \alpha_i = \beta$$

Remark: From Definitions 2 and 3, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

$$s_i \in S \Rightarrow (s_i, 0).$$

Comparison of 2-Tuples: let (s_k, α_k) and (s_l, α_l) be two 2-tuples, with each one representing a counting of information as follows:

1) If $k < l$ then $(s_k, \alpha_k) < (s_l, \alpha_l)$;

2) If $k = l$ ① $\alpha_k = \alpha_l$ then $(s_k, \alpha_k) = (s_l, \alpha_l)$;

② $\alpha_k < \alpha_l$ then $(s_k, \alpha_k) < (s_l, \alpha_l)$;

③ $\alpha_k > \alpha_l$ then $(s_k, \alpha_k) > (s_l, \alpha_l)$.

For a multiple attribute decision making problem, let $X=\{x_1,x_2,...,x_n\}$ be a discrete set of alternatives, $U=\{u_1, u_2,...,u_m\}$ be a set of attributes. For each alternative $x_j \in X$, the decision maker gives his/her preference value a_{ij} with respect to attribute $u_i \in U$, where a_{ij} takes the form of linguistic variable, that is $a_{ij} \in S$, then all the preference values of the alternatives consist of the decision matrix $A=(a_{ij})_{m \times n}$.

Definition 4. *Let* $\{(r_1,\alpha_1),(r_2,\alpha_2)...,(r_m,\alpha_m)\}$ $(r_i \in S, \alpha_i \in [-0.5,0.5]$ *,i=1,2,...,m) be a set of linguistic 2-tuples, then 2TLWA is called 2-tuple linguistic weighted averaging (2TLWA) operator, if*

$$2\text{TLWA}((r_1,\alpha_1),...,(r_m,\alpha_m)) = \Delta \left(\frac{\sum_{i=1}^{m} w_i \Delta^{-1}(r_i,\alpha_i)}{\sum_{i=1}^{m} w_i} \right) \quad (1)$$

where $w=(w_1,w_2,...,w_m)^T$ *is the weighting vector of the 2-tuples* (r_i,α_i) *(i=1,2,...,m).Generally,* $\sum_{i=1}^{m} w_i = 1$*,* $w_i \in [0,1]$*, i=1,2,...,m. Thus, Eq.(1) is reduced to the following:*

$$2\text{TLWA}((r_1,\alpha_1),...,(r_m,\alpha_m)) = \Delta \left(\sum_{i=1}^{m} w_i \Delta^{-1}(r_i,\alpha_i) \right) \quad (2)$$

Especially, if $w=(1/m,1/m...,1/m)^T$*, then the 2TLWA operator is reduced to the 2-tuple linguistic average (2TLA) operator.*

Definition 5[8]. *Let* $A = (a_{ij})_{m \times n}$ *be the linguistic decision matrix,* $a_j=(a_{1j},a_{2j},...,a_{mj})$*be the vector of attribute values corresponding to the alternative* x_j*, j=1,2,...,n, then we call*

$$z_j(w) = 2\text{TLWA}_w(a_{1j},a_{2j},...,a_{mj}) = \Delta \left(\sum_{i=1}^{m} w_i \Delta^{-1}(a_{ij}) \right) \quad (3)$$

the overall value of the alternative x_j*, where* $w=(w_1, w_2,...,w_m)^T$ *is the weighting vector of attributes.*

The linguistic multiple attribute decision making problems generally consist of finding the most desirable alternative(s) from a given alternative set. If the decision matrix $A=(a_{ij})_{m \times n}$ and the corresponding weight value of attributes are known, we can get the overall value of the alternative x_j by Eq.(3).

Definition 6. *Let* (r_k,α_k) *and* (r_l,α_l) *be two 2-tuples, then we call*

$$d((r_k,\alpha_k),(r_l,\alpha_l)) = |k + \alpha_k - l - \alpha_l| \quad (4)$$

the distance between (r_k,α_k) *and* (r_l,α_l)*.*

Obviously, the smaller the value of $d((r_k,\alpha_k),(r_l,\alpha_l))$, the closer (r_k,α_k) to (r_l,α_l), thus, $d((r_k,\alpha_k),(r_l,\alpha_l))$ can be used as a deviation measure of two 2-tuples and it is very

easy to compute. The distance between (r_k, α_k) and (r_l, α_l) is more intuitive .

