Correction of axis tilt error for 40Cr ring gear measurement based on collinear measuring heads

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**Abstract.** The ring gear and flywheel are interference fit. In order to verify whether magnitude of interference meets the processing requirements, the size change of the 40Cr ring gear before and after heating is measured. Measurement accuracy of ring gear measurement equipment should reach 0.01mm. Due to the machining errors of the components, assembly errors, elastic deformation and so on, the tilt of axis is almost universal. Therefore, the measurement accuracy of ring gear measurement equipment is reduced. The correction method of axis tilt error based on collinear measuring heads is proposed for the problem of measurement accuracy reduction due to the tilt of axis. Firstly, the physical model of axis tilt error is studied, and the corresponding mathematical model is established according to multi-body system theory. Secondly, the mathematical model of correction of axis tilt error based on collinear measuring heads is established and the tilt correction factor is solved. Finally, the system consisting of 2 eddy current sensors and 1 standard bar is used to identify the parameters of algorithm. Simulation results show that this algorithm can be very good for correction of axis tilt error when shaft inclination angle is in the range of 5\textdegree. The ring gear measurement accuracy after correction of tilt error can meet the processing requirements of interference magnitude.

**Introduction**

The ring gear and flywheel are interference fit. In order to verify whether magnitude of interference meets the processing requirements, the size change of the 40Cr ring gear before and after heating is measured. Measurement accuracy of ring gear measurement equipment should reach 0.01mm. Due to the machining errors of the components, assembly errors, elastic deformation and so on, the tilt of axis is almost universal. Therefore, the measurement accuracy of ring gear measurement equipment is reduced. The main reasons for the tilt of axis are perpendicularity error and wobble error. Therefore, many scholars at home and abroad had done a lot of research for measurement and error separation of perpendicularity error and wobble error causing tilt of axis. The method of two self collimation electronic theodolite with reflector to measure and the optimal datum fitting processing is proposed for perpendicularity error and wobble error of revolving spindle of laser gyro inertial stabilization platform [1]. Document [2] used the reversal method based on leveling instrument to measure the perpendicularity error of revolving spindle. The model of wobble error based on geometry and the method of error separation based on curve-fitting are proposed for wobble error of revolving spindle [3]. But correction of axis tilt error caused by perpendicularity error and wobble error has not been researched deeply. The correction method of axis tilt error based on collinear measuring heads is proposed for the problem of measurement accuracy reduction due to the tilt of axis. Firstly, the physical model of axis tilt error is studied, and the corresponding mathematical model is established according to multi-body system theory. Secondly, the mathematical model of correction of axis tilt error based on collinear measuring heads is established and the tilt correction factor is solved. Finally, the method of document [4] is used to identify the parameters of algorithm.
Modeling

The mathematical model which contains six error parameters is established according to multi-body system theory [5, 6]. It prepared for the following correction of axis tilt error. Fig. 1 shows schematic diagram of tilt of axis. The black line is the ideal location of axis and the red line is the actual location of axis. Perpendicularity error and wobble error is the main cause of tilt of axis. Perpendicularity error belongs to system error and wobble error belongs to random error.

The coordinate system \(o-xyz\) is established in benchmark plane. The shaft line of measured rigid body is as \(z\)-axis. \(x\)-axis and \(y\)-axis consist in benchmark plane which is perpendicular to \(z\)-axis. The intersection point of \(z\)-axis and benchmark plane is as the origin of coordinate system \(o\). The coordinate system \(o_i \sim x_iy_i\) is established in the rotary table. The shaft line of the spindle of rotary table is as \(z_i\)-axis. The laser beam which is perpendicular to \(z_i\)-axis is as \(x_i\)-axis. The line which is perpendicular to \(z_i\)-axis and \(x_i\)-axis is as \(y_i\)-axis. The intersection point of \(z_i\)-axis and plane that contains \(x_i\)-axis and \(y_i\)-axis is as the origin of coordinate system \(o_i\).

