

Equivalent Channel Parameters for Polar Coded Cooperative Relaying

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Abstract: In a polar coded cooperative relay system, the channel parameters play a very important role in the overall system performance. In order to find the optimal channel parameter which is suitable for polar encoding and decoding, and to optimize the system performance, a equivalent channel parameter (ECP) value which is universal to the AF cooperative relay system is given by rigorous mathematical derivation in this paper. The paper also analyzes the influence of the channel parameters on the ECP in a special case, and gives the simulation results.

Keywords: Polar codes; cooperative relaying; equivalent channel parameter (ECP).

1. Introduction

Cooperative relaying using relay channels [1] is a technology that has been widely used in wireless communication networks, which can significantly improve system capacity and error rate performance. The cooperative relaying obtains the diversity gain through the cooperation of each node, and then achieves the effect of a kind of virtual MIMO, which can be regarded as a special channel polarization process. Polar codes based on channel polarization, introduced by Arikan in [2], is the first coding scheme that theoretically achieve symmetric capacity of binary-input discrete memoryless channels (BDMCs). The excellent performance of the polar codes makes it attractive for scholars all over the world. At present, polar codes have been applied in many fields, e.g. image transmission [3], wiretap channel [4] and quantum channel [5], which have proved to be very advantageous.

Based on the similar channel polarization characteristics, the combination of the polar codes and the cooperative relay channel will result in better error performance and channel capacity. It is known that the polar codes is combined with the relay channel for the first time in the paper [6]. The paper describes the implementation of symmetric binary-input capacity in the physical degraded relay channel. After that, there are a lot of research on the combination of polar codes and relay channel, e.g. [7][8][9].

It is known that the polar codes are constructed based on the channel polarization characteristics. Therefore, channel parameters such as signal-to-noise ratio (SNR), noise variance, and so on, have a great impact on their performance. In cooperative relay systems with similar channel polarization, the influence of channel parameters is more obvious. In the usual research, scholars have adopted a simpler way, the default relay system parameters of the same channel. In practice, this setting is unrealistic, and has greater damage to the polar performance. In this paper, the ECP which can optimize the performance of the system are found from the general amplify and forward (AF) model using Rayleigh channel, and the ECP affected analysis in Gaussian channels are given.

The paper is organized as follows: In section 2, we briefly describe the relay system model based on polar codes in Rayleigh channel. The derivation of ECP is given in section 3. At the same time, the influence of ECP on Gaussian channel and its simulation analysis are also given in Section 3. Some concluding remarks are provided in section 4.

2. System model

Figure 1 shows the system chart of the polar coding scheme based on the AF protocol. In the first stage, at the source node S, the information is first polar encoded and then modulated and broadcasted; in the second stage, the relay R amplifies the received signal with amplification factor α , and then forwards it to the destination node D; D will be combined to decode the output of the two signals.

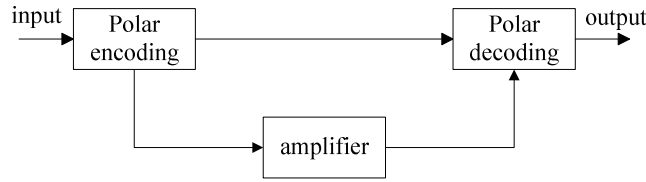


Figure 1. Polar coding scheme based on the AF protocol

Figure 2 shows the corresponding mathematical model of the cooperative polar coding system under AF protocol. In this model, all nodes are single-antenna systems. Assume that the signal transmission power at S and R is P_s and P_r , respectively. Each channel corresponds to the channel fading coefficient and noise is h_{ij} and n_{ij} ($i, j = S, R, D$) respectively.

