Frequency-dependent friction for non-Newtonian power-law fluid in transient pipe flow

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Abstract. The pressure wave transmitted in the non-Newtonian drilling fluid is widely used for the information transmission in Measurement While Drilling/Logging While Drilling technology. It is essential to get the friction model in the non-Newtonian drilling fluid to evaluate the signal attenuation. A model of transient non-Newtonian power-law pipe flow is developed by assuming a steady viscosity varied only with the radius, and the solution is derived analytically in the complex domain and time domain. The analysis results show that the shear stress which is the friction term in the motion equation increases with the n increase, and decreases with the frequency increase. The trend of signal attenuation increases with the frequency increase, but for different n , the signal attenuation varies significantly, which decreases with the n increase.

Introduction

Mud pulse telemetry has been the global standard for real-time data transmission in the Measurement While Drilling/Logging While Drilling (MWD/LWD) technology for the past thirty years[1]. In the mud-pulse-telemetry system, information from the vicinity of the drill bit is transmitted to the surface drilling platform via a flowing column of drilling fluid, i.e., mud. The information being converted into a binary code of pressure wave in the drilling fluid inside the drill string is sensed at the surface using a pressure transducer. The pressure wave is generated by periodically modulating the flowing mud column at the point downhole by a poppet valve or a rotary valve [2].

In order to guarantee a clear pressure wave appearance in the surface detector under various drilling conditions, theoretical and experimental works have been done in pressure wave transmission modeling. Chen and Aumann [3] developed a numerical solution of the unsteady fluid flow governing equations, in which a constant friction coefficient was used. Desbrandes et al. [4] used an attenuation equation which was based on Newtonian fluids derived by Lamb, where, the viscosity of the mud is considered to be a constant. Kytomaa and Crosso [5] developed the transfer functions for all the components that make up the drill rig configurations and assembled them in a modular fashion to represent the pressure wave transmission channel. When deriving the transfer function of the elastic drill string, a frequency-dependent friction term was utilized, however, this term was also based on the Newtonian fluids. Oliveira et al. [6,7] simulated the pressure propagation in the Bingham fluid for different geometries and boundary conditions; in their studies, they used the fluid conductance of the Bingham fluid, which was obtained from Melrose et al. [8], in the friction factor calculations.

The attenuation models of the pressure wave cited above have been limited to the Newtonian fluid or use an approximate non-Newtonian characteristic which cannot fully reflect the transmission characteristics of the pressure wave transmitted in the non-Newtonian fluid flowing. In this study, the frequency-dependent attenuation model for the non-Newtonian power-law fluid is developed based on the approach developed by Zielke[9] and Vardy and Brown [10] for Newtonian flows.

Mathematical Model

Governing Equation. In order to simplify the problem, the geometry is defined as a long horizontal pipeline in which the axial coordinate is x and radial coordinate is r . By assuming the fluid motion of the power-law fluid inside the drill string as incompressible and laminar, the motion equation for the unsteady-state flow can be written as

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial u}{\partial t} + \left(\frac{\partial \tau}{\partial r} + \frac{\tau}{r} \right) \quad (1)$$

where, u is the axial velocity; ρ , the drilling fluid density; p , the pressure; τ , the shear stress; t , the time; x , the axial coordinate; r , the radial coordinate.

The rheological equation used to represent power-law fluids in cylindrical coordinates can be written as

$$\tau = -K \left(\frac{\partial u}{\partial r} \right)^n = -\mu_a \frac{\partial u}{\partial r} \quad (2)$$

where, K is the consistency coefficient and n is the power-law index, μ_a is the apparent viscosity.

Substituting the Eq.(2) into the Eq.(1), we can get a nonlinear partial differential equation which is difficult to be solved analytically. In fact, for the information transmission, the amplitude of the pressure wave is small, so that the velocity disturbance and shear rate caused by the pressure wave is smaller than that of the steady-state flow. Therefore, in this paper, the shear rate in steady power-law flow is used in the apparent viscosity μ_a and can be written as

$$\left(\frac{\partial u}{\partial r} \right)_s = \left(\frac{f}{2} \frac{r}{K} \right)^{\frac{1}{n}} \quad (3)$$

As it is well-known in the art, in the non-Newtonian unsteady pipe flow, the apparent viscosity μ_a varies with the time and the radius of the flow layer in the pipe. After the assumption made above, the apparent viscosity just varies with the radius of the flow layer, which can be written as

$$\mu_a = K^{\frac{1}{n}} \left(\frac{f}{2} \right)^{\frac{n-1}{n}} r^{\frac{n-1}{n}} \quad (4)$$

Substituting Eq. (4) and Eq. (2) into the Eq. (1), the motion equation is written as

$$\frac{1}{\mu_{a0}} \frac{\partial p}{\partial x} = r^{\frac{n-1}{n}} \frac{\partial^2 u}{\partial r^2} + \frac{2n-1}{n} r^{\frac{-1}{n}} \frac{\partial u}{\partial r} - \frac{\rho}{\mu_{a0}} \frac{\partial u}{\partial t} \quad (5)$$

