

# Research on Quality Reliability of Rolling Bearing by Multi - weight Method (Part I : Theory)

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**Abstract.** Rolling bearing is an important connection basic part of mechanical, and its quality is very important. This paper puts forward the concept of the research on quality reliability of rolling bearing, introduces the weight calculation method of the various factors, and creates the rolling bearing quality reliability model. Then, the true value of rolling bearing quality reliability is estimated based on the poor information theory.

## Introduction

As an important basis of mechanical connection, the rolling bearing quality plays a very important role in the normal operation of the equipment [1-3].

The study on rolling bearing quality belongs to the poor information theory [4, 5]. Many efforts and achievements have been made on the rolling bearing quality by former researchers. Jiang K thought that bearing defective inspection played a vital role in bearing quality control [6]. Mitchell M R proposed a fuzzy hypothesis testing model to make variability analysis of a time series with poor information [7]. Xia X analyzed rolling bearing quality and its influencing factors using the methods of poor information [8]. Y.T. Shang put forward the fuzzy comprehensive evaluation method for evaluating bearing quality [9].

## Quality Reliability Model

**Data Collection.** Assuming that the vibration acceleration of the rolling bearing is the quality reliability assessment index, and the data sequence of the vibration acceleration,  $X_0$ , can be obtained by the test.

$$X_0 = (x_0(1), x_0(2), \dots, x_0(k), \dots, x_0(n)) \quad (1)$$

Data sequence of influencing factors affecting vibration acceleration of rolling bearing,  $X_i$ , is

$$X_i = (x_i(1), x_i(2), \dots, x_i(k), \dots, x_i(n)), i = 1, 2, \dots, \quad (2)$$

**Data Classification.** Assuming that  $Z_j$  is the standard value of  $P_j$  grade for the vibration acceleration and its influencing factor, and  $x_i(k)$  is the test state value. If  $x_i(k)$  satisfies

$$Z_{j-1} < x_i(k) \leq Z_j; \quad i = 0, 1, \dots, m; k = 1, 2, \dots, n; j = 1, 2, \dots, J \quad (3)$$

Then the vibration acceleration grade corresponding to the test state value is  $P_j$ .

Quality grade of all data are classified based on the Eq.3. Assuming that there are  $N_{ji}$  values satisfying Eq.3, the quality grade classification table can be obtained as shown in Table 1. The highest quality grade is defined as  $P_1$  grade, while the lowest quality grade is  $P_J$  grade. The higher the quality grade, the better the quality and its influence factors, that is, the better the bearing. Generally, take the number of rolling bearing quality grade  $J = 4 \sim 7$ .

Table 1 Status value quality grade of rolling bearing vibration acceleration and influence factors

Serial number	Quality grade	Quality grade standard value	The number of the meet
1	P <sub>1</sub>	Z <sub>1</sub>	N <sub>1i</sub>
2	P <sub>2</sub>	Z <sub>2</sub>	N <sub>2i</sub>
...	...	...	...
j	P <sub>j</sub>	Z <sub>j</sub>	N <sub>ji</sub>
...	...	...	...
J	P <sub>J</sub>	Z <sub>J</sub>	N <sub>Ji</sub>

It is known from Table 1 that the quality grade frequency sequence can be expressed as

$$Y_i^0 = (y_i^0(1), y_i^0(2), \dots, y_i^0(j), \dots, y_i^0(J)) \quad (4)$$

Wherein

$$y_i^0(j) = \frac{N_{ji}}{n}; j = 1, 2, \dots, J; i = 0, 1, 2, \dots, m \quad (5)$$

The distribution sequence of the bearing vibration acceleration and the influence factors of the state value quality grade can be expressed as

$$Y_i = (y_i(1), y_i(2), \dots, y_i(j), \dots, y_i(J)) \quad (6)$$

Wherein

$$y_i(j) = \sum_{s=1}^j y_i^0(s); j = 1, 2, \dots, J; i = 0, 1, 2, \dots, m \quad (7)$$

### The Method of Determining the Weights

**Gray Relative Correlation Degree and Gray Absolute Correlation Degree.** In the calculation of gray relative correlation degree, the initialization value of  $X_0$  and  $X_i$  can be expressed as

$$X_0' = \frac{X_0}{X_0(1)} = (x_0'(1), x_0'(2), \dots, x_0'(k), \dots, x_0'(n)) \quad (8)$$

$$X_i' = \frac{X_i}{X_i(1)} = (x_i'(1), x_i'(2), \dots, x_i'(k), \dots, x_i'(n)) \quad (9)$$

