Position inner loop impedance control of flexible link and flexible joint

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Abstract. When space robot arm implement the task, there was a certain requirement to tracking and contacting force in order to ensure the accuracy of the mission, and the structure of the robot arm was also affect the controlling effect. For this reason, we took considering the effect of the flexible joint /flexible manipulators, established the dynamics model by Kane equation and design impedance controller. In the end, the control simulation platform was established in MATLAB/Simulink. And we contrasted the response of the flexible manipulator to the different controller. The simulation results show that flexibility of characteristics could cause jitter in some degrees. The design of the impedance controller can realize the trajectory tracking and contact force control of the flexible manipulator.

Introduction

With the rapid development of space technology, the use of robotic arm to assist the astronauts to complete various operations has become a hot research topic. The space manipulator is generally considered as slender rod, because of the large ratio of length to breadth, which will produce elastic deformation during its operation, and the presence of flexible intra-articular elements such as the joint of the harmonic reducer and the torque sensor causes the joint to produce the flexible deformation. Elastic deformation coupling of flexible joint / flexible link during motion will cause difficulties in the active control of the manipulator, thus designing a controller about flexible joint / flexible manipulator is needed.

Impedance control has been gradually applied to various fields. Hongan [1] proposed robot impedance control, which incorporated free motion space and constrained motion space into a unified control framework. After that many scholars have carried out research on impedance control. An adaptive impedance controller was designed to meet the requirements of robot contacting with the environment, and for the environmental uncertainty, the contact motion of a planar two-link flexible manipulator on a triangular concave surface was reviewed [2]. Qingqing Wei [3] proposed the impedance control method of flexible manipulator helping with the docking of large load space cabin, used Lagrange method to derive the joint input torque of space manipulator as feedforward input, established impedance control program of space manipulator with dynamics feedforward, and Linear motion simulation and follow-up motion simulation being made. Aimed at the problem that auxiliary force could not adjust in real time during the process of robot-assisted limb rehabilitation, a new fuzzy self-adaptive impedance control method was proposed by Guozheng Xu [4], which indicated that the impedance control method is effective for the control of the force. For the manipulator needing to be controlled by the operators directly during the cooperative task, Fanny Ficuciello designed the Cartesian space impedance controller to control the end effector, and used the KUKA LAWRA manipulator to make simulation and verification, which proved the feasibility of Cartesian impedance control algorithm [5]. Zhao-Hui Jiang [6] proposed an impedance control for the end trajectory control of the manipulator, to make the actual target impedance track the ideal impedance model. For the parameter uncertainty, the adaptive impedance control was designed and the effectiveness of the proposed method was proved by two-link flexible manipulator. But there is no
relevant report about the space manipulator with joint flexibility and link flexibility. But there is no relevant report about the space manipulator with flexible joint / flexible manipulator.

In this paper, the dynamic model of flexible manipulator and the control of contact operation impedance was designed. A Cartesian space impedance control algorithm was proposed for the coexistence of joint flexibility and manipulator flexibility. The control simulation platform was established in Matlab/Simulink for manipulator and the feasibility of the designed control algorithm was proved.

**Dynamic modeling of two flexible links/flexible joints**

Fig. 1 shows the two-link manipulator with two flexible joints/ flexible links.

![Figure 1. Sketch map of flexible manipulator.](image)

On the simplification of flexible joint modeling, the simplified model of flexible joint was first proposed by Spong [7], on the base a complete model was given by Readman [8]. In the process of modeling, the effects of friction and deformation of the gears between harmonic reducers were neglected, but only the mechanical dynamics was considered. The flexible joint was simplified as a linear torsion spring between the motor rotor and the connecting rod without considering the influence of the electrical dynamics. The ratio of the length and the diameter of the space manipulator was relatively large, and the flexible deformation often occurs in a wide range of motion, thus the deformation quantity was described by the hypothetical mode method considering the deformation of the arm.

According to the simplified description, the dynamic model was established by using the Kane method. Kane method is more suitable for the dynamic derivation of complex systems with multiple degrees of freedom, such as robot, manipulator, by introducing the concepts of deflection velocity and linear speed, which provides a new approach to the dynamics. According to the Kane equation of the particle system, the generalized coordinates of a manipulator were: \( y = \{ \alpha_1, \alpha_2, \theta_1, \theta_2, q_1, q_2 \} \)

Where, \( \alpha_i = \theta_{iw}/N_i \), \( \alpha_2 = \theta_{iw}/N_z \), \( \theta_{iw}, \theta_{ws} \) are motor angle, \( N_i, N_z \) are Reduction ratio, \( \theta_1, \theta_2, q_1, q_2 \) are Joint angle and modal coordinates.