Boundary Conditions. The following equations represent the boundary conditions for all time which must be applied to solve the governing equation

$$u|_{r=R} = 0, \quad \frac{\partial u}{\partial r} \Big|_{r=0} = 0, \quad u|_{r=0} = u_m \quad (6)$$

Solution. Eq. (5) can be converted into ordinary differential equations by Laplace transform

$$\frac{1}{\mu_{a0}} F(s) = r^{\frac{n-1}{n}} \frac{d^2 U(s, r)}{dr^2} + \frac{2n-1}{n} r^{\frac{-1}{n}} \frac{dU(s, r)}{dr} - \frac{\rho}{\mu_{a0}} sU(s, r) \quad (7)$$

where, $U(s, r)$ is the Laplace transform of u , and s is the Laplace transform variable.

After a series of transformations, the governing equation of the non-Newtonian fluid has the form of the modified Bessel's equation

$$\frac{d^2 V(s, z)}{dz^2} + \frac{1}{z} \frac{dV(s, z)}{dz} - \left(1 + \left(\frac{1-n}{1+n} \right)^2 \frac{1}{z^2} \right) V(s, z) = 0 \quad (8)$$

Where, $z = \sqrt{\frac{\rho s}{\mu_{a0}}} \frac{2n}{3n-1} \frac{1}{m} r^{\frac{n+1}{2n}}$, $V(s, z) (\sqrt{mz})^{\frac{m-1}{2m}} = U(s, z) + \frac{F(s)}{\rho s}$, $m = \frac{n+1}{3n-1}$.

The general solution to the Eq. (8) is

$$V(s, z) = C_1 I_\sigma(z) + C_2 K_\sigma(z) \quad (9)$$

in which C_1 and C_2 are constants of integration to be determined from the boundary conditions, I_σ and K_σ are modified Bessel functions of first and second kinds and σ order, and $\sigma = |(1-n)/(1+n)|$.

With the boundary conditions in Eq.s (6), the relations between r and z , and the relations between U and V , and let $y = z\sqrt{m}$, the Eq. (9) is transformed as

$$U(s, y) = \left(\frac{I_{-\sigma} \left(y / \sqrt{m} \right) y^{\frac{m-1}{2m}}}{I_{-\sigma} \left(y_R / \sqrt{m} \right) y_R^{\frac{m-1}{2m}}} - 1 \right) \frac{F(s)}{\rho s} \quad (10)$$

Frequency-Dependent Friction

Pressure Wave Transmitted Inside The Drill String. In the continuous-wave information transmission, the pressure wave inside the drill string can be seen as the periodic flow in the steady pipe flow, and only the periodic flow plays the leading role. The amplitude of velocity oscillation for the periodic flow is smaller than the velocity in the steady pipe flow; therefore, in the periodic flow, a viscosity varied with radius but not varied with time is used.

For the periodic flow, the pressure gradient varies sinusoidal with time

$$\frac{\partial p(t, x)}{\partial x} = \rho P_p \exp(i\omega t) \rightarrow F(s) = \frac{\rho P_p}{s - i\omega} \quad (11)$$

The inverse Laplace transform of the Eq. (10) is

$$u(r, t) = \frac{P_p}{i\omega} \left(\frac{2r^{h-1}}{R^{h-1}(\sigma-2)} + 1 \right) + \frac{P_p}{i\omega} \left(1 - \frac{J_{-\sigma} \left(\sqrt{\frac{-i\omega}{v}} \frac{1}{h} r^h \right) r^{h-1}}{J_{-\sigma} \left(\sqrt{\frac{-i\omega}{v}} \frac{1}{h} R^h \right) R^{h-1}} \right) \exp(i\omega t) - P_p \sum_{k=1}^{\infty} \frac{J_{-\sigma} \left(\lambda_k \left(\frac{r}{R} \right)^h \right) r^{h-1}}{J_{1-\sigma}(\lambda_k) \lambda_k R^{h-1} \left(\lambda_k^2 \frac{vh^2}{2R^{2h}} + \frac{i\omega}{2} \right)} \exp \left(-\frac{\lambda_k^2}{R^{2h}} vh^2 t \right) \quad (12)$$

where, P_p is the amplitude of the sinusoidal pressure gradient, and ω is the angular frequency.

For the periodic flow, only the real part of the Eq. (12) gives the physical value, and for a long time steady oscillation, the transient term which is the third term in the right side of the Eq. (12) can be neglected. Therefore, for the long time steady periodic flow of non-Newtonian fluid, the result can be simplified as

$$u(r, t) = \frac{P_p}{i\omega} \left(1 - \frac{J_{-\sigma} \left(r^h / h \sqrt{-i\omega/v} \right) r^{h-1}}{J_{-\sigma} \left(R^h / h \sqrt{-i\omega/v} \right) R^{h-1}} \right) \exp(i\omega t) \quad (13)$$

where, h is a parameter relating to the power-law index n , the larger the n deviates from 1, the more obvious the non-Newtonian characteristic will be. The velocity profiles for different n are plotted in Fig. 1. It can be seen that the distance between the overshoot of the velocity and the wall increases with the n increase, so that the non-Newtonian characteristic for the periodic flow is obvious.