The topology model of tilted axis is established according to multi-body system theory. The measurement equation of weekly scanning and measuring system can be obtained as follows:

\[
\begin{bmatrix}
{r_{p_0}} \\
\end{bmatrix} = \begin{bmatrix}
{A_{oo_1}} & {A_{oo_1}} \\
\end{bmatrix}
\begin{bmatrix}
{r_{p_1}} \\
\end{bmatrix}
\]

\(A_{oo_1}\)-transformation matrix from coordinate system \(o\) to coordinate system \(o_1\);

\(\{r_{p_0}\}, \{r_{p_1}\}\)-position vector of measured point \(p\) in coordinate system \(o\) and coordinate system \(o_1\).

Position and orientation transformation matrix from coordinate system \(o\) to coordinate system \(o_1\), as follows:

\[
\begin{align*}
{A_{oo_1}} & = {a_0}^T \begin{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{align*}
\]

\(a_{xyz}\)-the translational transformation matrix from coordinate system \(o\) to coordinate system \(o_1\);

\(R_{xyz}\)-the rotational transformation matrix from coordinate system \(o\) to coordinate system \(o_1\). 

\[
\begin{align*}
{a_0}^T & = \begin{bmatrix}
1 & 0 & 0 & p_x \\
0 & 1 & 0 & p_y \\
0 & 0 & 1 & p_z \\
\end{bmatrix} \\
{a_{xyz}} & = \begin{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{align*}
\]

\(\{p_x, p_y, p_z\}\)-position vector of origin of coordinate system \(o_1\) in coordinate system \(o\).

\[
\begin{align*}
{a_{xyz}} & = \begin{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{align*}
\]

In the equation, \(s, c\)-the sine and cosine functions; \(\alpha\)-the rotation angle of coordinate system \(o_1\) rotate about the \(x\)-axis of coordinate system \(o\); \(\beta\)-the rotation angle of coordinate system \(o_1\) rotate about the \(y\)-axis of coordinate system \(o\); \(\gamma\)-the rotation angle of coordinate system \(o_1\) rotate about the \(z\)-axis of coordinate system \(o\).

Position vector of measured point \(p\) in coordinate system \(o\) and in coordinate system \(o_1\) is \(\{r_{p_0}\}\) and \(\{r_{p_1}\}\) respectively. Specifically expressed as follows:

\[
\begin{align*}
\end{align*}
\]
\[
\{p_{ro}\} = \begin{bmatrix}
x_R \\
y_R \\
z_R \\
1
\end{bmatrix}, \quad \{p_{ro}\} = \begin{bmatrix}
t
0
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
x_R \\
y_R \\
z_R
\end{bmatrix}^T \text{ - position vector of measured point } p \text{ in coordinate system } o; \\
t \text{ - measurements of the displacement sensor}
\]

Measurement equations which contain six error parameters, as follows:

\[
\begin{bmatrix}
x_R \\
y_R \\
z_R
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & p_z \\
0 & 1 & 0 & p_y \\
0 & 0 & 1 & p_x
\end{bmatrix} \times \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \times \begin{bmatrix}
c(\gamma) & s(\gamma) & 0 & 0 \\
-s(\gamma) & c(\gamma) & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 \\
c(\beta) \\
-s(\beta) \\
0
\end{bmatrix} \times \begin{bmatrix}
1 \\
0 \\
0 \\
1
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 & 0
\end{bmatrix} 
\]

\[
1 \text{-measured rigid body } \\
2 \text{-benchmark plane} \\
3 \text{-axis}
\]

Fig. 1 schematic diagram of tilt of axis

Fig. 2 schematic diagram of weekly scanning and measuring system based on collinear measuring heads

**Correction Algorithm**

Fig. 2 shows schematic diagram of weekly scanning and measuring system based on collinear measuring heads. The coordinate system \(o'-x_1y_1z_1\) is established in rigid body consisting of 2 collinear measuring heads. The shaft line of first measuring head is as \(x_1\)-axis. The shaft line of second measuring head is as \(x_2\)-axis. The intersection point of \(x_1\)-axis and \(x_2\)-axis is as the origin of coordinate system \(o'\). The coordinate system \(o-xyz\) is established in benchmark plane. The shaft line of measured rigid body is as \(z\)-axis. \(x\)-axis and \(y\)-axis consist in benchmark plane which is perpendicular to \(z\)-axis. The intersection point of \(z\)-axis and benchmark plane is as the origin of coordinate system \(o\). When measuring \(i\) time, the pose matrix of coordinate system \(o'\) in coordinate system \(o\), as follows:

\[
{o'_{o}T} = \begin{bmatrix}
x_x & o_x & -x_x & p_x \\
y_y & o_y & -y_y & p_y \\
z_z & o_z & -z_z & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

