Synchronization of Underwater Glider with Uncertainties Using Adaptive Fuzzy Sliding Mode Control

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Abstract—In this paper we study the synchronization of an underwater glider by means of the adaptive fuzzy sliding mode control technology. The sufficient conditions for the synchronization are obtained even though the slave underwater glider is unknown. Based on the Lyapunov stability theorem, the stability the synchronization is proved theoretically. Numerical simulations further support the theoretical results of the paper and demonstrate the efficiency of the proposed methods.

Keywords—underwater glider; adaptive fuzzy sliding mode control; synchronization; master-slave system

I. INTRODUCTION

Over the past two decades, an underwater glider has been widely used in the marine-environment monitoring, the unenclosed aquaculture and ocean exploration, especially in the long-term, large-scale oceanographic monitoring [1]. The UG has a number of attractive features, such as a high efficiency, a low noise, a large range, lower maintenance cost, lower pollution and a compact structure [2], etc. Besides their numerous practical applications, AUVs present a challenging control problem since the kinematic dynamic models of AUV are highly nonlinear and coupled.

Recently, the control of the underwater glider has become an active research field and the various control techniques have been reported for the control of the underwater glider [3-5]. For example, an optimal applicable control project is developed for controlling the actual mechanical system of the glider [3]. Feedback control is introduced for the glider motion robust of the autonomous underwater gliders [4]. In addition, the synchronization is important for the underwater glider. For example, it is critical in sampling water columns, mapping the whole aquatic environment and seeking pollutant sources. Precise synchronization is very challenging due to dynamic nonlinearity and strong coupling among multiple control inputs. Despite that several scholars [6-7] have investigated the synchronization of general nonlinear system, it should be worth noting that these studies do not focus on the underwater glider. Therefore, it is still important and meaningful to design a novel controller for the synchronization of the underwater glider.

Inspired by the above-mentioned work, in this paper, we investigate the synchronization between the known master underwater glider and the unknown slave underwater glider.

The main difficulty to achieve this goal is how to overcome the bad effects of little knowledge about the slave underwater glider on the synchronization. An adaptive fuzzy sliding mode controller is designed to cope with the above difficulty and construct the synchronization of the unknown slave underwater glider.

The rest of this paper is organized as follows. The dynamic equation of an underwater glider is represented in Section II. Synchronization of an underwater glider is investigated theoretically and numerically by using the adaptive fuzzy sliding mode control method in Section III. Finally in Section VI, the conclusions are given.

II. SYNCHRONIZATION OF UNDERWATER GLIDER

A. Dynamic Equation of Underwater Glider

The dynamical equation of an underwater glider can be described as [8]

$$\begin{cases} \dot{x}_1 = -\frac{X_u}{m_{11}}x_1 - (\frac{X_{u|u|}}{m_{11}}x_1|x_1| - \frac{m_{22}}{m_{11}}x_2x_3) \\ \dot{x}_2 = -\frac{Y_v}{m_{22}}x_2 - (\frac{Y_{v|v|}}{m_{22}}x_2|x_2| + \frac{m_{11}}{m_{22}}x_1x_3) \\ \dot{x}_3 = -\frac{N_r}{m_{33}}x_3 - (\frac{N_{r|r|}}{m_{33}}x_3|x_3| - \frac{m_{11}-m_{22}}{m_{33}}x_1x_2) \end{cases} \quad (1)$$

where $m_{11} = m - X_u$, $m_{22} = m - Y_v$, $m_{33} = I_z - N_r$. In this paper, the values of parameters in the system (1) are respectively set as

$$X_u = 70 \text{ kg/s}, X_{u|u|} = 100 \text{ kg/m}, Y_v = 100 \text{ kg/s},$$

$$Y_{v|v|} = 200 \text{ kg/m}, N_r = 50 \text{ kgm}^2/\text{s}, N_{r|r|} = 100 \text{ kgm}^2,$$

$$\text{so } m_{11} = 215 \text{ kg}, m_{22} = 265 \text{ kg}, m_{33} = 80 \text{ kgm}^2.$$

