Image Enhancement Technology Based on Fuzzy Set Theory
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Abstract. Images will lose some of local features by using traditional image enhancement technology. So this paper introduces the fuzzy set theory to transform the image smoothly in local areas. By considering sub-peak effect, we give one more channel to strengthen local features. In this way, images are enhanced with less distortion from analyzing gray level distribution curve and subjective sensing. Therefore, fuzzy set theory is more suitable in image enhancement, for its ability to evaluate images continuously.

Introduction
Image enhancement technology is one of the most important topics in image processing technology. There are such already existing image enhancement methods as Gamma transformation, logarithm transformation and histogram equalization. The most obvious weakness of these methods is that they enhance image in a large scale, so the local detail is blurred. Recent years, researchers have already applied fuzzy theory to image enhancement. The two main reasons are: the mechanism of human brain to fetch image features is driven by fuzzy effect and the uncertainty of image is engendered by fuzziness.

Fuzzy Set Theory in Image Enhancement
Fuzzy set theory is firstly introduced by Zadeh[1965]. Let $Z$ be a set of a kind of element, $Z = \{z\}$. The fuzzy set $A$ of $Z$ is defined as a pair of the value of $z$ and the membership degree $\mu_A(z) \in [0,1]$, as $A = \{z, \mu_A(z) | z \in Z\}$. This concept provides a smooth gray value transition, thus retaining the local detail information. Firstly, we define three input fuzzy sets--dark, gray and bright. Then we apply the following ‘IF-THEN’ law to get the fuzzy output set.

- $R_1$: IF a pixel is dark, THEN we guess it is darker;
- $R_2$: IF a pixel is gray, THEN we guess it should have an intensified degree;
- $R_3$: IF a pixel is bright, THEN we guess it is brighter;

For a given pixel $z_0$, let $A_i$ and $\tilde{A}_j$ be the input and output fuzzy set of gray level membership function, $Q_i(v)$ be the guessing membership function. ($i=$dark, gray, bright & $j=$darker, grayer, brighter). Thus, $Q_i(v) = A_i \cap \tilde{A}_j = \min\{\mu_i(z_0), \mu_j(z_0, v)\}$. In order to get all the responses in the range of $v$, let the final response $Q(v) = \bigcup_{i=A} Q_i(v)$, which equals to $\max_i\{Q_i(v)\}$. Finally, we remove fuzziness by computing the gravity center of fuzzy set $Q(v)$, the final output: $v_0 = \frac{\sum v Q(v)}{\sum Q(v)}$.

Image Transformation by Using Fuzzy Set Theory
The Contrast between Fuzzy Set Theory and Tradition Transformations. To illustrate the effect by using fuzzy set theory, we choose the following input and output membership functions:
In this case, output membership function is constant to the fuzzy input, thus to make the final output as:

$$v_0 = \frac{v_d \times \mu_{dark}(z_0) + v_g \times \mu_{gray}(z_0) + v_b \times \mu_{bright}(z_0)}{\mu_{dark}(z_0) + \mu_{gray}(z_0) + \mu_{bright}(z_0)}$$

Then, we separately use histogram equalization and gama transformation to enhance the image. Gama is set as 1.2 in gama transformation to enhance the original gray level. And we can get such enhanced image as following:

![Enhanced images by different methods](image)

We use the gray value distribution diagram to analyse the effect of each enhancement method:

![Gray value distribution of respectively enhancement method](image)

From the chart, it can be found that by using fuzzy set enhancement, the gray value interval is expanded from \([32,178]\) to \([0,255]\), which intensifies image discrimination. Meanwhile, gray value of image is transformed in a smooth transition, which utmostly retains image details, rather than
excessively exposing the image as histogram equalization. Since the gray value distribution chart of fuzzy set enhancement keeps the similarity of the original image’s, this method is one of no-distortion image processes.

**The Number of Input Channels.** By analysing the gray value distribution of the original image (figure(3)), we can find that the original image has such properties. The distribution function has a main plateau around 32 to 100, which means that the main color tone of this image is concentrated in this range. And it has another four sub peaks around gray level 116, 133, 150 and 175, which represents the local feature details. The main target of the image enhancement function is to intensify the local features and details. At the same time, the main tone of the image is unnecessary to be intensified (or be enhanced in a low degree). So we add one (or more) input fuzzy variable(s) $\mu_{\text{char}}(z)$ to intensify these detail regions. Other images also have such gray value distributions:

![Image 1](image1)

![Histogram](histogram1)

![Image 2](image2)

![Histogram](histogram2)

Figure 4. Another two image gray value distributions

**The Expression and Parameters of Input Membership Function.** The bell type function is more conformed to people’s intuition of an image as the input fuzzy $\mu_{\text{char}}(z)$. But bell type function increases computational complexity too much. Therefore, we approximately replace bell type function with quadratic function in characteristic region.

The turning points of overall input $\mu(z)$ equal to the minimal and maximal gray value of the original image. The median value of $\mu_{\text{char}}(z)$ equals to one of the gray value where the probability is of the top. And the minimal and maximal thresholds are symmetric with it. The input fuzzy membership function and enhanced image are as follows:

![Input fuzzy membership function](figure5)

Figure 5. Input fuzzy membership function
Conclusions
Fuzzy set theory is more appropriate of processing image detail enhancement than other traditional method, which is proved in previous discussion. Then we give a mode to choose the parameters of the membership function in this paper. Meanwhile it is still faced with problems like the number of input channels and the parameter of output fuzzy set for further works.

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References