Through further zero point processing,  $X_0$  and  $X_i$  can be expressed as

$$X_0^0 = (x_0^0(1), x_0^0(2), \dots, x_0^0(k), \dots, x_0^0(n)) = (x_0'(1) - x_0'(1), x_0'(2) - x_0'(1), \dots, x_0'(k) - x_0'(1), \dots, x_0'(n) - x_0'(1)) \quad (10)$$

$$X_i^0 = (x_i^0(1), x_i^0(2), \dots, x_i^0(k), \dots, x_i^0(n)) = (x_i'(1) - x_i'(1), x_i'(2) - x_i'(1), \dots, x_i'(k) - x_i'(1), \dots, x_i'(n) - x_i'(1)) \quad (11)$$

make

$$|s_0'| = \left| \sum_{k=1}^{n-1} x_0^0(k) + \frac{1}{2} x_0^0(n) \right|, |s_i'| = \left| \sum_{k=1}^{n-1} x_i^0(k) + \frac{1}{2} x_i^0(n) \right|, |s_i' - s_0'| = \left| \sum_{k=2}^{n-1} (x_i^0(k) - x_0^0(k)) + \frac{1}{2} (x_i^0(n) - x_0^0(n)) \right| \quad (12)$$

The gray relative correlation degree between  $X_0$  and  $X_i$  is

$$\gamma_{0i} = \frac{1 + |s'_0| + |s'_i|}{1 + |s'_0| + |s'_i| + |s'_i - s'_0|} \quad (13)$$

In the calculation of gray absolute correlation degree,  $X_0$  and  $X_i$  are not initialized, and other steps are the same with gray relative correlation degree, and the gray absolute correlation degree  $\varepsilon_{0i}$  can be obtained.

By the gray correlation degree weight method, the weight of influence factor  $X_i$  can be defined as

$$\omega_i^1 = \frac{\gamma_{0i}}{\sum_{i=1}^m \gamma_{0i}}, \omega_i^2 = \frac{\varepsilon_{0i}}{\sum_{i=1}^m \varepsilon_{0i}}; i = 1, 2, \dots, m \quad (14)$$

In the formula,  $\omega_i^1$  is the weight calculated by the gray relative correlation degree,  $\omega_i^2$  presents the weight calculated by the gray absolute correlation degree.

**Gray Comprehensive Correlation Degree.** Assuming the parameter  $\theta \in [0, 1]$ , then the gray comprehensive correlation degree between  $X_0$  and  $X_i$  is

$$\rho_{0i} = \theta \gamma_{0i} + (1 - \theta) \varepsilon_{0i} \quad (15)$$

The gray comprehensive correlation degree not only reflects the similarity degree between  $X_0$  and  $X_i$ , but also reflects their relative to the starting point of change degree. Generally take the parameter  $\theta$  as 0.5.

By the gray comprehensive correlation degree weight method, the weight of influence factor  $X_i$  can be defined as

$$\omega_i^3 = \frac{\rho_{0i}}{\sum_{i=1}^m \rho_{0i}}; i = 1, 2, \dots, m \quad (16)$$

**Gray Equivalent Relationship Coefficient.** According to the gray least-information theory, it can be assumed that the reference sequence  $X_C$ , which is consisted of the first data in  $X_0$ , is the initial value constant sequence

$$X_C = (x_C(1), x_C(2), \dots, x_C(k), \dots, x_C(n)) = (x_0(1), x_0(1), \dots, x_0(1)) \quad (17)$$

The reference sequence  $X_C$ , consisting of the mean sequence of  $X_0$  and  $X_i$ , is the mean constant sequence

$$X_C = (x_C(1), x_C(2), \dots, x_C(k), \dots, x_C(n)) = ((\sum_{k=1}^n x_0(k) + \sum_{k=1}^n x_i(k)) / (2n), (\sum_{k=1}^n x_0(k) + \sum_{k=1}^n x_i(k)) / (2n), \dots, (\sum_{k=1}^n x_0(k) + \sum_{k=1}^n x_i(k)) / (2n)) \quad (18)$$

The gray correlation degree between  $X_0$  and  $X_i$  is

$$\mu_{Ci} = \mu(X_C, X_i) = \frac{1}{n} \sum_{k=1}^n \mu(x_C(k), x_i(k)); i = 0, 1, 2, \dots, m \quad (19)$$

The gray relationship coefficient can be expressed as

$$\mu(x_C(k), x_i(k)) = \frac{\min_k |x_i(k) - x_C(k)| + \xi \max_k |x_i(k) - x_C(k)|}{|x_i(k) - x_C(k)| + \xi \max_k |x_i(k) - x_C(k)|} \quad (20)$$

In the formula,  $\xi$  is the resolution coefficient,  $\xi \in (0, 1]$ .

The gray distance between  $X_0$  and  $X_i$  is

$$d_{0i}(\xi) = |\mu_{C0} - \mu_{Ci}| \quad (21)$$

The maximum gray distance can be expressed as

$$d(x_0, x_i) = \max_{\xi \rightarrow \xi^*} d_{0i}(\xi) \quad (22)$$

In the formula,  $\xi^*$  is the optimal resolution coefficient.

