Performance Analysis of Full-Duplex Decode-and-Forward Two-Way Relay System

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Abstract: This paper considers a full-duplex two-way relaying (FD-TWR) system, where two terminals transmit and receive simultaneously with the help of a full-duplex relay node. We propose the transmission scheme for decode-and-forward (DF) protocol using the differential binary phase-shift keying (DBPSK) modulation to analyse the bit error ratio (BER), which do not need the channel state information (CSI). We consider the residual self-interference (RSI) in relay and source. By using the maximum a posteriori (MAP) and maximum likelihood (ML) decoder at the relay and the source node, respectively, we obtain the symbols. The simulation results show that the full-duplex decode-and-forward two-way relaying (F2DF-TWR) system can achieve the bit error performance of the half-duplex decode-and-forward two-way relaying (H2DF-TWR) system.

Keywords: full-duplex; two-way; decode-and-forward; residual self-interference; non-coherent maximum likelihood.

1. Introduction

In conventional cooperative communication systems, all the nodes operate in the half-duplex (HD) mode, which have to transmit and receive signals in different frequency bands and/or different time slots. Traditional half-duplex one-way relay system requires four time slots to exchange one frame, which leads to a loss in spectral efficiency [1]. In order to improve the spectral efficiency, TWR, two-path successive relaying and the FD are proposed. TWR is applicable only to bidirectional communication, and two-path successive relaying is a promising protocol [2], but suffer from severe inter-relay interference. The FD mode can efficiently improve the spectral efficiency by transmitting and receiving signals in the same time slot and frequency band. However, since the FD mode suffer from the strong RSI, it was not considered for a long time. With the development in digital and analog domain self-interference cancellation (SIC) technologies [3], the SI can be greatly eliminated, so the FD mode draws our attention [4].

Most of the previous cooperation schemes on FDF-TWR system mainly focus on the outage probabilities [5] and spectral-efficient [6]. And also a lot of works assume CSI is known at the receiver [6], [7], which is hard to satisfy in practice. But long-term CSI is easier to get and requires less overhead [8]. Therefore, this motivates the schemes using non-coherent modulation that do not require instant CSI. [9] proposes new schemes for AF and DF protocols where do not need the CSI known and derives ML detectors for both protocols. [10] proposes a DF TWR scheme and a denoise-and-forward (DNF) TWR scheme using non-coherent multiple differential phase-shift keying modulation in the half-duplex system, which do not need CSI. [11] uses the DNF scheme with DBPSK modulation to recover the spectral efficiency loss caused by the half-duplex relay, which do not need CSI either. Motivated by these, we try to combine the non-coherent modulation with FD wireless communication to analyse the BER.

In this paper, we study the F2DF-TWR scheme with DBPSK to analyse the BER, which do not need instantaneous CSI. And we consider the RSI. The simulation results show that the proposed FDF-TWR system can achieve the bit error performance of the H2DF-TWR system.

Notations: matrices and variables are denoted by bold capital and lower-case letters, respectively; $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ stand for conjugate, transpose, and conjugate transpose, respectively.

2. System model

In this paper, we investigate a FD-TWR system, as shown in Figure 2. The typical three-node model is applied to the FD enabled TWR system, which comprises two terminal nodes A , B and one relay node R with transmitting power P_A , P_B and P_C , respectively. Here, we assume $P_A = P_B = P_C = P$. Each node has two

antennas that one is for transmitting and another is used for receiving. And the RSI is taken into consideration. The physical-layer network coding (PNC) based on DF TWR [12], as shown in Figure 1, only needs two time slots to complete the bidirectional transmission. We assume the time, phase and carrier of all transmitted signals are synchronized.

Figure 1. Bi-directional information exchange using PNC

There are L frames to be exchanged at the source nodes A and B , and it needs $L + 1$ time slots to transmit the $2L$ frames. The signals to be transmitted by A and B are denoted by $c'_A(n)$ and $c'_B(n)$, respectively, and the signals decoded and to be sent in the relay are denoted by $c'_R(n)$, where n denotes the symbol index in one frame, t denotes the time slot.

Figure 2. The proposed full-duplex decode-and-forward two-way relay system

According to the Figure 2, there are the $A \rightarrow R$ channel, the $B \rightarrow R$ channel, the $R \rightarrow A$ channel, the $R \rightarrow B$ channel, and the RSI channels at A , B and R in the system. And the corresponding channel fading coefficients are h_{AR} , h_{BR} , h_{RA} , h_{RB} , h_{AA} , h_{BB} and h_{RR} .

