

Effects of Measuring Instrument and Measuring Points on Circular Coordinate Measurement Precision

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Abstract—There are many uncertainties influencing the precision of coordinate measurement, most of which can be detected and compensated according to ISO 10360. However the effects of measuring instrument accuracy and information of measuring points including number and distribution are not duly researched. There's few reference for measurement personnel to reduce the effects. In this paper three experiments are provided to explore the effects of measuring instrument accuracy and measuring points' distribution on circular coordinate measurement precision. With the analysis of the data acquired by hundred thousand simulations with Matlab we find out the precision variation tendency of each impact factor. The analysis offers a standard of measuring instrument accuracy and measuring point's selection.

Keywords—coordinate metrology, circular feature, uncertainty, measuring point, measuring instrument.

I. INTRODUCTION

This paper deals with the effects of measuring errors on circular feature via coordinate metrology. Although other devices, like laser scanners, are also available for such a task, Coordinate Measuring Machines (CMMs) are preferred because of their superiority in terms of accuracy in the measurement of point coordinates and level of process automation. The conclusions of this paper can apply widely, nearly any metrological task where sampling is required [1] [2].

Coordinate metrology technology is an important symbol of advanced manufacturing and measurement level [3] [4]. A geometry measurement technique for error evaluation after the acquisition and geometric elements fitting of the work piece contour surface is carried out [5]. Figure 1 illustrates the principle of coordinate measurement. Measure the work piece with CMM, collecting the coordinate values of point set named P. Fitting the points and compare with the nominal element to carry out the error evaluation [6].

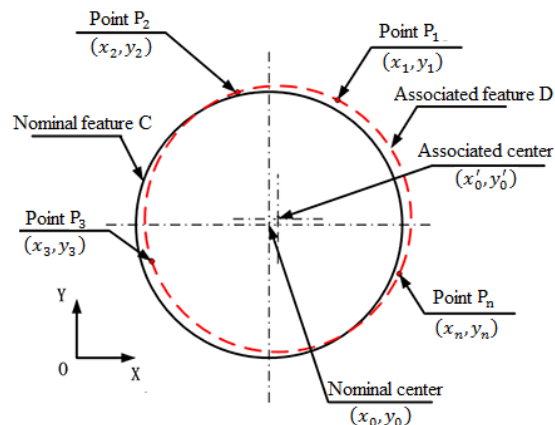


Figure 1. Principle of coordinate measurement.

Coordinate metrology is a large and complex technology. It involves many uncertainties, all of which will influence the reliability of the verification result [7]. Uncertainties in a measuring process will be a mix of known or unknown errors from a number of sources. Though sources are not the same in each case, it is still possible to take a systematic approach. There are always several sources or a combined effect of the ten different ones indicated in Figure 2 [8].

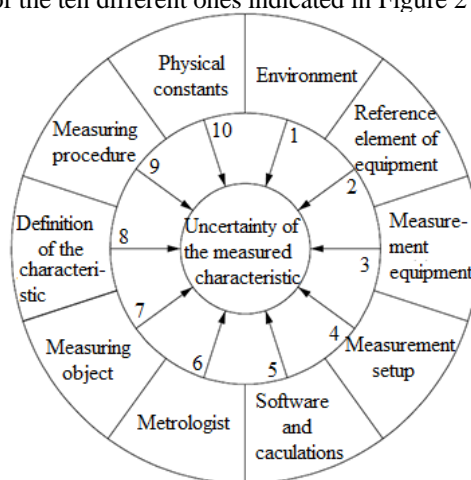


Figure 2. Uncertain components in measurement.

In general measurement most sources can be detected and compensated according to ISO 10360 [9] [10]. However

there are few written records referred to the impact of measuring instrument accuracy on the reliability of the verification result. Information of measuring points including number and distribution also has a significant impact which is not duly researched. BS 7172 [11] provides the recommended minimum number of points of each element but lacks of practical value. Cao [12] researched on the number of linear measurement points. This paper carries out three experiments to explore the impact of measuring instrument accuracy and measuring points.