\[
\begin{align*}
F_{yi} + F_{yi}^* &= 0 \quad (i = 1, \ldots, 6) \\
F_{yi}^* &= \int v_p^i a_p dm \quad (j = 1, 2) \\
F_{yi} &= \begin{cases} 
\tau_m & (1 \leq i \leq 2) \\
k \epsilon & (1 \leq i \leq 2) \\
-k \epsilon & (3 \leq i \leq 4) \\
\int EI \frac{\partial^2 \theta}{\partial x^2} & (3 \leq i \leq 6)
\end{cases}
\end{align*}
\]

Where, \( v_p^i \) is the partial velocity of the arm to the generalized coordinates, \( a_p^j \) is the acceleration of manipulator, \( \omega \) is the deformation of points, \( \tau \) is motor driving torque, \( k \) is linear torsional stiffness, \( \epsilon_i = \alpha_i - \theta_i \) (\( i = 1, 2 \)) is linear twist, \( EI \) is flexural rigidity of section, \( F_{yi} \) is generalized active force. \( F_{yi}^* \) is generalized inertial force.
As can be seen from the Fig. 1, \( P \) is the position of manipulator \( i \), \( \xi_i = r_i + \omega_i \) and \( \omega_i(x,t) = \phi(x)q(t) \) is elastic deformation. Thus the derivative velocity can be calculated according to the generalized coordinates. Bring the result to the Eq. 2, then the generalized inertial force of the flexible manipulator can be solved. The kinetic equation of the manipulator is established by the Kane equation. After removing the high-order coupling term of the kinetic equation, the dynamic equation can be written as follows:

\[
\begin{align*}
M(\theta, q)(\ddot{\theta}, \ddot{q}) + C(\theta, \dot{q}, \dot{q}) + H(\theta, q) &= \tau + \tau_{el} \\
J_{mi} \ddot{\omega} + \tau &= \tau_m \\
\tau &= k(\alpha - \theta)
\end{align*}
\]

(4)

The Eq. 4 can be abbreviated as:

\[
M \ddot{p} + C \dot{p} + Kp = Q
\]

(5)

Where, \( p = p(\theta, q) \), \( J_m \) is inertia of motor, \( M, C, K \) are generalized mass matrix, generalized damping matrix, generalized stiffness matrix. \( Q \) is generalized column matrix.

**Impedance control algorithm for the flexible joint/ flexible link**

Constrained by the external environment, link not only need precise position control, but also appropriate control of contact force in the process of assisting astronauts to carry out space operations, to avoid excessive impact damage to external cabin or body of the robotic arm. In order to meet this requirement, we adopted the control algorithm based on position inner loop in Cartesian space. Impedance control provides a control framework for the unity of free movement space and constraint space, using the contact force as the information source of the deviation between the expected position of the end of the robot arm and the actual location of environmental constraints. The impedance mode is used in this paper:

\[
M \ddot{E} + B \dot{E} + K \dot{E} = F_{ext}
\]

(6)

Where \( M_d \) is target inertia, \( B_d \) is target damping, \( K_d \) is target stiffness. \( E = X_d - X \) is position error, \( X_d \) is end desired target position, \( X \) is actual location, \( F_{ext} = K_e(X_d - X) \) is contact force, \( K_e \) is environmental stiffness.

For the dynamic Eq. 4, motor rotation angle can be used to represent joint angle, the calculation formula:

\[
\alpha = k^{-1}(M \ddot{p} + C(\dot{p}, \dot{p}) + H(p)) - k^{-1}J^T \dot{F}_{ext} + \theta
\]

(7)

The angular velocity and angular acceleration of the motor rotation angle can be obtained from the derivation of the Eq. 7, the kinetic formula of joint angle is:

\[
J_{mi}(k^{-1}M \ddot{p} + 2k^{-1}M \dot{p} \ddot{p} \dot{p} + k^{-1}M \dot{p} + k^{-1}(C(\dot{p}, \dot{p}) + H(p)) - k^{-1}J^T(\dot{p})F_{ext} - 2k^{-1}J^T(\dot{p})F_{ext})
\]

\[
- k^{-1}J^T(\dot{p})F_{ext} + \dot{\theta} + M \ddot{p} + C(\dot{p}, \dot{p}) + K(p) - J^T(\dot{p})F_{ext} = \tau_m
\]

(8)

According to Cartesian coordinates and definition of Jacobian Matrix \( J(p) \), The transformation relationship between joint space and operation space:

\[
\dot{x} = J(p) \dot{p}
\]

(9)

\[
\ddot{x} = J(p) \ddot{p} + J(p) \dot{J}(p) \dot{p}
\]

(10)

The formula of the position-based impedance control law of Cartesian space:

\[
\tau = J_{mi}k^{-1}M(p)J(p) \dot{p} - \beta_1 \dot{F}_{ext} - \beta_2 F_{ext} - G(p)
\]

(11)