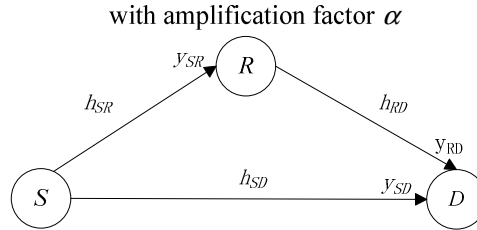


Figure 2. Mathematical model of single relay polar code cooperative system

As shown in Figure 2, x denotes a transmission signal, and y_{ij} ($i, j = S, R, D$) denotes a reception signal the received signal can be expressed as

$$y_{SR} = h_{SR}x + n_{SR} \quad (1)$$

$$\begin{aligned} y_{RD} &= h_{RD} \cdot \alpha y_R + n_{RD} \\ &= \alpha h_{SR} h_{RD} x + \alpha h_{RD} n_{SR} + n_{RD} \end{aligned} \quad (2)$$

$$y_{SD} = h_{SD}x + n_{SD} \quad (3)$$

The detailed derivation of the combined signal is given in the next section.

3. The Derivation of ECP

In this part, we derive the system ECP from the known information of the system in formula level, which can make the system get the best BER performance. The ECP specific derivation process is described in detail below.

3.1. Derivation of the component parameters

a) Channel SR.

Using σ_1^2 denotes the noise variance on the channel SR, and SNR_1 represents the SNR at the signal receiving side, combining (1), there is

$$SNR_1 = \frac{|h_{SR}|^2 P_s}{\sigma_1^2} \Rightarrow \sigma_1 = \sqrt{\frac{|h_{SR}|^2 P_s}{SNR_1}}. \quad (4)$$

b) Amplification times of relay node.

The transmit signal of relay node is αy_R , so the signal power

$$P_R = \alpha^2 |h_{SR}|^2 P_s + \alpha^2 \sigma_1^2. \quad (5)$$

Therefore,

$$\alpha = \sqrt{\frac{P_R}{|h_{SR}|^2 P_s + \sigma_1^2}} \quad (6)$$

c) Channel RD.

Similarly, Using σ_2^2 denotes the noise variance on the channel RD, and SNR_2 represents the SNR at the signal receiving side, combining (2)(6), there is

$$SNR_2 = \frac{|\alpha h_{SR} h_{RD}|^2 P_s}{|h_{RD}|^2 \cdot \alpha^2 \sigma_1^2 + \sigma_2^2} \quad (7)$$

$$\Rightarrow \sigma_2 = \sqrt{\frac{\alpha^2 |h_{RD}|^2 |h_{SR}|^2 P_s}{SNR_2} - |h_{RD}|^2 \cdot \alpha^2 \sigma_1^2} \quad (8)$$

d) Channel SD.

In the channel SD, the noise variance and SNR are denoted by σ_3^2 and SNR_3 , respectively. The same can be concluded as follows

$$SNR_3 = \frac{|h_{SD}|^2 P_s}{\sigma_3^2} \quad (9)$$

$$\Rightarrow \sigma_3 = \sqrt{\frac{|h_{SD}|^2 P_s}{SNR_3}} \quad (10)$$

So far, we obtain the channel parameters needed to derive the ECP for all three channels. In the next step, we will use these parameters to get what we want ECP.

3.2. ECP derivation

At the destination node D, we have received the relay channel and the direct channel transmission information y_{RD} and y_{SD} , in practice, we need to merge according to certain rules to combine the two signals, and then perform the decoding operation. In the process of decoding the combined signal, the noise variance plays a very important role in the decoding performance. Therefore, we derive the EPC for the whole system based on the component parameters obtained in 3.1, in order to obtain the optimal performance of the system. Specific derivation process is as follows.