To better express the this paper, we make the following assumptions: (1) All the channel coefficients remain static over one time slot; (2) The channels between two nodes and the RSI channels are subjected to Rayleigh fading; (3) The CSI is known at the node A , B and R , i.e., $h'_{AR} \sim \text{CN}(0, \sigma_{AR}^2)$, $h'_{BR} \sim \text{CN}(0, \sigma_{BR}^2)$, $h'_{RA} \sim \text{CN}(0, \sigma_{RA}^2)$, $h'_{RB} \sim \text{CN}(0, \sigma_{RB}^2)$, $h'_{AA} \sim \text{CN}(0, \sigma_{AA}^2)$, $h'_{BB} \sim \text{CN}(0, \sigma_{BB}^2)$ and $h'_{RR} \sim \text{CN}(0, \sigma_{RR}^2)$, where σ_{AR}^2 , σ_{BR}^2 , σ_{RA}^2 , σ_{RB}^2 , σ_{AA}^2 , σ_{BB}^2 and σ_{RR}^2 denote the corresponding channel gain, and $\sigma_{AR}^2 = \sigma_{BR}^2 = \sigma_{RA}^2 = \sigma_{RB}^2 = \sigma^2$, $\sigma_{AA}^2 = \sigma_{BB}^2 = \sigma_{RR}^2 = \sigma_{RSI}^2$; (4) There is no direct link between A and B due to the strong shadowing effect and the long communication distance; (5) The noises n_A , n_B , and n_R over the node A , B and R are independently and identically distributed (i.i.d) Gaussian noises with zero mean and variance of σ_n^2 , i.e., $n_i \sim \text{CN}(0, \sigma_n^2)$, $i = \{A, B, R\}$.

In the t th ($t = 1, 2, \dots, L$) time slot, the two source nodes A and B firstly generate the t th frame BPSK symbols $c'_A(n) \in \{1, -1\}$ and $c'_B(n) \in \{1, -1\}$ respectively. Then nodes A and B generate the DBPSK symbols $x_A^1 = c'_A(n) \times x'_A(n)$ and $x_B^1(n+1) = c'_B(n) \times x'_B(n)$ respectively, with $x'_A(1) = 1$ and $x'_B(1) = 1$ being the reference symbol. Finally, A and B transmit $x'_A(n)$ and $x'_B(n)$ to R , respectively. And in the t th ($t = 2, 3, \dots, L + 1$) time slot, R broadcast $x'_R(n)$ to A and B , where $x'_R(n+1) = c'_R(n) \times x'_R(n)$.

3. FDF-TWR Scheme

Based on the system model of FDF-TWR, as demonstrated in the Figure 2 in section 2, we will discuss the signals process in this section.

In the 1st time slot, the two source nodes transmit their first frame symbols to R , and the signals received at R can be expressed as

$$y_R^1(n) = \sqrt{P_A} h_{AR}^1 x_A^1(n) + \sqrt{P_B} h_{BR}^1 x_B^1(n) + z_R^1(n) \quad (1)$$

In the t th ($t = 2, 3, \dots, L$) time slot, the two source nodes transmit their t th frame symbols to R , and simultaneously, R forwards the symbols received and decoded in the previous time slot. The signals received at A , B and R can be respectively expressed as

$$y_R^t(n) = \sqrt{P_A} h_{AR}^t x_A^t(n) + \sqrt{P_B} h_{BR}^t x_B^t(n) + \sqrt{P_R} h_{RR}^t x_R^t(n) + z_R^t(n) \quad (2)$$

$$y_A^t(n) = \sqrt{P_A} h_{AA}^t x_A^t(n) + \sqrt{P_R} h_{RA}^t x_R^t(n) + z_A^t(n) \quad (3)$$

$$y_B^t(n) = \sqrt{P_B} h_{BB}^t x_B^t(n) + \sqrt{P_R} h_{RB}^t x_R^t(n) + z_B^t(n) \quad (4)$$

In the $(L+1)$ th time slot, R forwards the symbols received and decoded in the L th time slot, and the signals received at A and B can be respectively expressed as

$$y_A^{L+1}(n) = \sqrt{P_R} h_{RA}^{L+1} x_R^{L+1}(n) + z_A^{L+1}(n) \quad (5)$$

$$y_B^{L+1}(n) = \sqrt{P_R} h_{RB}^{L+1} x_R^{L+1}(n) + z_B^{L+1}(n) \quad (6)$$

Table 1 and Table 2 summary the transmitted and received signals at each node during one time slot, respectively.