In the real world, the information about attribute weights is incompletely known. Let $w=(w_1, w_2, ..., w_m)^T \in H$ be the weight vector of attributes, where $w_i \in [0, 1]$, $i=1, 2, ..., m$, $\sum_{i=1}^{m} w_i = 1$, H is a set of the known weight information, which can be constructed by the following forms[12-14,19,23,32,33], for $i \neq j$.

Form1. A weak ranking: $\{w_i \geq w_j\}$;

Form2. A strict ranking: $\{w_i - w_j \geq \alpha_i (>0)\}$;

Form3. A ranking of differences: $\{w_i - w_j \geq w_k - w_l\}$, for $j \neq k \neq l$;

Form4. A ranking with multiples: $\{w_i \geq \alpha_i w_j\}$, $0 \leq \alpha_i \leq 1$;

Form5. An interval form: $\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i$, $0 \leq \alpha_i < \alpha_i + \varepsilon_i \leq 1$.

Forms 1-2 and Forms 4-5 are well known types of imprecise information, and Form 3 is ranking of differences of adjacent parameters obtained by ranking between two parameters, which can be constructed based on Form1.

3. Models and Approach

Motivated the ideal by Ref. 31 and 32, we define the following concepts, which will be used in the following sections.

Definition 7. *The overall z_j^* value of alternative x_j which the decision maker expects to achieve is called the expectation level of alternative x_j.*

Definition 8[32]**.** *Let $a_i^- = s_0 = (s_0, 0)$, for all j, then we call $a^- = (a_1^-, a_2^-, ..., a_m^-)$ the 2-tuple linguistic negative ideal point.*

Especially, we get the overall value corresponding to the linguistic negative ideal point as follows,

$$z^-(w) = \Delta \left(\sum_{i=1}^{m} w_i \Delta^{-1}(s_0, 0) \right) = s_0 \qquad (5)$$

Definition 9. *Let $f(z_j(w)) = d(z_j(w), z^-(w))$ be the distance between the overall value $z_j(w)$ of the alternative x_j and the overall value $z^-(w)$ corresponding to the linguistic negative point a^- .*

Obviously, the greater the distance between the overall value $z_j(w)$ of the alternative x_j and the overall value $z^-(w)$ corresponding to the linguistic negative point a^-, the higher value of $f(z_j(w))$, that is, the value $f(z_j(w))$ of the alternative x_j is a strictly monotone increasing function. The higher $f(z_j(w))$, the better the alternative x_j is. As a result, we establish the following optimization model:

(M-1)

$$\max \quad f(w) = (f(z_1(w), f(z_2(w)), ..., f(z_j(w)))$$

$$\text{s.t.} \quad w \in H ,$$

$$\sum_{i=1}^{m} w_i = 1 , \ w_i \geq 0 , \ i = 1, 2, ..., m .$$

Definition 10[32]**.** *Let $d(z_j^*, z^-(w))$ be the distance between the expectation level of alternative x_j and the overall value $z^-(w)$ corresponding to the linguistic negative ideal point a^- , then we call*

$$\mu(z_j(w)) = \frac{d(z_j(w), z^-(w))}{d(z_j^*, z^-(w))} \qquad (6)$$

the achievement scale of the alternative x_j.

Obviously, function $\mu(z_j(w))$ has the following features:

(1) the alternative achievement scale is that the percentage of the achieved overall value of alternative to the expectation level, which is based on the premise of the minimum of the overall value. If the overall value $z_j(w)$ farther way from the minimum, the achievement scale of alternative is higher, otherwise, the achievement scale of alternative is lower.

(2) for w', $w'' \in H$, if $d(z_j(w'), z^-(w')) > d(z_j(w''), z^-(w''))$, then $\mu(z_j(w')) > \mu(z_j(w''))$, that is, $\mu(z_j(w))$ is a strictly monotone increasing function on $z_j(w)$.

Definition 11. *Let $c(w) = \sum_{j=1}^{n} d(z_j(w), z^-(w))$, then $c(w)$ is called the alternative comprehensive scale.*

The alternative achievement scale denotes the situation of each alternative achieved. The alternative comprehensive scale denotes the overall value of all alternatives. Taking into account the above two faces, then, in the interactive decision process, we could not only achieve the decision maker's request in the overall value of all alternatives, but could lead each alternative to achieve its better status as possible, and thus rank the alternative reasonably. The main idea is following. In the first, let the alternative comprehensive scale achieve the maximum, which is to construct the following single objective model:

(M-2) $\max c(w)$

$$\text{s.t.} \quad w \in H$$

$$\sum_{i=1}^{m} w_i = 1 , \ w_i \geq 0 , \ i = 1, 2, ..., m .$$

Solving this model, we get the best solution $w^{(0)}$, the overall value of each alternative $z_j(w^{(0)})$ and the comprehensive scale $c(w^{(0)})$, and also we get the achievement scale $\mu(z_j(w^{(0)}))$ ($j=1,2,...,n$). In the course of decision making, the decision maker then provides

the initial lower bounds $\lambda_j^{(0)}$ ($j=1,2,\ldots,n$) of the achievement scale of the alternatives x_j ($j=1,2,\ldots,n$) according to $\mu(z_j(w^{(0)}))$ and the lower bound c_0 of the alternative comprehensive scale.