II. EXPERIMENTS AND SIMULATIONS

As described in the principle of coordinate measurement, coordinate values acquisition of points is fundamental to coordinate measuring task. This paper takes the circular feature for the example. Fitting measuring points by the least square method, minimizing the sum of the squares of the distance from each measuring point to the fitted ideal datum plane, line, circle, or cylinder. This is the most popular method for the time being [13]. As shown in Figure 3, the evaluation of circle mainly includes the tolerances of dimension and location [14].

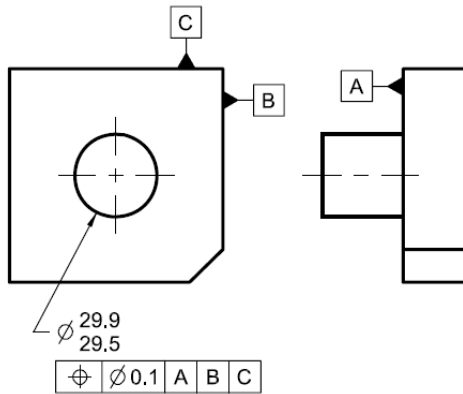


Figure 3. Tolerances of dimension and location of circular feature.

A. Experimental Method

First of all, set dimensioning tolerance, location tolerance and measuring instrument accuracy as S , Q , P . η means the proportion of location tolerance and dimensioning tolerance giving by $\eta = Q/S$. Similarly θ means the proportion of measuring instrument accuracy and dimensioning tolerance giving by $\theta = P/S$. As shown in Figure 1, C is a nominal feature and D is an associated one. The dimensioning tolerances and measuring instrument accuracy obey normal distribution [15]. Measure the circle with three measuring points at first and add one point in each iteration. Set $(x_1, y_1), (x_2, y_2), \dots, (x_i, y_i), \dots, (x_n, y_n)$ as the x and y co-ordinates of n points of the measured circle. Set center of the nominal circle is (x_0, y_0) and the diameter is d . We have to find the center (x'_0, y'_0) and diameter d' of an associated circle which best fits the measuring points [16]. The dimensioning measurement is qualified result if $d' - d$ is in the range of dimensioning tolerance. Make 100,000 simulations for each iteration with the software Matlab and

define δ as the percentage of qualified dimensioning measuring results. Similarly the location measurement is qualified result if $\sqrt{(x'_0 - x_0)^2 + (y'_0 - y_0)^2}$ is in the range of location tolerance and σ is the percentage of qualified location measuring results.

B. Uniformly Distributed Points without Instrument Accuracy

In this experiment the measuring points are uniformly distributed around the circle as shown in Figure 4. The measuring instrument accuracy is ideal, so $P = 0$ and $\theta = 0$. Measure the circle with three measuring points at first and add one point in each iteration. Evaluate the results according to the method provided in section A.

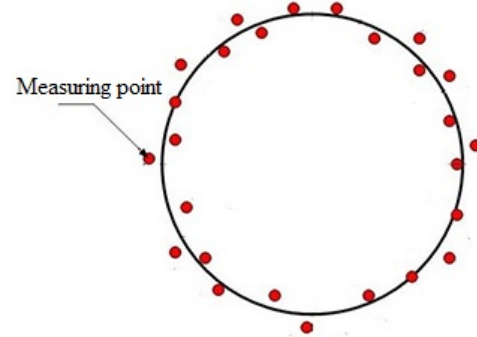


Figure 4. Uniformly distributed measuring points.

C. Uniformly Distributed Points with Instrument Accuracy

In this experiment the measuring points are still uniformly distributed around the circle. However the proportions of measuring instrument accuracy and dimensioning tolerance are listed in TABLE I. Perform the experiment according to the method provided in section A.

TABLE I. PROPORTIONS OF MEASURING INSTRUMENT ACCURACY AND DIMENSIONING TOLERANCE

No.	1	2	3	4	5	6
θ	1	2/3	1/2	1/3	1/5	1/10

D. Unevenly Distributed Points without Instrument Accuracy

The points are unevenly distributed in this experiment and the measuring instrument accuracy is ideal. In this experiment measuring points are distributed in four different ranges as shown in Figure 5. The ranges are 90° , 180° and 270° .