In the equation, \(n_x,n_y,n_z\) - directional vector of \(x_1\)-axis in coordinate system \(o\); \(o_x,o_y,o_z\) - directional vector of \(y'\)-axis in coordinate system \(o\); \((-n_x,-n_y,-n_z)\) - directional vector of \(x_2\)-axis.
in coordinate system \( o ; (p_x, p_y, p_z)^T \) - position vector of origin of coordinate system \( o \), in coordinate system \( o \).

Parametric equations of \( x_1 \)-axis in coordinate system \( o \), as follows:

\[
\begin{align*}
\begin{cases}
  x = n_x t + p_x \\
  y = n_y t + p_y \\
  z = n_z t + p_z
\end{cases}
\end{align*}
\]  

We supposed equations of measured rigid body:

\[
\begin{align*}
\begin{cases}
  x^2 + y^2 = R^2 \\
  z = z_0
\end{cases}
\end{align*}
\]  

\( R \) - radius of measured rigid body when measuring \( i \) time

Parameters when \( x_1 \)-axis and measured rigid body intersect can be obtained by combination of Eq. (8) and Eq. (9).

\[
t_1 = \frac{- (n_x p_x + n_y p_y) + \sqrt{R^2 (n_x^2 + n_y^2) - (n_x p_x - n_y p_y)^2}}{n_x^2 + n_y^2}
\]  

\( t_1 \) - \( x_1 \)-axis measurements of the displacement sensor

When \( p_x, p_y, t_1 \) and \( R \) in Eq. (10) is known, Eq. (10) can be seen as a function of \( n_x \) and \( n_y \).

\[
f_1(n_x, n_y; t_1, p_x, p_y, R) = 0
\]  

Similarly, \( x_2 \)-axis measurements of the displacement sensor can be solved.

\[
t_2 = \frac{(n_x p_x + n_y p_y) + \sqrt{R^2 (n_x^2 + n_y^2) - (n_x p_x - n_y p_y)^2}}{n_x^2 + n_y^2}
\]  

\( t_2 \) - \( x_2 \)-axis measurements of the displacement sensor

Eq. (12) can be seen as a function of \( n_x \) and \( n_y \).

\[
f_2(n_x, n_y; t_2, p_x, p_y, R) = 0
\]  

Eq. (10) and Eq. (12) are measurement equations. Eq. (11) and Eq. (13) are parameter identification equations. Parameters when measuring \( i \) time can be solved.

\[
\begin{align*}
\begin{cases}
  n_x = n_{xi} \\
  n_y = n_{yi}
\end{cases}
\end{align*} (i = 1, 2, \ldots, N)
\]  

The tilt correction factor \( k_i \) can be solved according to Eq. (8) and Eq. (14).

\[
k_i = \frac{n_{xi, n_{yi, n_{zi, n_{zi}}}} (n_{xi, n_{yi, n_{zi}}}, 0)}{n_{xi, n_{yi, n_{zi}}, 0}} = \sqrt{n_{xi}^2 + n_{yi}^2} \quad (i = 1, 2, \ldots, N)
\]  

The measurements of the displacement sensor after correction, as follows:

\[
t_i = k_i \frac{- (n_{xi} p_{xi} + n_{yi} p_{yi}) + \sqrt{R^2 (n_{xi}^2 + n_{yi}^2) - (n_{xi} p_{xi} - n_{yi} p_{yi})^2}}{n_{xi}^2 + n_{yi}^2} \quad (i = 1, 2, \ldots, N)
\]  

The axis tilt error can be corrected by Eq. (16).

