Research on the Numerical Method and Parametric Study of Constitutive Model of Modified Double Base Propellant

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Abstract. In order to characterize uniaxial mechanical properties of modified double base propellant, a one dimensional form of viscoelastic-viscoplastic constitutive model is derived. In order to solve the first-order ordinary differential equations (ODEs) consisted by viscoplastic flow rule and internal state variables evolution, the backward difference formulas (BDF) is applied and achieved by MATLAB procedure. The effect of viscoplastic parameters on viscoplastic creep strain is analyzed by adjusting viscoplastic parameters based on a set of the given parameters, the results show that the greater values of viscoplastic potential energy parameter Γ correspond to more flow while N correspond to more flow, the greater values of the hardening function parameters result in less flow.

Introduction

Solid rocket motors is widely used in military weapon and aerospace fields. The structural integrity of the solid propellant grain affect the safety and reliability of the rocket weapon, and is currently the critical task to solid rocket motor. In order to obtain the mechanical properties of solid rocket propellant, many scholars studied the constitutive model of the propellant. XU[1] establishes the thermal-damage-viscoelastic constitutive model to describe the mechanical properties of HTPB composite propellant. DENG[2] studies the temperature dependent viscoelastic mechanics of solid propellant by shear relaxation experiments. Modified double-base propellant, with its very complex mechanical properties is a kind of commonly used solid propellant. SUN [3] deeply analyses quasi-static and dynamic compression mechanics of modified double-base propellant, experimental results show that the yield stress, the initial modulus and the maximum strain are significantly affected by strain rate and temperature. Meng Gonglei[4] proposes a nonlinear viscoelastic constitutive model which includes cumulative damage for nonlinear mechanical behavior of modified double-base propellant.

Modified double-base propellant is a kind of granular material, experimental results show that it has viscoelastic and viscoplastic mechanics. Many scholars studied the viscoelastic and viscoplastic constitutive model of solid material, in the establishment of DARABI[5,6] establish a thermal-viscoelastic-viscoplastic constitutive model to describe the mechanical properties of heat asphalt mixtures. KIM[7] compare the Schapery-Perzyna model and Schapery-Valanis model to describe the viscoelastic and viscoplastic mechanics of particles reinforced material. ZHU[8] and ZHU[9] use backward difference formulas to solve viscoelastic - viscoplastic model in order to obtain optimum model parameters.

In this study, one dimensional form of viscoelastic-viscoplastic constitutive model of modified double base propellant is derived, and backward difference formulas is used to solve viscoplastic model. Parametric study of constitutive model focused on the creep response of modified double base propellant is analyzed.

Viscoelastic-Viscoplastic Constitutive Model

In researches on constitutive model of solid materials, the total deformation subjected to an applied stress can be decomposed into recoverable and irrecoverable components. Elastic and viscoelastic models are used to describe recoverable deformation, while plastic and viscoplastic models are employed to model irrecoverable deformation.

Experimental research shows that the deformation of modified double base propellant include recoverable and irrecoverable deformation, where the extent of each is mainly affected by time, temperature, and loading rate. In this study, the total strain of modified double base propellant is decoupled into recoverable viscoelastic strain and irrecoverable viscoplastic strain:

$$\varepsilon_{ij} = \varepsilon_{ij}^{ve} + \varepsilon_{ij}^{vp} \quad (1)$$

where ε_{ij} is the total strain tensor; ε_{ij}^{ve} is the viscoelastic strain tensor; and ε_{ij}^{vp} is the viscoplastic strain tensor.

Viscoelastic Constitutive Model

In this study, the Schapery's linear viscoelasticity theory is used to describe the viscoelastic response of modified double base propellant, when the loading stress is σ^r :

$$\varepsilon_{ij}^{ve,t} = D_0 \sigma_{ij}^t + \int_0^t \Delta D^{(\psi^t - \psi^\tau)} \frac{d(\sigma_{ij}^\tau)}{d\tau} d\tau \quad (2)$$

where D_0 is the instantaneous compliance; ΔD is the transient compliance; $\psi^t = t$ when temperature effect isn't considered. The Prony series is used to represent the transient compliance ΔD for numerical convenience, such that:

$$\Delta D^t = \sum_{n=1}^N D_n [1 - \exp(-\lambda_n t)] \quad (3)$$

Viscoplastic Constitutive Model

Differentiating Eq. (1) with respect to the time implies:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{ve} + \dot{\varepsilon}_{ij}^{vp} \quad (4)$$

where the viscoplastic strain rate is expressed as follows:

$$\dot{\varepsilon}_{ij}^{vp} = \Gamma \langle \phi(f) \rangle^N \frac{\partial g}{\partial \sigma_{ij}} \quad (5)$$

where $\langle \cdot \rangle$ is McCauley bracket, Γ is the viscoplastic viscosity parameter, ϕ is the overstress function, N is the viscoplastic rate sensitivity exponent, g is the viscoplastic potential function, overstress function is expressed as follows:

$$\phi(f) = (f / \sigma_y^0)^N \quad (6)$$

Where σ_y^0 is reference stress. The Drucker-prager yield surfaces is used:

$$f = \tau - \alpha I_1 - k(\varepsilon_e^{vp}) \quad (7)$$

where $\tau = \sqrt{3S_{ij}S_{ij}/2}$, $I_1 = \sigma_{kk}$, α is a material parameter, k is the isotropic hardening function, LEMAITRE and CHABOCHE [10] proposed a isotropic hardening model as follows:

$$k(\varepsilon_e^{vp}) = k_0 + k_1(1 - \exp(-k_2\varepsilon_e^{vp})) \quad (8)$$

where k_0 , k_1 and k_2 are material parameters, ε_e^{vp} is effective viscoplastic strain. The effective viscoplastic strain rate $\dot{\varepsilon}_e^{vp}$ is expressed as follows[11]:

$$\dot{\varepsilon}_e^{vp} = \left(1 + 2\left(\frac{0.5 + \beta/3}{1 - \beta/3}\right)\right)^{-1/2} \sqrt{\dot{\varepsilon}_{ij}^{vp}\dot{\varepsilon}_{ij}^{vp}} \quad (9)$$

It can be shown that for the uniaxial state of stresses, $\dot{\varepsilon}_e^{vp} = \dot{\varepsilon}_{11}^{vp}$.

The non-associated flow rules are commonly used to describe the viscoplasticity in the studies of material viscoplastic mechanics[11,12,13], in this study the non-associated flow rule is expressed as follows:

$$g = \tau - \beta I_1 \quad (10)$$

where β is material parameter, then one can write:

$$\frac{\partial g}{\partial \sigma_{ij}} = \frac{\partial \tau}{\partial \sigma_{ij}} - \frac{1}{3}\beta\delta_{ij} \quad (11)$$

Numerical Method of Constitutive Model

In this paper, the strain-style constitutive model is adopted, the uniaxial creep conditions are studied here for numerical convenience. Under the condition of uniaxial compression, the viscoelastic-viscoplastic model can be transformed to one dimensional form as follows:

$$\text{Fundamental equation} \quad \dot{\varepsilon}_{\text{total}} = \dot{\varepsilon}_{\text{ve}} + \dot{\varepsilon}_{\text{vp}} \quad (12)$$

$$\text{Rules of flow} \quad \begin{cases} \varepsilon_{\text{ve}} = \sigma \left\{ D_0 + \sum_{n=1}^N D_n [1 - \exp(-\lambda_n t)] \right\} \\ \dot{\varepsilon}_{\text{vp}} = (1 - \beta/3) \Gamma(f / \sigma_y^0)^N \\ f = (1 - \alpha/3) \sigma - k(\varepsilon_e^{vp}) \\ g = (1 - \beta) \sigma \end{cases} \quad (13)$$

$$\text{Internal state variables evolution laws} \quad k(\varepsilon_e^{vp}) = k_0 + k_1(1 - \exp(-k_2\varepsilon_e^{vp})) \quad (14)$$

It is obviously shown from Eq. (13) that viscoelastic strain can be obtained directly as soon as viscoelastic parameters are determined. The viscoplastic flow rule and internal state variables evolution consists of a first-order ordinary differential equations (ODEs) which cannot be treated

analytically, in this study the backward difference formulas (BDF) is applied to solve the stiff ODEs, the procedure for the determination of viscoplastic strain is discussed below.