Figure 1 Velocity profiles for different n due to oscillating pressure gradient

Figure 2 The amplitude of shear stress and velocity gradient varied with n

Frequency-Dependent Friction. Base on the Eq. (13), the mean velocity and the wall shear stress are

$$\bar{U}(t) = \left(1 - \frac{1}{\Pi_{1-\sigma} \left(R^h / h \sqrt{-i\omega/v} \right)} \frac{2}{h} \right) \frac{P_p}{i\omega} \exp(i\omega t), \quad \tau_w(t) = K \left(\frac{R^{2h-1} P_p}{vh} \frac{\exp(i\omega t)}{\Pi_{1-\sigma} \left(R^h / h \sqrt{-i\omega/v} \right)} \right)^n \quad (14)$$

in which the function $\Pi_{1-\nu}(z) = zJ_{-\sigma}(z)/J_{1-\sigma}(z)$ is defined as a modified quotient of Bessel functions of $1-\sigma$ order.

The steady viscosity is used here and according to the Eq.s (2) and (4), τ_w can be derived as

$$\tau_w(t) = \frac{\omega \rho R}{2} \frac{1}{\eta} \text{Re} \bar{U}(t) + \frac{\rho R}{2} \xi \frac{\partial \text{Re} \bar{U}(t)}{\partial t} \quad (15)$$

$$\frac{1}{\eta} = \text{Re} \left(\frac{2i/h}{\Pi_{1-\sigma}(R^h/h\sqrt{-i\omega/\nu}) - 2/h} \right), \xi = \text{Im} \left(\frac{2i/h}{\Pi_{1-\sigma}(R^h/h\sqrt{-i\omega/\nu}) - 2/h} \right) \quad (16)$$

Figure 3 Shear stress varied with ω for different n

Eq. (15) is the frequency-dependent friction of the non-Newtonian power-law pipe flow, which varied with time, frequency, power-law index, pipe radius, consistency coefficient and acceleration of the fluid. Fig.2 shows the amplitude of the shear stress and the velocity gradient at the pipe wall for $n > 1$. It can be seen that, when $n < 1$, the amplitude of the velocity gradient at the pipe wall is less than 1 and increases with the n increase, so that the value of the shear stress increases little. The shear stress varied with the ω is shown in Fig.3, it can be seen that the shear stress decreased with the increase of the ω .

Frequency-Dependent Friction Used in the Pressure Wave Attenuation. In this section, the pressure attenuation is studied, and only the non-Newtonian characteristic is considered. For the other parameters, such as, wave speed, the depth of the well, etc., are not discussed in this paper; and there are not reflections at the two ends of the pipe.

The equations of the pressure wave propagation are used in a simplified form by neglecting the nonlinear terms, then the motion equation and continuity equation become

$$\frac{\partial p}{\partial x} + \frac{\rho}{\pi R^2} \frac{\partial Q}{\partial t} + \frac{2\tau_w}{R} = 0, \quad \frac{\partial p}{\partial t} + \frac{\rho a^2}{\pi R^2} \frac{\partial Q}{\partial x} = 0 \quad (17)$$

Where, Q is the flow rate. a is the wave speed in the drilling fluid.

Substituting Eq. (15) into Eq. (17), the motion equation becomes

$$\frac{\partial p}{\partial x} + \frac{\rho}{\pi R^2} (1 + \xi) \frac{\partial Q}{\partial t} + \frac{\rho \omega}{\pi R^2 \eta} Q = 0 \quad (18)$$

The propagation constant $\gamma = \sqrt{C_f \omega (-L_f \omega + iR_f)}$ becomes

$$\gamma = \frac{\omega}{a} \sqrt{-(1 + \xi) + \frac{i}{\eta}} \quad (19)$$

where, R_f , L_f , C_f are fluid resistance, fluid capacitance and fluid inductance respectively. The attenuation constant is the real part of the propagation constant, which can be written as

$$\alpha = \frac{\omega}{a\sqrt{2}} \sqrt{(\xi + 1)^2 + \frac{1}{\eta^2} - (1 + \xi)} \quad (20)$$

where, ξ and η are related to the ω , n and K . The attenuation of pressure wave as it propagate through the drilling fluid channel can be expressed as

$$p(x) = p(0) \exp(-\alpha x) \quad (21)$$

Figure 4 Signal attenuation of different n

Fig. 4 shows the non-Newtonian characteristic of the signal attenuation. The signal attenuation increases with the increase of the n and decreases with the frequency increase. Therefore, the power-law index which stands for the non-Newtonian characteristic is an important influencing factor on the pressure wave attenuation.

Summary

A model of the frequency-dependent friction has been developed. When $n < 1$, the value of the shear stress increases little with the n increase; The shear stress decreases with the increase of the frequency. Frequency-dependent friction has been used in the pressure wave attenuation. The attenuation of the pressure wave increases with the increase of the n and decreases with the frequency increase.

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