Table 1. Transmitted signals at each node

Time slot	1	2	...	L-1	L	L+1
A	x_A^1	x_A^2	...	x_A^{L-1}	x_A^L	
B	x_B^1	x_B^2	...	x_B^{L-1}	x_B^L	
R		x_R^2	...	x_R^{L-1}	x_R^L	x_R^{L+1}

Table 2. Received signals at each node

Time slot	1	2	...	L-1	L	L+1
A		y_A^2	...	y_A^{L-1}	y_A^L	y_A^{L+1}
B		y_B^2	...	y_B^{L-1}	y_B^L	y_B^{L+1}
R	y_R^1	y_R^2	...	y_R^{L-1}	y_R^L	

3.1 PNC mapping

The realization of the PNC scheme based on DF relay is closely related to the demodulation mapping. As shown in Figure 1, the nodes A and B transmit the BPSK-modulated signal to the relay node R at the same time slot, the received superimposed signal is demodulated at R .

Table 3. PNC Mapping: modulation mapping at A and B ;
demodulation and modulation mappings at R

Modulation mapping at A and B				Demodulation mapping at R		
				Input	Output	
Input		Output			Modulation mapping at R	
					Input	Output
s_A^t	s_B^t	c_A^t	c_B^t	$c_A^t + c_B^t$	$s_A^t \oplus s_B^t$	c_R^t
1	1	-1	-1	-2	0	1
0	1	1	-1	0	1	-1
1	0	-1	1	0	1	-1
0	0	1	1	2	0	1

The BPSK modulation-based PNC demodulation mapping scheme [12] is shown in Table 3. The relay node R transmits the signals c_R^t to the nodes A and B in the second time slot, according to the modulation/demodulation mapping scheme proposed in Table 3. Nodes A and B carry on BPSK demodulation to the received data again, can get the $s_A^t \oplus s_B^t$, then do the exclusive-OR operation with the information that they have sent respectively, complete the information exchange.

3.2 Decode-and-forward

At the relay node R , we define $\mathbf{y}_R^t(n) = [\mathbf{y}_R^t(n+1) \quad \mathbf{y}_R^t(n)]^H$ ($t = 2, 3, \dots, L+1$). According to (2), we have the conditional probability density function (PDF) of $\mathbf{y}_R^t(n)$ given $c_A^t(n)$, $c_B^t(n)$ and $c_R^t(n)$, i.e.

$$f(\mathbf{y}_R^t(n) | c_A^t(n), c_B^t(n), c_R^t(n)) = \frac{1}{\pi^2 |\mathbf{K}_R^t(n)|} \exp\{-\mathbf{y}_R^t(n)^H \mathbf{K}_R^t(n)^{-1} \mathbf{y}_R^t(n)\} \quad (7)$$

where $\mathbf{K}_R^t(n)$ denotes the covariance matrix can be expressed as

$$\mathbf{K}_R^t(n) = \begin{bmatrix} P_A \sigma_{AR}^2 + P_B \sigma_{BR}^2 + P_R \sigma_{RR}^2 + \sigma_n^2 & P_A \sigma_{AR}^2 c_A^t(n) + P_B \sigma_{BR}^2 c_B^t(n) + P_R \sigma_{RR}^2 c_R^t(n) \\ P_A \sigma_{AR}^2 c_A^t(n)^* + P_B \sigma_{BR}^2 c_B^t(n)^* + P_R \sigma_{RR}^2 c_R^t(n)^* & P_A \sigma_{AR}^2 + P_B \sigma_{BR}^2 + P_R \sigma_{RR}^2 + \sigma_n^2 \end{bmatrix} \quad (8)$$

Since $c_R^t(n)$ are known at R , and we let $c_{AB}^t(n) = c_A^t(n) + c_B^t(n)$, so (7) and (8) can be rewrite as

$$f(\mathbf{y}_R^t(n) | c_{AB}^t(n)) = \frac{1}{\pi^2 |\mathbf{K}_R^t(n)|} \exp\{-\mathbf{y}_R^t(n)^H \mathbf{K}_R^t(n)^{-1} \mathbf{y}_R^t(n)\} \quad (9)$$

$$\mathbf{K}_R^t(n) = \begin{bmatrix} 2P\sigma^2 + P\sigma_{RSI}^2 + \sigma_n^2 & P\sigma^2 c_{AB}^t(n) + P\sigma_{RSI}^2 c_R^t(n) \\ P\sigma^2 c_{AB}^t(n)^* + P\sigma_{RSI}^2 c_R^t(n)^* & 2P\sigma^2 + P\sigma_{RSI}^2 + \sigma_n^2 \end{bmatrix} \quad (10)$$

As several $c_A^t(n)$ and $c_B^t(n)$ pairs may result the same value of $c_{AB}^t(n) = c_A^t(n) + c_B^t(n)$, we use the MAP detector by maximizing $f(c_{AB}^t(n) | \mathbf{y}_R^t(n)) = \frac{f(\mathbf{y}_R^t(n) | c_{AB}^t(n)) f(c_{AB}^t(n))}{f(\mathbf{y}_R^t(n))}$. Because $f(\mathbf{y}_R^t(n))$ do not affect the result, we

can obtain

$$\hat{c}_{AB}^t(n) = \arg \max f(\mathbf{y}_R^t(n) | c_{AB}^t(n)) \cdot f(c_{AB}^t(n)) \quad (11)$$

According to the demodulation and modulation mappings at R , the corresponding c_R^t will be used as the forwarded symbols of the relay in the next time slot, i.e., $c_R^{t+1}(n)$.