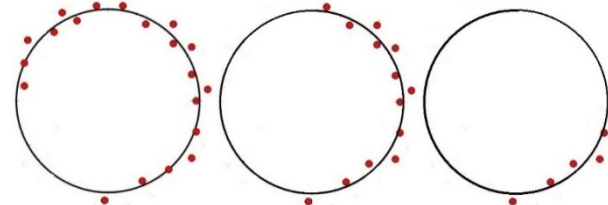


Figure 5. Unevenly distributed measuring points.

III. DATA

In experiment 1, adding the number of measuring points from three to thirty. The measuring points are uniformly distributed and the measuring instrument accuracy is ideal. The percentage of qualified dimensioning measuring results δ and the percentage of qualified location measuring results α are given in TABLE II. For the percentage of qualified location measuring results, the proportion of location tolerance and dimensioning tolerance is 1/2.

TABLE II. PERCENTAGE OF QUALIFIED DIMENSIONING AND LOCATION MEASURING RESULTS WITH INCREASING POINTS

No.	3	4	5	6	7	8
$\delta(\%)$	97.62	99.12	99.62	99.81	99.94	99.96
$\alpha(\%)$	26.91	33.93	40.28	45.92	52.12	56.38
No.	9	10	11	12	13	14
$\delta(\%)$	99.98	99.98	99.99	100	100	100
$\alpha(\%)$	60.61	63.99	68.42	71.82	73.50	76.57
No.	15	16	17	18	19	20
$\delta(\%)$	100	100	100	100	100	100
$\alpha(\%)$	78.99	81.27	82.71	84.72	86.13	87.25
No.	21	22	23	24	25	26
$\delta(\%)$	100	100	100	100	100	100
$\alpha(\%)$	88.97	90.26	91.12	92.45	92.61	93.46
No.	27	28	29	30		
$\delta(\%)$	100	100	100	100		
$\alpha(\%)$	94.21	94.67	95.22	95.79		

The data in TABLE III show the percentage of qualified location measuring results with different proportions of location tolerance and dimensioning tolerance. In the experiment the proportions value 1, 5/6, 5/7, 5/8, 5/9, 1/2.

TABLE III. PERCENTAGE OF QUALIFIED LOCATION MEASURING RESULTS WITH DIFFERENT PROPORTIONS OF LOCATION TOLERANCE AND DIMENSIONING TOLERANCE

η $\alpha(\%)$ No.	1	5/6	5/7	5/8	5/9	1/2
3	70.98	58.18	47.53	39.65	31.30	26.91
4	81.44	68.73	56.54	47.41	39.51	33.93
5	87.75	76.94	65.63	55.34	53.52	40.28
6	91.99	82.24	71.66	61.99	47.00	45.92
7	94.36	87.13	77.15	68.25	58.70	52.12
8	95.89	89.97	81.82	72.72	64.60	56.38
9	97.67	93.18	85.65	77.42	68.03	60.61
10	98.50	94.69	88.36	79.98	71.97	63.99
11	99.05	95.86	90.33	83.19	76.49	68.42
12	99.31	96.69	92.13	85.99	78.22	71.82
13	99.56	97.58	93.89	87.70	80.59	73.50
14	99.76	98.31	95.35	89.66	83.48	76.57
15	99.87	98.70	96.34	91.21	85.20	78.99

In experiment 2, Set $\eta = 1/2$, the proportions of measuring instrument accuracy and dimensioning tolerance value 1, 2/3, 1/2, 1/3, 1/5, 1/10. The percentages of qualified dimensioning measuring results δ are shown in TABLE IV. And the percentages of qualified location measuring results α are shown in TABLE V.

TABLE IV. PERCENTAGE OF QUALIFIED DIMENSIONING MEASURING RESULTS WITH PROPORTIONS OF MEASURING INSTRUMENT ACCURACY AND DIMENSIONING TOLERANCE

θ $\delta(\%)$ No.	1	1/2	1/3	1/5	1/10
3	92.87	96.16	97.03	97.25	97.34
4	96.60	98.69	98.74	99.02	99.04
5	98.14	99.35	99.55	99.65	99.55
6	98.88	99.68	99.70	99.90	99.76
7	99.42	99.90	99.93	99.94	99.96
8	99.71	99.94	99.95	99.98	99.98
9	99.82	99.96	99.96	99.99	99.99
10	99.96	100.00	99.99	100.00	99.99
11	99.97	100.00	100.00	100.00	100.00
12	99.98	100.00	100.00	100.00	100.00
13	100.00	100.00	100.00	100.00	100.00
14	100.00	100.00	100.00	100.00	100.00
15	100.00	100.00	100.00	100.00	100.00