For the concision of the contents and the convenience of readers, we define the following parameters

$$\begin{cases} f_1(\mathbf{x}, t) = -\frac{X_u}{m_{11}} x_1 - \left(\frac{X_{u|u|}}{m_{11}} x_1 |x_1| - \frac{m_{22}}{m_{11}} x_2 x_3\right) \\ f_2(\mathbf{x}, t) = -\frac{Y_v}{m_{22}} x_2 - \left(\frac{Y_{v|v|}}{m_{22}} x_2 |x_2| + \frac{m_{11}}{m_{22}} x_1 x_3\right) \\ f_3(\mathbf{x}, t) = -\frac{N_r}{m_{33}} x_3 - \left(\frac{N_{r|r|}}{m_{33}} x_3 |x_3| - \frac{m_{11} - m_{22}}{m_{33}} x_1 x_2\right) \end{cases}$$

and

$$\mathbf{f} = [f_1(\mathbf{x}, t), f_2(\mathbf{x}, t), f_3(\mathbf{x}, t)]^T \in \mathbb{R}^{3 \times 1}$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$.

So Eq. (1) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \quad (2)$$

B. Theoretical Results of Synchronization

Let us define the master underwater glider as (2), and define the slave underwater glider as follow

$$\dot{\mathbf{y}} = \mathbf{g}(\mathbf{y}, t) + \mathbf{b}\mathbf{u} + \mathbf{d} \quad \mathbf{y} \in \mathbb{R}^{n \times 1} \quad (3)$$

where $\mathbf{u} = (u_1, u_2, \dots, u_n)^T \in \mathbb{R}^{n \times 1}$ is a control input.

$\text{diag}(\cdot)$ is a diagonal function. $\mathbf{d} = (d_1, d_2, \dots, d_n)^T \in \mathbb{R}^{n \times 1}$ is

the external disturbance. It is assumed that $|d_i| \leq \bar{d} < +\infty$ for

all t , where $\bar{d}$ is not necessarily known previously.

$\mathbf{y} = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^{n \times 1}$ is the state vector of the slave system.

$\mathbf{g}(\mathbf{y}, t) = [g_1(\mathbf{y}, t), g_2(\mathbf{y}, t), \dots, g_n(\mathbf{y}, t)]^T \in \mathbb{R}^{n \times 1}$ and

$$\begin{cases} g_1(\mathbf{y}, t) = -\frac{X_u}{m_{11}} y_1 - \left(\frac{X_{u|u|}}{m_{11}} y_1 |y_1| - \frac{m_{22}}{m_{11}} y_2 y_3\right) \\ g_2(\mathbf{y}, t) = -\frac{Y_v}{m_{22}} y_2 - \left(\frac{Y_{v|v|}}{m_{22}} y_2 |y_2| + \frac{m_{11}}{m_{22}} y_1 y_3\right) \\ g_3(\mathbf{y}, t) = -\frac{N_r}{m_{33}} y_3 - \left(\frac{N_{r|r|}}{m_{33}} y_3 |y_3| - \frac{m_{11} - m_{22}}{m_{33}} y_1 y_2\right) \end{cases}$$

The synchronization error between (2) and (3) is defined as follow

$$\mathbf{e} = \mathbf{y} - \mathbf{x} \quad (4)$$

where $\mathbf{e} = (e_1, e_2, \dots, e_n)^T \in \mathbb{R}^{n \times 1}$.