First of all, we combine the received signals using maximum-ratio-combing (MRC). As known

$$\hat{y} = \frac{h_0^* y_0 + h_1^* y_1 + \dots + h_n^* y_n}{|h_0|^2 + |h_1|^2 + \dots + |h_n|^2}, \quad (11)$$

so there is

$$y_{MRC} = \frac{h_{SD}^* y_{SD} + \alpha h_{SR}^* h_{RD}^* y_{RD}}{|h_{SD}|^2 + |\alpha h_{SR} h_{RD}|^2}. \quad (12)$$

Combining equations (2) (3), (12) can be rewritten as

$$y_{MRC} = \frac{h_{SD}^* h_{SD} x + h_{SD}^* n_{SD} + (\alpha h_{SR} h_{RD})^* [\alpha h_{SR} h_{RD} x + \alpha h_{RD} n_{SR} + n_{RD}]}{|h_{SD}|^2 + |\alpha h_{SR} h_{RD}|^2} \quad (13)$$

$$= \frac{|h_{SD}|^2 x + h_{SD}^* n_{SD} + \alpha^2 |h_{SR}|^2 |h_{RD}|^2 x + \alpha^2 |h_{RD}|^2 h_{SR}^* n_{SR} + \alpha h_{SR}^* h_{RD}^* n_{RD}}{|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2} \quad (14)$$

$$= \frac{(|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2) x + h_{SD}^* n_{SD} + \alpha h_{SR}^* h_{RD}^* n_{RD} + \alpha^2 |h_{RD}|^2 h_{SR}^* n_{SR}}{|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2} \quad (15)$$

$$= x + \frac{h_{SD}^* n_{SD} + \alpha h_{SR}^* h_{RD}^* n_{RD} + \alpha^2 |h_{RD}|^2 h_{SR}^* n_{SR}}{|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2} \quad (16)$$

Therefore, the equivalent SNR parameter of the total signal can be obtained

$$SNR_{eq} = \frac{P_s}{\frac{|h_{SD}|^2 \sigma_3^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2 \sigma_2^2 + \alpha^4 |h_{RD}|^4 |h_{SR}|^2 \sigma_1^2}{(|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2)^2}} \quad (17)$$

and the equivalent noise variance parameter is

$$\sigma_{eq}^2 = \frac{|h_{SD}|^2 \sigma_3^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2 (\sigma_2^2 + \alpha^2 |h_{RD}|^2 \sigma_1^2)}{(|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2)^2}. \quad (18)$$

That is, σ_{eq} is composed of $\sigma_1, \sigma_2, \sigma_3$ according to the above function relationship. Furthermore, substituting $\sigma_1, \sigma_2, \sigma_3$ obtained by the formula (4) (8) (10) into the formula (18), respectively, there is

$$\sigma_{eq}^2 = \frac{\frac{|h_{SD}|^2 \cdot |h_{SD}|^2 P_s}{SNR_3} \sigma_3^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2 \left(\frac{\alpha^2 |h_{RD}|^2 |h_{SR}|^2 P_s}{SNR_1} + \frac{\alpha^2 |h_{RD}|^2 |h_{SR}|^2 P_s}{SNR_2} - \frac{\alpha^2 |h_{RD}|^2 |h_{SR}|^2 P_s}{SNR_1} \right)}{(|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2)^2} \quad (19)$$

$$= \frac{\frac{|h_{SD}|^4 P_s}{SNR_3} + \alpha^2 |h_{SR}|^2 |h_{RD}|^2 \cdot \frac{\alpha^2 |h_{RD}|^2 |h_{SR}|^2 P_s}{SNR_2}}{(|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2)^2} \quad (20)$$

$$= \frac{\frac{(|h_{SD}|^2)^2}{SNR_3} + \frac{(\alpha^2 |h_{RD}|^2 |h_{SR}|^2)^2}{SNR_2}}{(|h_{SD}|^2 + \alpha^2 |h_{SR}|^2 |h_{RD}|^2)^2} \cdot P_s. \quad (21)$$

Further, from (4) (6),

$$\alpha^2 = \frac{P_R}{|h_{SR}|^2 P_s + \sigma_1^2} = \frac{P_R}{|h_{SR}|^2 P_s (1 + \frac{1}{SNR_1})} \quad (22)$$