TABLE V. PERCENTAGE OF QUALIFIED LOCATION MEASURING RESULTS WITH PROPORTIONS OF MEASURING INSTRUMENT ACCURACY AND DIMENSIONING TOLERANCE

θ $\alpha(\%)$ No.	1	2/3	1/2	1/3	1/5	1/10
3	19.24	23.05	23.72	25.97	26.72	27.39
4	24.66	28.96	31.29	32.66	33.43	34.43
5	29.56	34.64	37.21	39.64	40.76	40.16
6	33.18	39.74	41.73	44.02	45.13	45.98
7	38.45	45.39	46.60	49.62	50.54	52.05
8	42.06	49.71	51.76	55.13	56.46	56.19
9	46.49	53.94	56.52	59.35	60.61	60.75
10	50.17	57.11	59.70	63.23	64.64	64.37
11	53.23	61.55	63.29	65.40	66.42	68.09
12	56.97	63.75	67.16	69.10	70.21	70.67
13	59.31	67.46	70.13	72.04	73.42	74.04
14	61.39	68.85	72.56	74.76	75.87	76.32
15	64.84	72.56	75.49	77.35	78.40	79.12

In experiment 3, the measuring points are unevenly distributed and the measuring instrument accuracy is ideal. Measure the circle with three measuring points at first and the distribution ranges of the measuring points are 90° , 180° and 270° . Then add the number of measuring points from three to thirty. The percentages of qualified dimensioning measuring results δ are listed in TABLE VI. Set $\eta = 1/2$,

the percentages of qualified location measuring results σ are listed in TABLE VII.

TABLE VI. PERCENTAGE OF QUALIFIED DIMENSIONING MEASURING RESULTS WITH UNEVENLY DISTRIBUTED MEASURING POINTS

$R(\%)$ $\delta(\%)$ No.	90	180	270	360
3	29.56	92.72	97.30	97.62
4	30.53	92.82	98.94	99.12
5	31.46	93.58	99.53	99.62
6	32.17	94.06	99.81	99.81
7	33.51	94.93	99.89	99.94
8	34.92	95.74	99.97	99.96
9	35.37	96.28	99.99	99.98
10	36.46	96.78	99.98	99.98
11	38.45	97.37	99.97	99.99
12	38.69	97.85	99.99	100.00
13	40.08	98.26	100.00	100.00
14	41.34	98.39	100.00	100.00
15	42.05	98.58	100.00	100.00

TABLE VII. PERCENTAGE OF QUALIFIED LOCATION MEASURING RESULTS WITH UNEVENLY DISTRIBUTED MEASURING POINTS

$R(\%)$ $\sigma(\%)$ No.	90	180	270	360
3	4.83	21.72	25.28	26.91
4	5.01	24.44	33.54	33.93
5	5.77	27.43	40.00	40.28
6	6.75	30.21	45.35	45.92
7	7.43	32.79	50.10	52.12
8	7.78	34.76	54.35	56.38
9	7.77	38.37	58.07	60.61
10	9.20	38.84	61.89	63.99
11	9.66	41.97	65.71	68.42
12	10.82	45.02	68.22	71.82
13	10.97	46.91	70.00	73.50
14	11.42	49.42	72.82	76.57
15	11.98	50.14	74.98	78.99

IV. RESULT AND DISCUSSION

The effect of measuring points' number on the dimensioning tolerance and location tolerance is shown in Figure 6 and Figure 7.

