

An Optimal Model of Traffic Signal Control System

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Abstract. With the economic development and urbanization, the number of motor vehicles increased rapidly, because the contradictions between urban traffic supply and demand imbalances are becoming increasingly acute. We used the cell transmission model (CTM) to optimize on the basis of a single intersection by observing the delay time and the traffic capacity, research into the intelligent urban traffic signal control system, improve the operating efficiency of the existing transportation system. To a certain extent, it addresses the issue of urban traffic congestion.

Introduction

The cell transmission model method [1][2][3] divide traffic roads into to the piecemeal which have the same nature (the piecemeal to be also called cellular), subdivide the time into the equally small time interval (time step) simultaneously. Each small time interval records k , the small time interval's length records T_s , this method needs to satisfy that cellular length l is exactly equal to the distance in a small time interval T_s goes on the free travel traffic flow, is $l = v_f T_s$, it requests that: vehicles enter cellular in some small time interval unable in the same small time interval to leave this cellular.

Establishment of Model

In this section, we will base in the single intersection[4] as shown in Fig. 1, marked with 1 to 12.

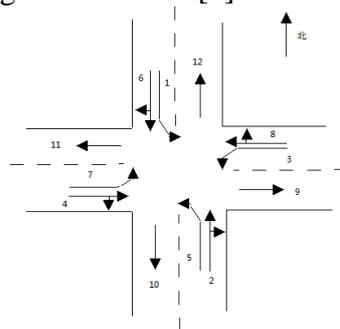


Figure 1. Distributed diagram of traffic flow on a single intersection

Regarding the street intersection in the Fig. 1, in general, transportation turning right is quite small, will not have the influence to other directions' transportation[5][6][7]. This article supposes that the left-turn transportation for protection, the right-turn transportation is not controlled by the signal light, and then the road intersection signal control may be divided into four phases of demonstration like Fig. 2 shown.

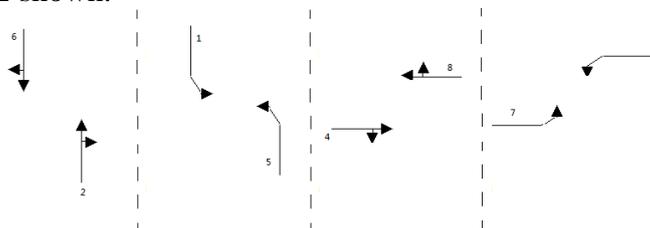


Figure 2. Four phases of demonstration for the road intersection signal control

Distributed diagram of traffic flow on a single intersection based on the CTM model as shown in Fig. 3.

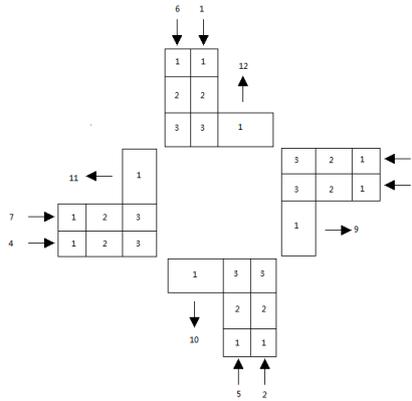


Figure 3. Distributed diagram of traffic flow on a single intersection based on the CTM model

Various transplanted in Fig. 3, may be separated into mutually independent two types [8][9]: source and course collection and the outlet flow set.

Some specific significant structural cells of the two collections' description following:

(1) The head-stream's first cell saves all the traffic demands have been going to enter the street intersection when $k = 1$, it's expressed $\sum D_i(k)$; the second cell's maximum flow is $Q_{i2}(k), i = 1, \dots, 8$; supported $D_i(k)$ as uncertain time-variable demand for the outside. If space in second structure cell may use, the vehicles will enter the network according to the change demand. Otherwise, the vehicles will wait in the first cell

$$n_{i1}(1) = \sum_k D_i(k), i = 1, \dots, 8 \tag{1}$$

$$Q_{i2}(k) = D_i(k), i = 1, \dots, 8 \tag{2}$$

Where, $n_{i1}(1)$ is the number of vehicles when the cellular $(i,1)$ when $k = 1$, $Q_{i2}(k)$ is the maximum input capacity of traffic flow when the cellular $(i,2)$ in the time of k , $D_i(k)$ is uncertain time-variable demand for the outside i in the time of k .

(2) As for the third cell, this cell has the following function:

This cell has simulated a control signal, during the time k occupies the green light, the maximum current capacity hypothesis is the saturated current capacity. It's described that

$$\begin{cases} Q_{i3}(k) = S_i, \text{ when } k \text{ occupies the green light} \\ Q_{i3}(k) = 0, \text{ when } k \text{ occupies the red light} \end{cases} \tag{3}$$

Where, S_i is the saturated current capacity, $Q_{i3}(k)$ is the maximum input capacity of traffic flow when the cellular $(i,3)$ in the time of k .

(3) Assumed storage capacity of the cellular from the outlet flow is the infinity, in order to solve export structure cell space limited problem: $N_1 \rightarrow \infty$. As shown in Fig. 4.