Then the gray equivalent relationship coefficient between the  $X_0$  and  $X_i$  is

$$\tau_{0i} = 1 - d(x_0, x_i) \quad (23)$$

Based on the gray equivalent relationship coefficient weight method, the the weight of influence factor  $X_i$  can be defined as

$$\omega_i^{4,5} = \frac{\tau_{0i}}{\sum_{i=1}^m \tau_{0i}}; i = 1, 2, \dots, m \quad (24)$$

In the formula,  $\omega_i^4$  is the weight calculated by the initial value constant sequence,  $\omega_i^5$  is the weight calculated by the mean constant sequence.

**Quality Reliability Model.** By synthesizing the states of all influencing factors through the decomposition and synthesis methods of factors, the state synthetic value of quality grade influence factors,  $x_j$ , can be obtained

$$x_j = \sum_{i=1}^m \omega_i^d y_i(j); d = 1, 2, \dots, 5; j = 1, 2, \dots, J \quad (26)$$

The quality reliability function of rolling bearing,  $r_j(d)$ , is defined by the organization reliability theory [10].

$$r_j(d) \approx 1 - \exp(-ax_j^b); j = 1, 2, \dots, J \quad (27)$$

In the formula,  $a$  and  $b$  represent quality influence coefficient.

From Eq.27, it can be found that the larger the state synthetic value of quality grade influence factors, the greater the value of the quality reliability. This means that the lower the quality requirements of rolling bearings, the greater the ability to achieve this quality grade; otherwise, the smaller the ability to achieve the quality grade.

### True Value Estimation of Quality Reliability

**Quality Reliability Bootstrap Sample.** When the rolling bearing quality grade is  $P_j$ , the reliability sequence,  $R_j$ , can be expressed as

$$R_j = (r_j(1), r_j(2), \dots, r_j(d), \dots, r_j(D)); d = 1, 2, \dots, D; D = 5 \quad (28)$$

By the bootstrap method, the quality reliability samples,  $R_b$ , can be obtained through sampling with equal probability and where sample is replaceable.

$$R_b = (r_b(1), r_b(2), \dots, r_b(k), \dots, r_b(D)) \quad (29)$$

The mean value of the bootstrap sample,  $R_b$ , is expressed as

$$r_b = \frac{1}{D} \sum_{k=1}^D r_b(k) \quad (30)$$

The data sequence is sampled for  $B$  times, and the  $B$  samples obtained are expressed as vectors

$$R = [r_1, r_2, \dots, r_b, \dots, r_B]^T \quad (31)$$

**Maximum Entropy Probability Density Function of Rolling Bearing Quality Reliability.** The origin moment of each order of the quality reliability,  $m_l$ , is

$$m_l = \frac{1}{B} \sum_{b=1}^B r_b^l; l = 1, 2, \dots, L \quad (32)$$

In the formula,  $l$  represents the origin moment order, and  $L$  represents the highest order.

By the maximum entropy principle, the origin moment of the bearing quality reliability,  $m_l$ , need to satisfy

$$m_l = \frac{\int_{\Omega} r^l \exp(\sum_{l=1}^M \lambda_l r^l) dr}{\int_{\Omega} \exp(\sum_{l=1}^M \lambda_l r^l) dr} \quad (33)$$

In the formula,  $r$  represents the continuous random variable on  $r_b$ ,  $\Omega$  represents the viable domain of  $r$  and  $\lambda_l$  represents the Lagrangian multiplier.

The maximum entropy probability density function of bearing quality reliability can be expressed as

$$f = f(r) = \exp(\lambda_0 + \sum_{l=1}^M \lambda_l r^l) \quad (34)$$

Wherein

$$\lambda_0 = -\ln(\int_{\Omega} \exp(\sum_{l=1}^M \lambda_l r^l) dr) \quad (35)$$

The maximum entropy probability distribution of the quality grade is

$$F = F(r) = \int_{\Omega_0}^r f(r) dr = \int_{\Omega_0}^r \exp(\lambda_0 + \sum_{l=1}^M \lambda_l r^l) dr \quad (36)$$

In the formula,  $\Omega_0$ —is the lower limit of integration.

**True Value Estimation of Quality Reliability.** The quality reliability true value of  $P_j$  level of bearing quality,  $r_j$ , is

$$r_j = \int_{\Omega} r f(r) dr \quad (37)$$

## Summary

In this paper, the concept of rolling bearing quality reliability is put forward based on poor information theory. The method of determining the influencing factors weight is briefly introduced, and the quality reliability model is established. The true value of rolling bearing quality reliability is estimated by bootstrap re-sampling. This study provides theoretical basis and research methods for the research of rolling bearing quality reliability, and has certain theoretical and practical significance.

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