Then, from (4), we can obtain the error dynamic equation as following

$$\dot{\mathbf{e}} = \mathbf{g}(\mathbf{y}, t) - \mathbf{f}(\mathbf{x}, t) + \mathbf{u} + \mathbf{d} \quad (5)$$

The object of this paper is that for the given master-slave system with uncertainties, such as (2) and (3), the adaptive fuzzy sliding mode controllers are designed in spite of external disturbances, so that the asymptotic stability of system (5) can be achieved in the sense that

$$\lim_{t \rightarrow +\infty} e = 0 \quad \text{and} \quad \lim_{t \rightarrow +\infty} \dot{e} = 0 \quad (6)$$

The switching sliding surface is defined as $s_i(t) = c_i e_i(t)$ ($i = 1, 2, 3$), where c_i is a positive constant. Then the first derivative of $s_i(t)$ with respect to time t is

$$\dot{s}_i(t) = c_i [g_i(\mathbf{y}, t) - \lambda_i f_i(\mathbf{x}, t) + u_i + d_i] \quad (i = 1, 2, 3) \quad (7)$$

It is generally known that when the system operates in the sliding mode, it satisfies the following equations^[9]

$$s_i(t) = 0 \quad \text{and} \quad \dot{s}_i(t) = 0 \quad (i = 1, 2, 3) \quad (8)$$

Therefore, the equivalent controller is

$$u_i = b_i^{-1} (-g_i + f_i - d_i) \quad (i = 1, 2, \dots, n) \quad (9)$$

However, in real-world applications, if both the system (3) is unknown, the equivalent controller (9) is failure. To control and identify the system (3), the fuzzy systems $\hat{g}_i(\mathbf{y}, t)$ and $\hat{d}_i$ are used to approach the functions $g_i(\mathbf{y}, t)$ and d_i , respectively. In this paper, the fuzzy rules of the fuzzy systems are defined as follows

R(j): If y_i is $A_1^{(j)}$ and $D_t^{q_i} y_i$ is $A_2^{(j)}$, then $\hat{g}_i$ is $G^{(j)}$;

If s_i is $C^{(j)}$, then $\hat{h}_i$ is $H^{(j)}$.

Based on the singleton fuzzifier, the product inference, and the weighted average defuzzifier^[10], the fuzzy systems

$\hat{g}_i(y_i, D_t^{q_i} y_i)$ and $\hat{h}_i(s_i)$ are respectively described by

$$\hat{g}_i = \alpha_i^T \xi \left(y_i, \dot{y}_i \right) \quad \text{and} \quad \hat{h}_i = \gamma_i^T \phi(s_i)$$

where α_i and γ_i vary with the designed adaptive update laws. Both $\xi(\cdot)$ and $\phi(\cdot)$ are the fuzzy vectors and defined respectively as follows

$$\xi(y_i, \dot{y}_i) = (\xi^{(1)}(y_i, \dot{y}_i), \xi^{(2)}(y_i, \dot{y}_i), \dots, \xi^{(N)}(y_i, \dot{y}_i))^T$$

and

$$\phi(s_i) = (\phi^{(1)}(s_i), \phi^{(2)}(s_i), \dots, \phi^{(N)}(s_i))^T,$$

where
$$\xi^{(j)}(y_i, \dot{y}_i) = \frac{\mu_{A_1^{(j)}}(y_i) \mu_{A_2^{(j)}}(\dot{y}_i)}{\sum_{j=1}^N \mu_{A_1^{(j)}}(y_i) \mu_{A_2^{(j)}}(\dot{y}_i)}$$

and

$$\phi^{(j)}(s_i) = \frac{\varphi_{C^{(j)}}(s_i)}{\sum_{j=1}^N \varphi_{C^{(j)}}(s_i)}$$

$\mu_{A_1^{(j)}}(y_i)$, $\mu_{A_2^{(j)}}(\dot{y}_i)$ and $\varphi_{C^{(j)}}(s_i)$ are respectively the membership functions of y_i , $\dot{y}_i$ and s_i . N is the number of the fuzzy rules.