The better of the alternative comprehensive scale, the more satisfies to the decision maker's request in the whole of all the alternatives. But it will be lower the achievement scale of some alternatives, and thus far way from its better status. On the other hand, if we only utilize the achievement scale, it could not realize the equivalent of different alternatives effectively, and also all the alternatives are fairly competitive. As a result, we establish the following optimization model:

(M-3) $\quad \max J = \sum_{j=1}^{n} \lambda_j$

$\text{s.t.} \quad c(w) \geq c_0$

$\quad\quad \mu(z_j(w)) \geq \lambda_j \geq \lambda_j^{(0)},$

$\quad\quad w \in H , \sum_{i=1}^{m} w_i = 1 , w_i \geq 0 , i=1,2,\ldots,m .$

Solving the model (M-3), if there exists no optimal solution, then the decision maker needs to reconsider the lower bounds c_0 of the comprehensive scale and the lower bounds $\lambda_j^{(0)}$ ($j=1,2,\ldots,n$) of the achievement scale of the alternatives x_j ($j=1,2,\ldots,n$) till the optimal solution is obtained.

Theorem 1. *The optimal solution of the model (M-2) is the Pareto solution of the model (M-1).*

Proof. Suppose that w^* is the optimal solution of the model (M-2), and w^* is not the Pareto solution of the model (M-1), then there exists w' such that $f(z_j(w'))>$ $f(z_j(w^*))$, for all $x_j \in X$, and there exists $x_{j_0} \in X$, such that $f(z_{j_0}(w')) > f(z_{j_0}(w^*))$. As $c(w)$ is a strictly monotone increasing function on $f(z_j(w))$, thus

$\quad\quad c(w') > c(w^*)$

which contradicts to the supposition condition. This completes proof of Theorem 1. $\quad\quad\square$

Theorem 2 . *The optimal solution of the model (M-3) is the Pareto solution of the model (M-1).*

Proof. Suppose that w^* is the optimal solution of the model (M-3), and w^* is not the Pareto solution of the model (M-1), then there exists w' such that $f(z_j(w'))>$ $f(z_j(w^*))$, for all $x_j \in X$, and there exists $x_{j_0} \in X$, such that $f(z_{j_0}(w')) > f(z_{j_0}(w^*))$. Because $\mu(z_j(w))$ is also a strictly monotone increasing function on $f(z_j(w))$, so

$\mu(z_{j_0}(w')) > \mu(z_{j_0}(w^*))$. Then $\mu(z_j(w')) \geq \lambda_j \geq \lambda_j^{(0)}$, for all $x_j \in X$, and there exists λ_{j_0}' , such that $\mu(z_i(w'')) > \lambda_{j_0}' > \lambda_{j_0} > \lambda_j^{(0)}$. Thus

$$\sum_{j=1, j \neq j_0}^{n} \lambda_j + \lambda_{j_0}' > \sum_{j=1}^{n} \lambda_j ,$$

which contradicts to the supposition condition. This completes proof of Theorem 2. $\quad\quad\square$

Theorem 1 and Theorem 2 denote that the optimal solution of the model (M-2) and (M-3) is the Pareto solution of the model (M-1). If the decision maker satisfies the optimal solution of (M-3), then we can utilize the Eq.(3) to get the overall value $z_j(w)$ ($j=1, 2,\ldots,n$) of each alternative. Then we rank $z_j(w)$ ($j=1, 2, \ldots,n$) in descending order.