Fig. 3 shows the basic flowchart of correction of axis tilt error. The basic flowchart includes the steps as follows:
(1) \( p_x \) and \( p_y \) \((i=1,2,\ldots,N)\) is solved by the method of document [4].

(2) The measured data \( t_i \) \((i=1,2,\ldots,N)\) is obtained by scanning a circle.

(3) The semi-minor axis \( b \) of the ellipse is obtained by fitting \( t_i \) according to least square ellipse fitting method and it is used as initial value of \( R \).

(4) \( n_x \) and \( n_y \) \((i=1,2,\ldots,N)\) is solved by Eq.(10) and Eq.(12).

(5) The tilt correction factor \( k_i \) \((i=1,2,\ldots,N)\) is solved by Eq.(15).

(6) Modified \( t_i \) \((i=1,2,\ldots,N)\) is solved by Eq.(16).

(7) The semi-minor axis \( b \) and semi-major axis \( a \) of the ellipse is obtained by fitting modified \( t_i \) according to least square ellipse fitting method.

(8) If the cutoff condition \( \Delta R = (a - b) / 3 < \varepsilon \) \((\varepsilon \) is correction accuracy) is met, the correction process is over. Otherwise, the correction process continues.

![Flowchart](image)

**Fig. 3 Basic flowchart of correction of axis tilt error.**

**Simulation and Results Analysis**

**Materials.** 40Cr steel is medium carbon steel. Its tensile strength is more than 980Mpa and hardness of HRC is 32~36. 40Cr steel has good quenching performance. After quenching and tempering treatment, it has good comprehensive mechanical properties. The coefficient of thermal expansion of 40Cr steel is \( 6 \times 10^{-6} \) at 20-200 Celsius degrees.

**Methods.** In order to verify that this algorithm can improve the measurement accuracy of ring gear measurement equipment, the simulation is done with MATLAB. We set radius of measured rigid body \( R = 100mm \), radial error \( p_x = 0.03 + 0.1 \times \text{rand}(1,1)mm \), and \( p_y = 0.05 + 0.1 \times \text{rand}(1,1)mm \). For reasons of space we only listed partial data. Table.1 show Partial data of before and after correction. Among them, \( R_i \) stands for the measured value. Simulation results show that this algorithm can be very good for correction of axis tilt error when shaft inclination angle is in the range of 5°.
Table 1. Partial data of before and after correction.

<table>
<thead>
<tr>
<th>angle of Shaft inclination [°]</th>
<th>$R$ [mm]</th>
<th>Before correction</th>
<th>After correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$R_i$ [mm]</td>
<td>$(R_i - R)/R$ [%]</td>
</tr>
<tr>
<td>1.5</td>
<td>100</td>
<td>100.0175</td>
<td>0.0175</td>
</tr>
<tr>
<td>2.0</td>
<td>100</td>
<td>100.0305</td>
<td>0.0305</td>
</tr>
<tr>
<td>2.5</td>
<td>100</td>
<td>100.0476</td>
<td>0.0476</td>
</tr>
<tr>
<td>3.0</td>
<td>100</td>
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<td>0.0686</td>
</tr>
<tr>
<td>3.5</td>
<td>100</td>
<td>100.0934</td>
<td>0.0934</td>
</tr>
<tr>
<td>4.0</td>
<td>100</td>
<td>100.1220</td>
<td>0.1220</td>
</tr>
<tr>
<td>4.5</td>
<td>100</td>
<td>100.1545</td>
<td>0.1545</td>
</tr>
<tr>
<td>5.0</td>
<td>100</td>
<td>100.1908</td>
<td>0.1908</td>
</tr>
</tbody>
</table>

Summary

In this paper, the correction algorithm of axis tilt error based on collinear measuring heads is deduced in detail and the method of document [4] is used to identify the parameters of algorithm. Then, the tilt correction factor was solved for each location and the correction was completed. The simulation analysis is carried out before and after correction. Simulation results show that this algorithm can be very good for correction of axis tilt error when shaft inclination angle is in the range of 5°.

The above analysis results are based on theoretical analysis, formula derivation and simulation, and the results will be verified by experiments. The ring gear measurement accuracy after correction of tilt error can meet the processing requirements of interference magnitude.

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References