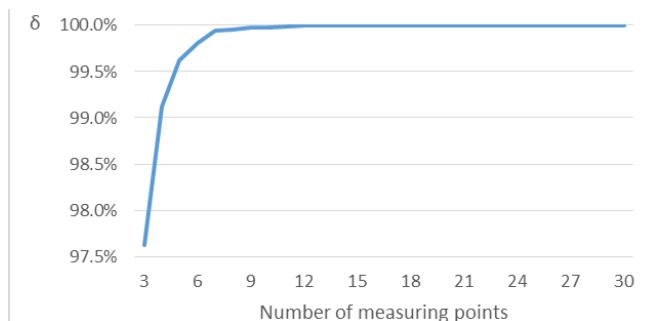


Figure 6. Effect of measuring points' number on dimensioning tolerance

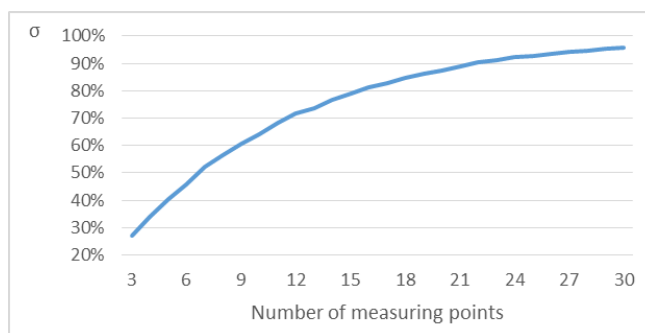


Figure 7. Effect of measuring points' number on location tolerance.

For the case of dimensioning tolerance, the percentage of qualified dimensioning measuring results can reach 97.62% with three measuring points and there are only small differences between three points and thirty points. But for location tolerance the percentage of qualified location measuring results is just 26.91% and will be over 90% with more than twelve points. The effect on the dimensioning tolerance and location tolerance both increase with increasing number of measuring points. The effect on dimensioning tolerance is weak while the effect on location tolerance is strong enough to be mentioned.

The effect of the proportions of location tolerance and dimensioning tolerance on location tolerance is shown in Figure 8. The percentages of qualified location measuring results increase with increasing value of η . When the proportion is less than 1/2, the control of location tolerance measurement accuracy is much more difficult.

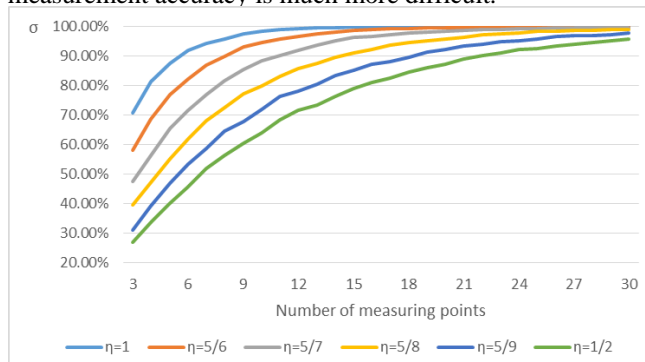


Figure 8. Effect of proportions of location and dimensioning tolerance on location tolerance.

The effect of measuring instrument accuracy on the dimensioning tolerance and location tolerance is shown in

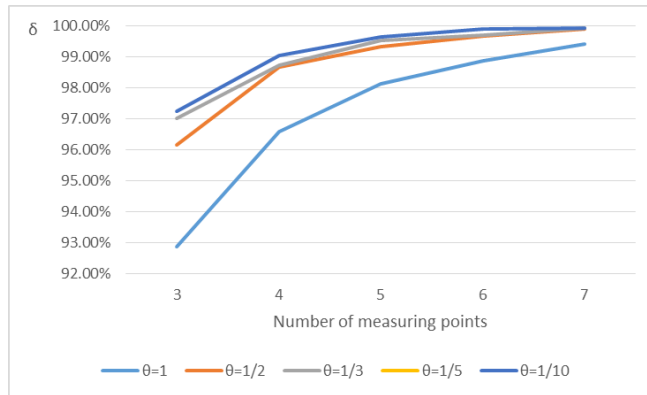


Figure 9. Effect of measuring instrument accuracy on dimensioning.

Figure 8.