Write $\dot{\epsilon}_{11}^{vp}$ to $\dot{y}$ implies:

$$\dot{y} = \frac{y_{i+1} - y_i}{\Delta t} = (1 - \beta/3) \Gamma \left(\frac{(1 - \alpha/3) \sigma - k(y_{i+1})}{\sigma_y^0} \right)^N \quad (15)$$

Rewrite Eq. (15) implies:

$$y_{i+1} - \Delta t (1 - \beta/3) \Gamma \left(\frac{(1 - \alpha/3) \sigma - k(y_{i+1})}{\sigma_y^0} \right)^N - y_i = 0 \quad (16)$$

Write y_{i+1} to z implies:

$$f(z) = z - \Delta t (1 - \beta/3) \Gamma \left(\frac{(1 - \alpha/3) \sigma - k(z)}{\sigma_y^0} \right)^N - y_i = 0 \quad (17)$$

Newton–Raphson scheme is used here in order to solve Eq. (17):

$$\begin{aligned} z_{new} &= z - f(z) / f'(z) \\ &= z - \frac{z - \Delta t (1 - \beta/3) \Gamma \left(\frac{(1 - \alpha/3) \sigma - k(z)}{\sigma_y^0} \right)^N - y_i}{1 + N \Delta t (1 - \beta/3) \Gamma \left(\frac{(1 - \alpha/3) \sigma - k(z)}{\sigma_y^0} \right)^{N-1} \frac{k_1 k_2 \exp(-k_2 z)}{\sigma_y^0}} \end{aligned} \quad (18)$$

The values of z getting from Eq. (18) is the uniaxial viscoplastic strain, the numerical computation wrote above can be achieved by MATLAB procedure.

Parametric study

The procedure for the determination of the viscoelastic-viscoplastic model parameters of modified double base propellant has been thoroughly discussed by WANG[14] based on creep-recovery tests. The obtained viscoelastic and viscoplastic model parameters at the reference temperature are listed in Table 1 and Table 2.

TABLE 1. Viscoelastic model parameters

n	0	1	2	3
$D_n (\text{MPa}^{-1})$	0.00369	0.00276	0.00212	0.00192
$\lambda_n (\text{s}^{-1})$		0.1	0.01	0.001

TABLE 2. Viscoplastic model parameters

Γ	k_0 (MPa)	k_1 (MPa)	k_2	N	α	β
0.0010	0.0599	5.011	20.782	2.6544	0.30	0.15

ABU AL-RUB[15] presents the results of a parametric study for all of the viscoplastic material parameters, mainly analyses the effect on strain-stress response under constant strain rate. In this section, the parametric study focus on the creep response of modified double base propellant.

In order to more clearly analyse the effect of various parameters on the viscoplastic creep strain, the viscoplastic parameters are adjusted based on the given parameters in Table 1 and Table 2. As shown in Fig. 1, Fig. (a) shows that the greater values of viscoplastic potential energy parameter Γ correspond to more flow, especially in the first creep stage; Fig. (b), Fig. (c) and Fig. (d) show that the greater values of the hardening function parameter k_0 , k_1 and k_2 result in less flow, and are best understood by understanding the hardening function k , which is shown in Eq. (7); Fig. (e) shows that the greater values of viscoplastic potential energy parameter N correspond to less flow.

Fig. (a) Effect of parameter Γ

Fig. (b) Effect of parameter k_0

Fig. (c) Effect of parameter k_1

Fig. (d) Effect of

parameter k_2

Fig. (e) Effect of parameter N

FIGURE 1. Effect of the viscoplastic parameters to viscoplastic creep strain

Conclusion

1) One dimensional form of viscoelastic-viscoplastic constitutive model of modified double base propellant is derived for the convenience of numerical calculation, which is adapted to the uniaxial creep conditions.

2) The backward difference formulas (BDF) were applied to solve the stiff ODEs consisted by the viscoplastic flow rule and internal state variables evolution, the creep strain can be obtained through MATLAB procedure.

3) Parametric study on the effect of viscoelastic material parameters show that the greater values of viscoplastic potential energy parameter Γ correspond to more flow while N correspond to more flow, the greater values of the hardening function parameter k_0 , k_1 and k_2 result in less flow.

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