So we have

$$\alpha^2 |h_{RD}|^2 |h_{SR}|^2 = \frac{P_R}{P_s \cdot |h_{SR}|^2 (1 + \frac{1}{SNR_1})} \cdot |h_{RD}|^2 |h_{SR}|^2 = \frac{|h_{RD}|^2 P_R}{P_s} \cdot \frac{SNR_1}{1 + SNR_1} \quad (23)$$

Combinations (23), (21) can be written as

$$\sigma_{eq}^2 = \frac{\frac{|h_{SD}|^4}{SNR_3} + \left[\frac{|h_{RD}|^4 P_R^2}{P_s^2} \cdot \left(\frac{SNR_1}{1 + SNR_1} \right)^2 \right] / SNR_2}{(|h_{SD}|^2 + \frac{|h_{RD}|^2 P_R}{P_s} \cdot \frac{SNR_1}{1 + SNR_1})^2} \cdot P_s \quad (24)$$

And there is

$$SNR_{eq} = \frac{P_s}{\sigma_{eq}^2} = \frac{(|h_{SD}|^2 + \frac{|h_{RD}|^2 P_R}{P_s} \cdot \frac{SNR_1}{1 + SNR_1})^2}{\frac{|h_{SD}|^4}{SNR_3} + \left[\frac{|h_{RD}|^4 P_R^2}{P_s^2} \cdot \left(\frac{SNR_1}{1 + SNR_1} \right)^2 \right] / SNR_2} \quad (25)$$

From (25), it can be seen that SNR_{eq} is composed of SNR_1 , SNR_2 and SNR_3 according to the defined function relationship. In the next section, we will analyze the channel fading coefficient of 1 (i.e. with Gaussian channel), the impact of each component channel parameters on the ECP.

3.3. ECP affected analysis in Gaussian channels

In the previous section, we present the ECP formulation for any Rayleigh fading channel. Using the ECP, the system can achieve better performance. In this section, we simplify the system model by defining all the channel fading coefficients as 1, $P_s = P_R$ and the amplification factor is also 1, in order to more intuitively express the influence of each component parameter on ECP. In this case, the influence of the component channel parameters on ECP is expressed as follows.

In this particular case, there is

$$SNR_{eq} = \frac{P_s}{\sigma^2} = \frac{(1 + \frac{SNR_1}{1 + SNR_1})^2}{\frac{1}{SNR_3} + (\frac{SNR_1}{1 + SNR_1})^2 / SNR_2} \quad (26)$$

$$\sigma_{eq}^2 = \frac{\sigma_3^2 + \sigma_2^2 + \sigma_1^2}{4} \quad (27)$$

For SNR_{eq} and σ_{eq} have a one-to-one relationship $SNR_{eq} = P_s / \sigma_{eq}^2$, so we will take SNR_{eq} as an example. Figure 3, Figure 4, Figure 5 show the influence of the other two component channel parameters on SNR_{eq} when fixed SNR_1 , SNR_2 and SNR_3 respectively.

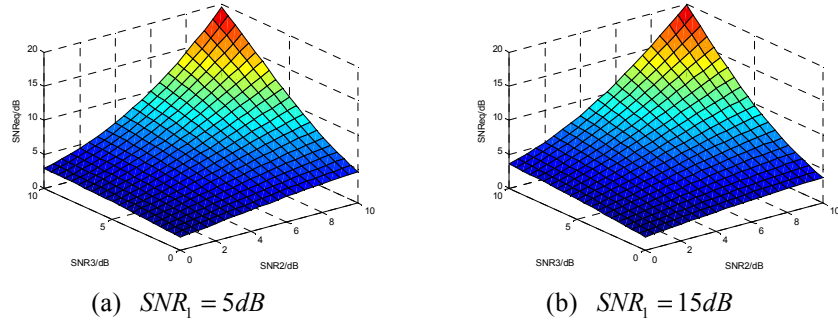


Figure 3. The effect of SNR_1 , SNR_2 and SNR_3 on SNR_{eq} when fixed SNR_1

Figure 3 shows the influence of the other two parameters on the SNR_{eq} when SNR_1 is taken. The purpose of selecting the two values (5dB and 10dB) is to study the influence of the other two parameters on the ECP when the fixed channel parameters are within and outside the fluctuation range. We selected the same parameters in the next two simulations. From Figure 3, we can see that when fixed SNR_1 , SNR_{eq} increases with the increase of SNR_2 and SNR_3 , and the effect of SNR_2 and SNR_3 on SNR_{eq} is approximately equal. In addition, we were surprised to find that in this case, the change in the value of SNR_1 has almost no effect on the ECP.