Based on the above models and theorems, we develop a practical method for solving the linguistic multiple attribute decision making problems, in which the information about attribute weights is incompletely known, and the attribute values take the form of linguistic variables, and the decision procedure need the decision maker to modify his/her request constantly until the decision maker satisfies the result. The method involves the following steps:

Step1. Let $X=\{x_1,x_2,\ldots,x_n\}$ be a discrete set of alternatives, $U=\{u_1,u_2,\ldots,u_m\}$ be a set of attributes, and $w=(w_1,w_2,\ldots,w_m)^T \in H$ be the weight vector of attributes, where $\sum_{i=1}^{m} w_i = 1$, $w_i \in [0,1]$, $i=1,2,\ldots,m$, H is a set of the known weight information, which can be constructed by the Forms 1-5. For each alternative $x_j \in X$, the decision maker gives his/her preference value a_{ij} with respect to attribute $u_i \in U$, where a_{ij} takes the form of linguistic variable, that is $a_{ij} \in S$, then all the preference values of the alternatives consist of the decision matrix $A=(a_{ij})_{m \times n}$. The decision maker gives his/her expectation level z_j^* of each alternative. Let k^* be the maximum iteration number, and $k=0$.

Step 2. Utilize the model (M-2) to derive the original optimal solution $w^{(0)} = (w_1^{(0)}, w_2^{(0)},\ldots, w_m^{(0)})^T$, then calculate the alternative comprehensive scale $c(w^{(0)})$, and also calculate the achievement scale $\mu(z_j(w^{(0)}))$ ($j=1,2,\ldots,n$). The decision maker gives the lower bounds $\lambda_j^{(0)}$ ($j=1,2,\ldots,n$) of the achievement scale of the alternatives x_j ($j=1,2,\ldots,n$) according to the achievement scale $\mu(z_j(w^{(0)}))$ ($j=$

1,2,…,*n*), and also gives the lower bound c_0 of the alternative comprehensive scale. Let $k=1$.

Step 3. Utilize the model (M-3) to derive the weight vector $w^{(k)} = (w_1^{(k)}, w_2^{(k)}, ..., w_m^{(k)})^T$ and calculate the alternative comprehensive scale $c(w^{(k)})$, and also calculate the achievement scale $\mu(z_j(w^{(k)}))$ (*j*=1,2,…,*n*).

Step 4. If the decision maker is satisfied with the result obtained by Step 3 or $k=k^*$, then calculate the overall values $z_j(w^{(k)})$ (*j*=1,2,…,*n*) of the alternatives x_j (*j*=1,2,…,*n*) by using Eq.(3), and rank all alternatives according to the values of $z_j(w^{(k)})$ (*j*=1,2,…,*n*) in descending order, and then go to Step 5; if there exists no solution for the model (M-3) or the result does not satisfy the decision maker, then the decision maker should increase the achievement scale of some alternatives, and decrease the achievement scale of some alternatives, and also if necessary, adjust the lower bound of the comprehensive scale c_k. Let $k=k+1$, and return to Step 3.

Step 5. End.

4. Illustrative Example

In this section, a MADM problem of evaluating university faculty for tenure and promotion is used to illustrate the developed procedure. The attributes used at some universities are (adapted from Xu[32]): u_1: teaching, u_2: research, and u_3:service. Five faculty candidates (alternatives) x_j (*j*=1,2,3,4,5) are evaluated using the linguistic label set

$S = \{s_0 = $ extremely poor $, s_1 = $ very poor $, s_2 = $ poor $,$

$s_3 = $ slightly poor $, s_4 = $ fair $, s_5 = $ slightly good $,$

$s_6 = $ good $, s_7 = $ very good $, s_8 = $ extremely good$\}$

by the decision maker under these three attributes, as listed in Table 1.

Suppose that the known weight information is as follows:

$H = \{0.25 \le w_1 \le 0.4, 0.15 \le w_2 \le 0.3, w_3 > w_2\}$

To get the most desirable alternative(s), the following steps are involved:

Table 1. Linguistic decision matrix A

u_i	x_1	x_2	x_3	x_4	x_5
u_1	s_6	s_7	s_4	s_8	s_6
u_2	s_7	s_8	s_7	s_3	s_7
u_3	s_4	s_2	s_6	s_6	s_5

Step 1. The decision maker gives his expectation level z_j^* (*j*=1,2,…,5) of each alternative x_j (*j*=1,2,…,5) as follows:

$z_1^* = (s_6, 0.2)$, $z_2^* = (s_7, -0.5)$, $z_3^* = (s_6, 0)$,

$z_4^* = (s_6, 0.4)$, $z_5^* = (s_7, -0.4)$.

Step 2. Utilize the model (M-2) to establish the following optimization model:

$$\max 11w_1 + 12w_2 + 3w_3 + 20$$

s.t. $0.25 \le w_1 \le 0.4$, $0.15 \le w_2 \le 0.3$, $w_3 > w_2$,

$w_1 + w_2 + w_3 = 1$, $w_j \ge 0$, $j = 1,2,3$.