Where, \( u = k_e(x - \dot{x}) + k_d(\dot{x} - \ddot{x}) \), \( x = x_0 - E \), \( \beta_1 = J^T(p) + J_{mi}k^{-1}J^T(\dot{p}) \), \( \beta_2 = 2J_{mi}k^{-1}J^T(p) \), \( \beta_3 = J_{mi}k^{-1}J^T(p) \)

\[
G(p) = C(p, \dot{p}) + H(p) + J_{mi}k^{-1}(C(p, \dot{p}) + H(p)) + J_{mi}k^{-1}M \ddot{p} + M \dddot{p} + H(p) + 2J_{mi}k^{-1}M \dddot{p}
\]
Compared with reference [9], Position inner loop and flexible joint factors are increased in the impedance control algorithm. The modal coordinates are introduced and the influence of arm deflection is considered. Impedance control flow chart can be obtained from Eq. 11, shown in Fig. 2:

\[
\begin{align*}
F_d &= \alpha \dot{q} + \beta q + \gamma \\
q &= q_d + \delta \\
\end{align*}
\]

Where PD is controlling position inner loop, \( k_p \) is proportional coefficient, \( k_d \) is differential coefficient. When the manipulator is not in contact with the external objects, \( F = 0 \), \( x_e = x_d \) actual trajectory tracking desired trajectory. When the end of the manipulator contact with the environment, the contact force generated and the information source of the position correction amount is provided to achieve the purpose of control.

**Simulation experiment based on Simulink platform**

In order to verify the effectiveness of the proposed algorithm which was included the flexible joints/ flexible manipulator. Took two link flexible manipulator were taken as object and we analysed the corresponding results of different controllers. The basic parameters of the flexible manipulator are shown in Table 1.

<table>
<thead>
<tr>
<th>Project</th>
<th>Index</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic parameters of flexible manipulator</td>
<td>Density 2.7*10^3Kg/m³</td>
<td>Inertia of joint 0.75*10^-4Kg.m²</td>
</tr>
<tr>
<td></td>
<td>Modal stiffness 11.36N.m²</td>
<td>Inertia of motor 0.106*10^-4Kg.m²</td>
</tr>
<tr>
<td></td>
<td>Cross-sectional area 1.2*10^-4m²</td>
<td>Rigidity 7500Nm/rad</td>
</tr>
</tbody>
</table>

The impedance control algorithm shown in Fig. 2 was inputted in Matlab, based on the function called System Function. The fourth-order Runge-Kutta method was adopted in the algorithm, where the simulation step was set to 0.01s and the simulation time was 5s. The initial joint position of the manipulator was that \( \theta_1 = \pi / 2 \) and \( \theta_2 = -\pi / 2 \), when the initial angular velocity of the joint was zero. In the simulation process, the desired trajectory of the manipulator is the Eq. 12. The desired contact force was 5N.

\[
\begin{align*}
\dot{x}_d &= \frac{\sqrt{2}}{10} t + \frac{\sqrt{2}}{2} \\
\dot{y}_d &= \frac{\sqrt{2}}{10} t + \frac{\sqrt{2}}{2} \\
\end{align*}
\]

There are three methods to obtain impedance control parameters, which are adaptive method, neural network learning method and acquiring from teaching data. The third method was used in the
paper, and the control parameters were adjusted through many simulation experiments. This paper took $k_p$ as 16, $k_d$ as 55. Control parameters were taken as:

$$
M_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}, \quad K_d = \begin{bmatrix} 123 & 0 \\ 0 & 123 \end{bmatrix}
$$

Fig. 3 is the actual trajectory of the end of the manipulator, it can be seen that the position of the inner loop impedance controller can achieve the basic trajectory tracking; while Fig. 5 and Fig. 6 show the error between the desired trajectory and the actual trajectory in the middle of motion. The maximum error $X$ direction of the impedance controller reaches 0.0075m and $Y$ direction reaches 0.024m, while the maximum error of the inner loop impedance controller was around 0.0015, and the error gradually tended to be stable.

Fig. 4 shows the deformation of the end of the robot arm during the movement, it can be seen that the deformation of the end of the impedance controller is larger Fig. 7 and Fig. 8 show the force control effect of $X$ direction and $Y$ direction in the process of flexible manipulator, and the position of the inner loop impedance controller is basically stable near the desired contact force.
Conclusions

In this paper, the impedance control algorithm is designed and applied to the flexible manipulator, which is used to solve the compliant control problem. We can conclude that the vibration caused by flexible arm / flexible joint included in the control process can’t be ignored. Comparative analysis of the response of the manipulator under different control algorithms we can draw that the design of impedance control algorithm in the force contact space can realize the force control of the manipulator, and prove the feasibility and effectiveness of the impedance control algorithm.

Acknowledgements

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References


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