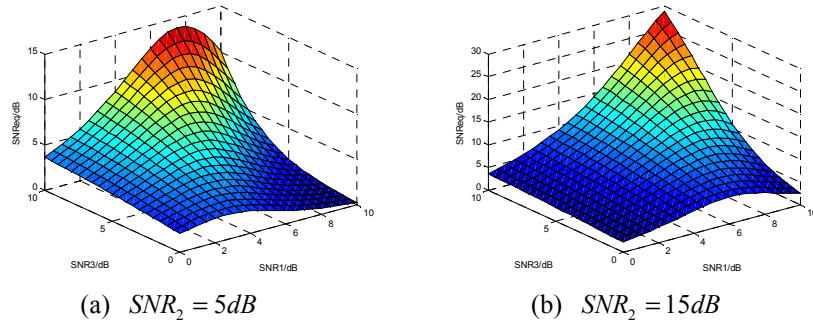


Figure 4. The effect of SNR_1 , SNR_2 and SNR_3 on SNR_{eq} when fixed SNR_2

Figure 4 shows a very interesting phenomenon. That is, when SNR_2 is within the range of SNR_1 and SNR_3 , SNR_{eq} increases with the increase of SNR_3 , but not with the increase of SNR_1 presents an increasing trend. There will be a maximum value, and when the value of SNR_1 and SNR_2 are approximately equal to achieve this extreme. What's more, when the value of SNR_2 exceeds the range of SNR_1 and SNR_3 , this phenomenon does not exist.

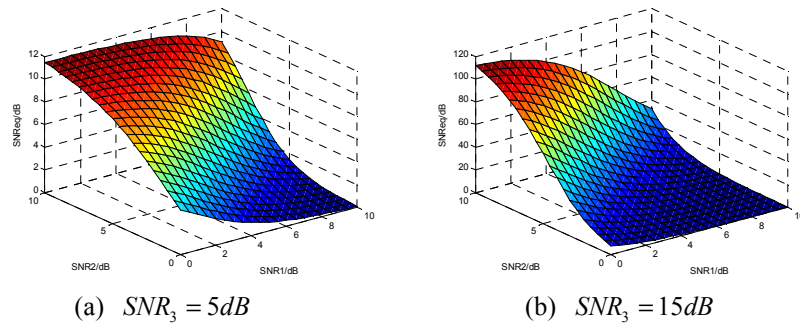


Figure 5. The effect of SNR_1 , SNR_2 and SNR_3 on SNR_{eq} when fixed SNR_3

The effect of SNR_1 , SNR_2 and SNR_3 on SNR_{eq} when fixed SNR_3 is shown in Figure 5. As shown in the figure, when fixed SNR_3 , the appropriate reduction of SNR_1 will get a larger SNR_{eq} . And for different values of SNR_3 , the impact of ECP is very significant.

4. Conclusion

The performance of polar codes based on channel polarization is very sensitive to channel parameters, especially in cooperative relay channels. And in general, it is not practical to set the relay channel and direct passing channel parameter to be the same. In order to find the optimal channel parameter which is suitable for polar encoding and decoding, and to optimize the system performance, a ECP value which is universal to the AF cooperative relay system is given by rigorous mathematical derivation. This ECP varies with the variation of the channel parameters (arbitrary) of the system. The paper analyzes the influence of the channel parameters on the ECP in a special case, and gives the simulation results.

Acknowledgments

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