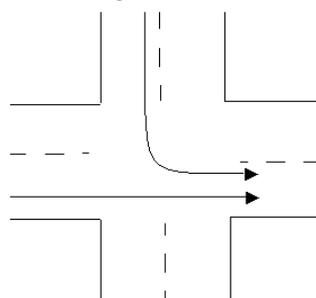


Figure 4. Distributed diagram of intersection confluence

Owing to we assumed the structure cell's storage capacity is the infinity, the direct acting and left-turn stream of vehicles inflow confluence cell's current capacity by below mode:

$$f_{i3}(k) = \min\{n_{i3}(k), Q_{i3}(k)\}, i = 1, \dots, 8 \quad (4)$$

Takes the cell (9, 1) in the Fig. 2 as the example, has the following formula:

$$n_{91}(k+1) = n_{91}(k) + f_{43}(k) + f_{13}(k) \quad (5)$$

Traffic Signal Control Optimized Model Based on the CTM Method

The research's purpose is to make the traffic signal control optimization, but in the case of supersaturated, not only to meet the above requirements, but also to take full advantage of the capacity.

We need to solve the following problems:

(1) The calculation of total vehicle delay

The volatility of each traffic flow can be used by a group of random events which probability is $p^y (y=1,2,\dots,Y)$, If the probability P_y of a random event y occurred, the corresponding flow of traffic demand can be expressed as q_i^y . So at this point the total delay corresponding to various traffic flows of vehicles can determine. Reference literature, CTM model provides a simple method of the vehicle delays [10]:

$$D^y = \sum_k \sum_i \sum_j d_{i,j}(k) \quad (6)$$

Where, $d_{i,j}(k) = n_{i,j}(k) - n_{i,j+1}(k)$ it means the total delays of all the vehicles in cell (i, j) . You can imagine that every car stay in the cell will produce a time step delay.

(2) The calculation of the total capacity

$$Pa^y(k) = n_{9,1}(k) + n_{10,1}(k) + n_{11,1}(k) + n_{12,1}(k) \quad (7)$$

Where, $n_{i,1}(k), i = 9,10,11,12$ respectively are vehicles of entering export cells $(i,1), i = 9,10,11,12$.

(3) The constraint

Suppose we study L cycles times, through the optimization method we can dynamically determine the green light duration $g_1(l), g_2(l), g_3(l), g_4(l)$ of each cycle, each phase and the signal cycle $C(l)$, and this optimization need to meet the conditions:

i External demand needs to satisfy

$$Q_{i2}^y(k) = D_i^y(k), i = 1, \dots, 8, y \in \Omega \quad (8)$$

Where, $D_i^y(k)$ is the external demand of the head-stream i when the time is k , $Q_{i2}^y(k)$ is the input flow in the second cell in the of the head-stream i .

ii The time of each signal display is not too little or too much, meanwhile, it is needed to meet the condition that the total time for the four phases in a period of time is equal to a circle time.

$$C(l) = g_1(l) + g_2(l) + g_3(l) + g_4(l), l = 1, \dots, L$$

$$g_{\min} \leq g_p(l) \leq g_{\max}, p = 1,2,3,4 \quad (9)$$

$$C_{\min} \leq C(l) \leq C_{\max}$$

The Optimization Model

Now we consider the total vehicle delay minimized and the aggregate capacity maximized as the most comprehensive evaluation index. First does linear normalization processing:

$$\frac{D^y}{D_0} - \mu \frac{Pa^y}{Pa_0} \quad (10)$$

Considering the uncertainty of traffic flow at the same time, to determine the optimal cycle length and each cycle phase green duration optimization model is as follows

$$\min Z = \min \sum_y p^y \left(\frac{D^y}{D_0} - \mu \frac{Pa^y}{Pa_0} \right), y \in \Omega$$

$$S.T. C(l) = g_1(l) + g_2(l) + g_3(l) + g_4(l), l = 1, \dots, L$$

$$g_{\min} \leq g_p(l) \leq g_{\max}, p = 1, 2, 3, 4 \quad (11)$$

$$C_{\min} \leq C(l) \leq C_{\max}$$

Above all, we need to establish the relationship with the objective function of the relationship between timing plan and the output $Q_{i3}(k)$.

For the first cycle in the four phases as shown in Fig. 2:

The initial phase: The green light starting time is $starp_1(l) = \sum_{x=1}^l C(x-1)$, where $C(0) = 0$. The end time is $endp_1(l) = starp_1(l) + g_1(l)$. Assuming that $starp_1(l) \leq k \leq endp_1(l)$.

$$\begin{cases} Q_{13}^y = S_1, Q_{53}^y = S_5 \\ Q_{23}^y = Q_{33}^y = Q_{43}^y = Q_{63}^y = Q_{73}^y = Q_{83}^y = 0 \end{cases} \quad (12)$$

Using the above method we can get the other phases timing plans.

Conclusions

This article based on cell transmission model (CTM) dynamic traffic signal control model, through optimizing the system of the absolute delay and total capacity to generate the optimal timing plan, which takes the uncertainty of traffic demand and dynamic change into consideration.

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