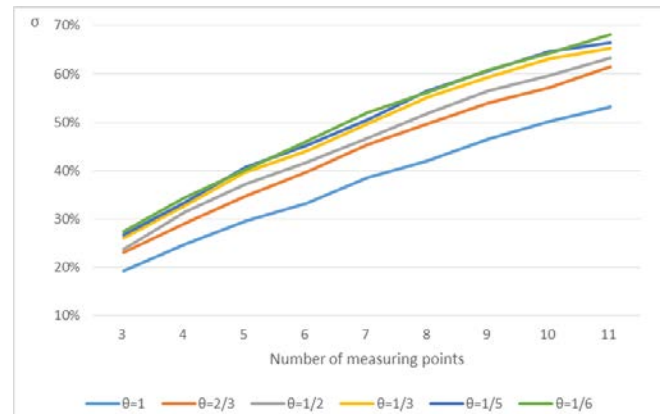


Figure 10. Effect of measuring instrument accuracy on location tolerance.

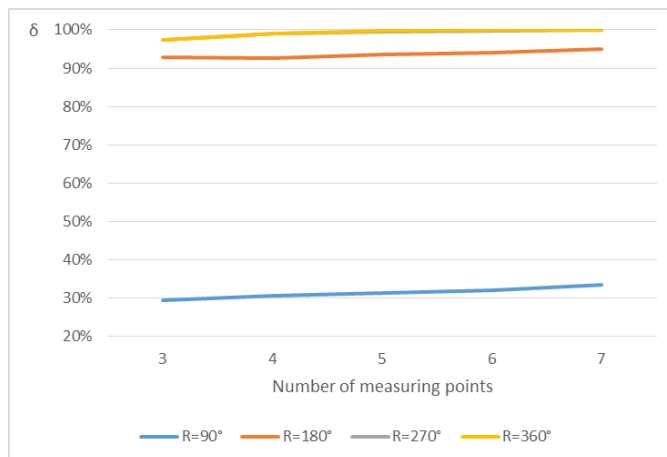


Figure 11. Effect of distribution range on dimensioning tolerance.

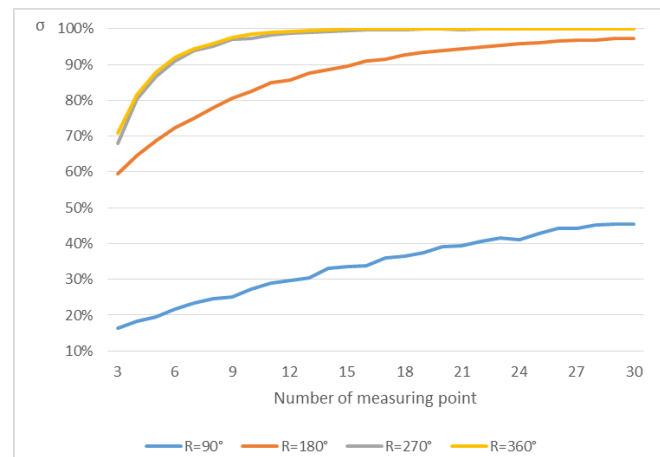


Figure 12. Effect of distribution range on location tolerance.

As shown in Figure 9, the percentage of qualified dimensioning measuring results can reach 99.93% with seven measuring points when θ values 1/3. For ideal measuring instrument accuracy, the percentage is 99.94%. For the case of location tolerance as shown in Figure 10, the percentage of qualified location measuring results is 63.23% with ten measuring points when θ values 1/3. While the percentage with ideal measuring instrument accuracy is 63.99%. According to analysis results, for measuring the proportion of measuring instrument accuracy and dimensioning tolerance should be 1/3~1/10.

The effect of distribution range on the dimensioning tolerance and location tolerance is shown in Figure 11 and Figure 12.

As shown in Figure 11 and Figure 12, for both the dimensioning and location tolerance, the percentage increases with the increasing distribution range. When the range is less than 180° , the measurement will lead to an unreliable result. The measuring distribution range should be as wide as possible.

V. CONCLUSION

The effects of measuring instrument and measuring points on measurement precision have been studied with three experiments in this paper. These can provide some guidance in selection of the accuracy of measuring instrument and distribution of measuring points. The uncertainty decreases with increasing number of measuring points. The paper provides the probable percentages of qualified results for measurement selection. For the case of measuring distribution range, the range should be as wide as possible. 1/3~1/10 is the recommended range for the proportion of measuring instrument accuracy and dimensioning tolerance. With the achievements in this paper and ISO 10360, the uncertainty of circle measurement can be controlled in an acceptable range. Further studies are required to investigate the effects on other features.

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