Thus, according to the linear feedback method, the controller is redesigned as following

$$u_i = \hat{b}_i^{-1}(-\hat{g}_i + f_i - \hat{h}_i) \quad (i=1,2,3) \quad (10)$$

To describe the new design and analysis, the following optimization parameters are needed

$$\alpha_i^* = \arg \min_{\alpha_i \in \Omega_g} \left[\sup_{y_i \in R} \left| \hat{g}_i[(y_i, D_t^{q_i} y_i) | \alpha_i] - g(y, t) \right| \right] \quad (11a)$$

$$\gamma_i^* = \arg \min_{\gamma_i \in \Omega_\gamma} \left[\sup_{s_i \in R} \left| \hat{h}_i(s_i | \gamma_i) - (\bar{d} + \delta) \text{sgn}(s_i) \right| \right] \quad (11b)$$

where $i=1,2,\dots,n$. Ω_g and Ω_γ are the sets of α_i and γ_i , respectively, and δ is a small positive number and $\text{sgn}(\cdot)$ is a symbol function.

The minimum approaching error is defined as follow $w_i = g_i - \hat{g}_i^*$, where $\hat{g}_i^* = \hat{g}_i[(y_i, \dot{y}_i) | \alpha_i^*]$. Then Eq. (7) can be rewritten as

$$\dot{s}_i = g_i - f_i + u_i + d_i = w_i + \tilde{\alpha}_i^T \xi(y_i, \dot{y}_i) + \tilde{\gamma}_i^T \phi(s_i) - \hat{h}_i^* + d_i$$

where $\tilde{\alpha}_i = \alpha_i^* - \alpha_i$ and $\tilde{\gamma}_i = \gamma_i^* - \gamma_i$.

Theorem: When $g_i(y, t)$ and d_i are unknown, and

$$\hat{h}_i(s_i | \gamma_i^*) = (\bar{d} + \delta) \text{sgn}(s_i) \quad (\delta \geq 0) \quad \text{and} \quad s_i \in R$$

$|d_i| \leq \bar{d}$ ($i=1,2,\dots,n$), if the controllers are designed as

(10) and the adaptive update laws satisfy the following equations

$$\begin{cases} \dot{\alpha}_i = r_i s_i \xi(y_i, D_t^{q_i} y_i) \\ \dot{\gamma}_i = \rho_i s_i \phi(s_i) \end{cases} \quad (12)$$

then systems (2) will synchronize systems (3).

Proof:

Construct a Lyapunov error function of the following

$$V = \sum_{i=1}^n V_i \quad (13)$$

where

$$V_i = s_i^2 + \frac{1}{2r_i} \tilde{\alpha}_i^T \tilde{\alpha}_i + \frac{1}{2\rho_i} \tilde{\gamma}_i^T \tilde{\gamma}_i \quad (14)$$

Then the first derivative $\dot{V}_i$ of with respect to time t is

$$\begin{aligned} \dot{V}_i = & s_i w_i + \tilde{\alpha}_i^T \left[s_i \xi(y_i, \dot{y}_i) - \frac{1}{r} \dot{\alpha}_i \right] \\ & + \tilde{\gamma}_i^T \left[s_i \phi(s_i) - \frac{1}{\rho_i} \dot{\gamma}_i \right] - (\bar{d}_i + \delta) |s_i| + d_i s_i \end{aligned} \quad (15)$$

where $\dot{\tilde{\alpha}}_i = -\dot{\alpha}_i$ and $\dot{\tilde{\gamma}}_i = -\dot{\gamma}_i$.

Substituting Eq.(12) into Eq.(15), we yield the equation

$$\dot{V}_i = s_i w_i - (\bar{d}_i + \delta) |s_i| + d_i s_i$$

According to a fuzzy approaching theory^[11], an adaptive update fuzzy system can accurately approach any unknown system, so the minimum approaching error w_i can approach zero.

$$\text{So } \dot{V}_i \leq s_i w_i + (|d_i| - \bar{d}_i - \delta) |s_i| < 0,$$

$$\text{then } \dot{V} = \sum_{i=1}^n \dot{V}_i \leq 0.$$

So $\dot{V} \leq 0$ confirms the presence of the reaching condition^[12], and means that s_i is toward the switching surface $s_i = 0$ ($i=1,2,\dots,n$). Furthermore, since the sliding modes are asymptotically stable, it clearly shows that the error dynamic responses of $\lim_{t \rightarrow +\infty} e = 0$ and $\lim_{t \rightarrow +\infty} D_t^q e = 0$ are satisfied. Hence, the proof is achieved completely.