Similarly, at the source nodes A and B (we only talk about the node B , because it is symmetric), we define $\mathbf{y}_A^t(n) = [\mathbf{y}_A^t(n+1) \quad \mathbf{y}_A^t(n)]^H$ ($t = 2, 3, \dots, L+1$) as well. According to (3), we have $\mathbf{y}_A^t(n) \sim \text{CN}(0, \mathbf{K}_A^t(n))$ given $c_A^t(n)$ and $c_R^t(n)$, where $\mathbf{K}_A^t(n)$ denotes the covariance matrix can be derived as

$$\mathbf{K}_A^t(n) = \begin{bmatrix} P_R \sigma_{RA}^2 + P_A \sigma_{AA}^2 + \sigma^2 & P_R \sigma_{RA}^2 c_R^t(n) + P_A \sigma_{AA}^2 c_A^t(n) \\ P_R \sigma_{RA}^2 c_R^t(n)^* + P_A \sigma_{AA}^2 c_A^t(n)^* & P_R \sigma_{RA}^2 + P_A \sigma_{AA}^2 + \sigma^2 \end{bmatrix} \quad (12)$$

The conditional probability density function (PDF) of $\mathbf{y}_A^t(n)$ given $c_A^t(n)$ and $c_R^t(n)$ is

$$f(\mathbf{y}_A^t(n) | c_A^t(n), c_R^t(n)) = \frac{1}{\pi^2 |\mathbf{K}_A^t(n)|} \exp\{-\mathbf{y}_A^t(n)^H \mathbf{K}_A^t(n)^{-1} \mathbf{y}_A^t(n)\} \quad (13)$$

Since $c_A^t(n)$ are known at R , we can respectively rewrite (12) and (13) as

$$f(\mathbf{y}_A^t(n) | c_R^t(n)) = \frac{1}{\pi^2 |\mathbf{K}_A^t(n)|} \exp\{-\mathbf{y}_A^t(n)^H \mathbf{K}_A^t(n)^{-1} \mathbf{y}_A^t(n)\} \quad (14)$$

$$\mathbf{K}_A^t(n) = \begin{bmatrix} P\sigma^2 + P\sigma_{RSI}^2 + \sigma_n^2 & P\sigma^2 c_R^t(n) + P\sigma_{RSI}^2 c_A^t(n) \\ P\sigma^2 c_R^t(n)^* + P\sigma_{RSI}^2 c_A^t(n)^* & P\sigma^2 + P\sigma_{RSI}^2 + \sigma_n^2 \end{bmatrix} \quad (15)$$

According to ML, we have

$$\hat{c}_R^t(n) = \arg \max f(\mathbf{y}_R^t(n) | c_R^t(n)) \quad (16)$$

Therefore, we get decoding result $\hat{c}_R^t(n)$ for $c_R^t(n)$. Then according to the Table 3, we get the corresponding $c_{AB}^t(n) = c_A^t(n) + c_B^t(n)$. Because A knows $c_A^t(n)$, the signals $\hat{c}_B^t(n)$ transmitted from the other can be decoded by

$$\hat{c}_B^t(n) = \hat{c}_R^t(n) \oplus c_A^{t-1}(n) \quad (17)$$

4. Simulation results

In this section, we analyse the BER performance of the full-duplex decode-and-forward two-way-relay (F2DF-TWR) system under the influence of self-interference and we compare it with the half-duplex decode-and-forward two-way-relay (H2DF-TWR) system. Figure 3 shows when the SNR is 20 and 30dB, the BER versus $1/\sigma_{RSI}^2$ curves will approach a value. Figure 4 shows the BER versus the SNR of the F2DF-TWR system with $1/\sigma_{RSI}^2=30$ and the H2DF-TWR, we can see that the curves of the two is similar.

Figure 3. The BER versus $1/\sigma_{RSI}^2$ curve of the F2DF-TWR

Figure 4. The BER comparison of F2DF-TWR with $1/\sigma_{RSI}^2=30$ and H2DF-TWR

5. Conclusion

In this paper, we investigate a full-duplex decode-and-forward two-way relaying (F2DF-TWR) system. To effectively use the spectrum resources and improve the rate, we apply the PNC based DF. By using DBPSK with MAP and ML decoder at the relay and the source node, respectively, we can correctly obtain the information from the other. The simulation results show that the F2DF-TWR system can achieve a good BER performance with the RSI cancelled well.

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