Solving this model, we obtain the original optimal solution:

$$w^{(0)} = (0.4, 0.3, 0.3)^T$$

and calculate the alternative comprehensive scale $c(w^{(0)})$, and also calculate the achievement scale $\mu(z_j(w^{(0)}))$ (*j*=1,2,…,5).

$c(w^{(0)}) = 28.9$, $\mu(z_1(w^{(0)})) = 0.9194$,

$\mu(z_2(w^{(0)})) = 0.8923$, $\mu(z_3(w^{(0)})) = 0.9167$,

$\mu(z_4(w^{(0)})) = 0.9219$, $\mu(z_5(w^{(0)})) = 0.9091$.

The decision maker gives the lower bounds $\lambda_j^{(0)}$ (*j*=1,2,…,5) of the achievement scale of the alternatives x_j (*j*=1,2,…,5) according to the achievement scale $\mu(z_j(w^{(0)}))$ (*j*=1,2,…,5), and gives the lower bound of the comprehensive scale c_0 as follows:

$\lambda_1^{(0)} = 0.9$, $\lambda_2^{(0)} = 0.87$, $\lambda_3^{(0)} = 0.92$, $\lambda_4^{(0)} = 0.92$,

$\lambda_5^{(0)} = 0.9$ and $c_0 = 28.7$.

Step 3. Using model (M-3), we establish the following optimization model:

$$\max J = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$$

s.t. $(2w_1 + 3w_2 + 4)/6.2 \ge \lambda_1 \ge 0.9$,

$(3w_1 + 4w_2 - 2w_3 + 4)/6.5 \ge \lambda_2 \ge 0.87$,

$(3w_2 + 2w_3 + 4)/6 \ge \lambda_3 \ge 0.92$,

$(4w_1 - w_2 + 2w_3 + 4)/6.4 \ge \lambda_4 \ge 0.92$,

$(2w_1 + 3w_2 + w_3 + 4)/6.6 \ge \lambda_5 \ge 0.9$,

$11w_1 + 12w_2 + 3w_3 + 20 \ge 28.7$,

$0.25 \le w_1 \le 0.4$, $0.15 \le w_2 \le 0.3$, $w_3 > w_2$,

$w_1 + w_2 + w_3 = 1$, $w_j \ge 0$, $j = 1,2,3$.

Solving this model, we get the attribute weight vector:

$$w^{(1)} = (0.388, 0.296, 0.316)^T$$

and calculate the alternative comprehensive scale $c(w^{(1)})$, and also the achievement scale $\mu(z_j(w^{(1)}))$ (*j*=1, 2,…,5).

$c(w^{(1)}) = 28.768$, $\mu(z_1(w^{(1)})) = 0.9135$,

$\mu(z_2(w^{(1)})) = 0.8794$, $\mu(z_3(w^{(1)})) = 0.92$,

$\mu(z_4(w^{(1)})) = 0.92$, $\mu(z_5(w^{(1)})) = 0.9061$.

Again, the decision maker is not satisfied with the result, then he gives the new lower bounds $\lambda_j^{(1)}$ (j=1, 2,…,5) of the achievement scale of the alternatives x_j (j=1,…,5) according to the achievement scale $\mu(z_j(w^{(1)}))$ (j=1,2,…,5), and gives the new lower bound of the comprehensive scale c_1 as follows:

$\lambda_1^{(1)} = 0.9$, $\lambda_2^{(1)} = 0.86$, $\lambda_3^{(1)} = 0.93$, $\lambda_4^{(1)} = 0.9$,

$\lambda_5^{(1)} = 0.9$ and $c_1 = 28.5$.

Using model (M-3), we establish the following optimization model:

$$\max \ J = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5$$

$$\text{s.t.} \ \ (2w_1 + 3w_2 + 4)/6.2 \geq \lambda_1 \geq 0.9 \ ,$$

$$(3w_1 + 4w_2 - 2w_3 + 4)/6.5 \geq \lambda_2 \geq 0.86 \ ,$$

$$(3w_2 + 2w_3 + 4)/6 \geq \lambda_3 \geq 0.93 \ ,$$

$$(4w_1 - w_2 + 2w_3 + 4)/6.4 \geq \lambda_4 \geq 0.9 \ ,$$

$$(2w_1 + 3w_2 + w_3 + 4)/6.6 \geq \lambda_5 \geq 0.9 \ ,$$

$$11w_1 + 12w_2 + 3w_3 + 20 \geq 28.5 \ ,$$

$$0.25 \leq w_1 \leq 0.4, \ 0.15 \leq w_2 \leq 0.3, \ w_3 > w_2 \ ,$$

$$w_1 + w_2 + w_3 = 1, \ w_j \geq 0, j = 1,2,3 \ .$$

Solving this model, we obtain the attributes weight vector:

$$w^{(2)} = (0.36, 0.3, 0.34)^T \ ,$$

then calculate the alternative comprehensive scale $c(w^{(2)})$, and also the achievement scale $\mu(z_j(w^{(2)}))$ (j=1, 2,…,5).

$c(w^{(2)}) = 28.58$, $\mu(z_1(w^{(2)})) = 0.9065$,

$\mu(z_2(w^{(2)})) = 0.8615$, $\mu(z_3(w^{(2)})) = 0.93$,

$\mu(z_4(w^{(2)})) = 0.9094$, $\mu(z_5(w^{(2)})) = 0.903$.

Step 4. The decision maker is satisfied with this result. Therefore, we can calculate the overall values $z_j(w^{(2)})$ (j=1,2,…,5) of the alternatives x_j (j=1,2,…,5) using Eq.(3)

$z_1(w^{(2)}) = (s_6, -0.38)$, $z_2(w^{(2)}) = (s_6, -0.4)$,

$z_3(w^{(2)}) = (s_6, -0.42)$, $z_4(w^{(2)}) = (s_6, -0.18)$,

$z_5(w^{(2)}) = (s_6, -0.04)$.

and rank all alternatives according to the values of $z_j(w^{(2)})$ (j=1,2,…,5).

$$x_5 \succ x_4 \succ x_1 \succ x_2 \succ x_3 \ .$$

Hence, the most desirable candidate is x_5.

From the above example, we can see that the higher comprehensive scale, it will not always be the higher achievement scale of each alternative, so the DM will not be satisfied with the result. On the other hand, if we only use the achievement scale, it can not achieve the equivalent between the different alternatives effectively.

As a result, taking into account the two indexes, it will not only consider the single alternative's request, but also achieve the equivalent between the different alternatives effectively.

In Ref. 32, Xu ranked all the alternatives as $x_5 \succ x_4 \succ x_2 \succ x_1 \succ x_3$. There is minor difference between ours. But both our method and Xu's method obtain the most desirable candidate is x_5. In Xu's model, it only considers the achievement scale (he called satisfactory degree) of each alternative. The method does not consider the comprehensive scale of all the alternatives. Furthermore, the overall values of Xu's method are expressed in virtual linguistic, and the virtual linguistic computed by the linguistic index, although they could rank the alternatives, the final virtual linguistic without syntax either semantics limits the interpretability. In our model, we not only consider the achievement scale, but the comprehensive scale. In the interactive process, the decision maker can know the satisfactory degree of each alternative, and also know the comprehensive degree of all the alternatives. The interactive process can be realized by giving and revising the achievement scale and comprehensive scale till an optimum satisfactory solution is achieved. Our final results are in 2-tuple linguistic form. It seems more reasonable that linguistic decision making should be in linguistic information.

5. Concluding Remarks

In this paper, we investigate the MADM problems, in which the attribute values are expressed in linguistic variables because of time pressure, lack of knowledge, and his/her limited information processing capabilities and the information about attribute weights is partly known because that: (1) a decision should be made under time pressure and lack of data, (2) many of the attributes are intangible or non-monetary because they reflect social and environmental impacts, and (3) a DM has limited attention and information processing capabilities. In the paper, we have defined some new concepts, such as the distance between two 2-tuple linguistic variables. We also define the expectation level of alternative, the achievement scale, the alternative comprehensive scale under linguistic settings. Based on these concepts, we have established an interactive procedure for solving the problem. The interactive process can be realized by giving and revising the achievement scale and comprehensive scale. The method not only uses the objective information adequately, but also considers the DM's interactive request in the maximum extend. Finally, the method has been applied to the evaluation of university faculty for tenure and promotion. The

theoretical analysis and the computational results have showed robust method for solving the MADM problems with linguistic informati- on. In the real world, the decision process is made not only by one DM, but also is made by a group DMs, or the decision information is expressed in the form of uncertain linguistic variables[22,23,34], and the interactive procedure for multiple attribute group decision making under uncertain linguistic environment will be left for future work.

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