C. Numerical Simulation Results of Synchronization

In this section, the numerical examples about the synchronization of an underwater glider will be provided. When numerically solving the equation of an underwater glider, we adopt the fourth order Runge-Kutta method^[13].

α_i is defined as 25×1 vectors, and γ_i is defined as 3×1 vectors, the initial values of α_i and γ_i are set as 0.1. The initial conditions of the master underwater glider and the slave underwater glider are selected as $(x_1(0), x_2(0), x_3(0)) = (-0.3, 0.5, 0.6)$ and $(y_1(0), y_2(0), y_3(0)) = (0.1, -0.1, -0.5)$, respectively. $(c_1, c_2, c_3) = (2, 5.5, 4)$. The coefficients of the adaptive update laws are set as $(r_1, r_2, r_3) = (2, 10, 30)$, $(p_1, p_2, p_3) = (1000, 200, 30)$ and $(\rho_1, \rho_2, \rho_3) = (1000, 200, 80)$, respectively.

Five kinds of membership degree functions are used to haze the errors variables y_i , y_i and s_i respectively defined as follows

$$\begin{aligned} \mu_{NM}(z) &= \exp\left[-\left(z + \frac{\pi}{6}\right)^2 / \frac{\pi^2}{24^2}\right], \mu_{NS}(z) = \exp\left[-\left(z + \frac{\pi}{12}\right)^2 / \frac{\pi^2}{24^2}\right] \\ \mu_{ZO}(z) &= \exp\left[-z^2 / \frac{\pi^2}{24^2}\right], \mu_{PS}(z) = \exp\left[-\left(z - \frac{\pi}{12}\right)^2 / \frac{\pi^2}{24^2}\right] \\ \text{and } \mu_{PM}(z) &= \exp\left[-\left(z - \frac{\pi}{6}\right)^2 / \frac{\pi^2}{24^2}\right]. \end{aligned}$$

The membership degree functions of the switching sliding surface s_i ($i = 1, 2, 3$) are defined as follows

$$\begin{aligned} \varphi_{NM}(z) &= 1 / [1 + \exp(5z + 15)], \varphi_{ZO}(z) = \exp(-z^2) \text{ and} \\ \varphi_{PM}(z) &= 1 / [1 + \exp(5z - 15)] \end{aligned}$$

Figure 1. With external disturbances: (a) the temporal evolution of synchronization errors; (b) the temporal evolution of synchronization errors.

To verify and demonstrate the robustness of the controlled system for the external disturbances, the time-varying external disturbances (16) are used to disturb the unknown slave system. The synchronization errors between the known master underwater glider and the unknown slave underwater glider are depicted in figure 1(a). By observing Fig.1(a), one can find that the synchronization errors are hardly affected by the external disturbances and converge to zero asymptotically, which implies that the synchronization between the known underwater glider and the unknown

underwater glider can be well realized even though the unknown slave underwater glider is subject to the uncertain external disturbances. The temporal evolution of the applied control inputs is depicted in figure 1(b). Therefore, the designed controllers (9) and (11) can resist all kinds of external disturbances and the synchronization is well achieved.

III. CONCLUSION

In this paper, we have studied the synchronization of the underwater glider, where the slave underwater glider is not necessarily known previously. Based on the designed adaptive update laws, the fuzzy system and the sliding mode strategy have been combined into an adaptive fuzzy sliding mode control strategy. To illustrate the effectiveness and feasibility of the proposed method, the numerical examples about the synchronization of an underwater glider are provided. The proposed scheme in this paper is remarkable, since it is valid even though the slave underwater glider is fully unknown. Therefore, the results of this paper are also very applicable and representative. We hope our work can contribute to the control of the underwater glider.

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