

Nonlinear Stability Calculation Method of Crane Lattice Boom with Large Slenderness Ratio

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Abstract. Aiming at the stability problem of crane lattice boom with large slenderness ratio, the applicability and practicability of four kinds of design theories and methods (the allowable stress method based on linear theory, the allowable stress method based on nonlinear theory, the limit state method based on linear theory and the limit state method based on nonlinear theory) are studied by taking VS2012 as development platform and programming multi-theoretical software for the stability calculation of crane lattice boom with large slenderness ratio using C# programming language. The optimal theory and method are obtained by comparing the generalized margin of above four methods. Practical example results show that the limit state method based on nonlinear theory has the minimum generalized margin and highest material utilization on the premise of meeting the stability of boom which illustrates the linear theory based on small displacement assumption is unsuitable for the stability design calculation of lattice boom with large slenderness ratio while the limit state method based on nonlinear theory is more suitable for the occasion that the relationship between load and internal force is nonlinear and the calculation results are more in line with actual situation. Finally, the optimal theory and methods are verified through ANSYS, it can be found that analytic results are in good agreement with finite element results which further explains the suitability and practicability of the limit state method based on nonlinear theory.

Introduction

Due to the demand in the fields of wind and electricity power, metallurgy, petrochemical industry, etc, and for large-scale lifting operation, the load capacity of a crane tends to be heavier; but itself tends to be lighter, also it will have the features of larger space and higher efficiency. The application of high-strength steel not only makes crane boom structure more lightweight but also makes the flexibility of structure increased and geometry deformation under heavy load nonlinear. As a result, the translational displacement, rotational displacement and strain are no longer infinite small, and the constitutive relation presents a strong nonlinear. If the linear theory based on the assumption of small displacement is still used to analyze the stability of boom structure, the results will deviates from actual stress state which will cause an unsafe or incorrect statistic. Moreover, from structural mechanics perspective, strictly speaking, linear problem is a special case of nonlinear problems, and linear assumption is only a simplification of practical engineering problems. Practical engineering problems such as the research on elastic-plastic dynamic response of the structure under earthquake, wind resistance of high-rise building, dynamic stability of large

span reticulated shell, form finding and cutting analysis of cable membrane structure, and wind-induced vibration of large bridges are just assumed as linear problem are not enough, and further consideration is required for nonlinear problems. Therefore, the nonlinear analysis of various engineering structures is necessary and increasingly important. The stability analysis of crane lattice boom structure, which belongs to biaxial bending, need to consider the effects of geometric non-linearity because of larger slenderness ratio and smaller stiffness [1, 2]. Meanwhile, as the increase of steel yield limit, the conversion slenderness ratio of boom rises constantly while the stability coefficient decreases, which have highlighted structural stability issues [3]. To solve the challenging problems above, it is necessary to study stability calculation method.

Stability calculation of crane lattice boom

The stability of crane metal structure is one of the important indicators to measure whether structure is out of operation [3]. Hence, when conducting stability calculation of crane metal structure, close calculation formulas with actual stress state should be given. Due to smaller stiffness, when subjected to heavy load, structure will have a larger deformation which will lead to a nonlinear geometric relationship between the load and displacement, however, the relationship between the stress and strain of material is still linear-elastic. Therefore, the stability of boom structure belongs to the category of geometric nonlinear mechanics so that the second order nonlinear analysis should be adopted.

Overall Stability. The lattice boom structure presents a significant second order effect when bearing both transverse and axial load so the additional bending moment caused by initial defect and axial compression must be involved in stability calculation. When using the limit state method based on nonlinear theory to calculate the overall stability, N_{Ex} and N_{Ey} , within the increase coefficient $(1 - \gamma_m N / N_{Ex}, 1 - \gamma_m N / N_{Ey})$ should be divided by the resistance coefficient γ_m . The computation formula is as follow [3]:

$$\frac{N}{\varphi A} + \frac{M_x(z)}{(1 - \gamma_m N / N_{Ex}) W_x(z)} + \frac{M_y(z)}{(1 - \gamma_m N / N_{Ey}) W_y(z)} \leq \lim \sigma \quad (1)$$

Where, N is the axial force of boom, A is the cross-sectional area, $\lim \sigma$ is the limit stress value of material, N_{Ex} is the critical force in amplitude plane, N_{Ey} is the critical force in rotary plane, $M_x(z)$ is the basic moment of section z produced in amplitude plane, $M_y(z)$ is the basic moment of section z produced in rotary plane, $W_x(z)$ is the section modulus in bending of lattice boom section to the x axis, $W_y(z)$ is the section modulus in bending of lattice boom section to the y axis, φ is the stability coefficient determined by cross section type and maximum slenderness ratio λ or hypothetical slenderness ratio λF .

The calculation formula of the stability coefficient φ is as follows:

$$\varphi = \begin{cases} 1 - \alpha_1 \lambda_n^2, \lambda_n = \frac{\lambda}{\pi} \sqrt{\sigma_s / E} \leq 0.215 \\ \frac{1}{2 \lambda_n^2} [(\alpha_2 + \alpha_3 \lambda_n + \lambda_n^2) - \sqrt{(\alpha_2 + \alpha_3 \lambda_n + \lambda_n^2)^2 - 4 \lambda_n^2}], \lambda_n > 0.215 \end{cases} \quad (2)$$

Where, α_1 , α_2 and α_3 are the calculation coefficient determined by the cross section type, λ_n is the regular slenderness ratio [3].

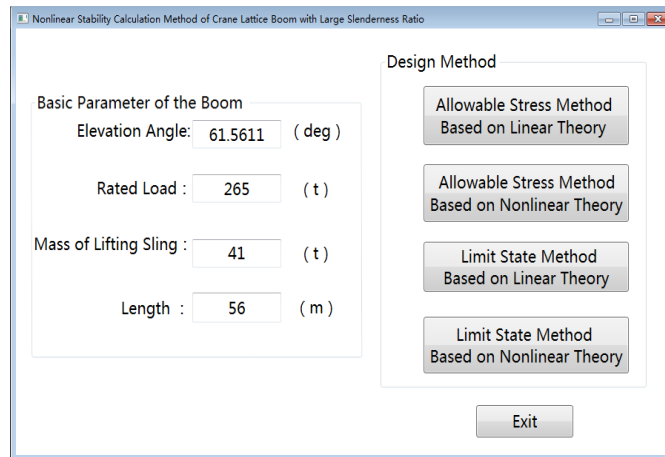


Fig. 1. Software interface

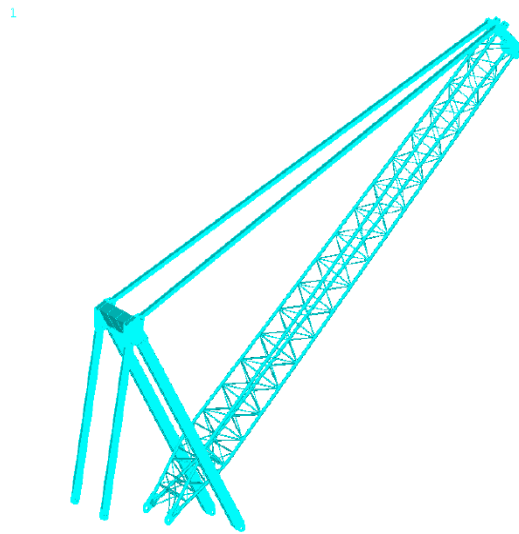


Fig.2. 290t lattice boom crane

Single Limb Stability. For statically determinate structure, the single limb instability will cause the evolution from the initial structure geometric construction to geometrically unstable system. Structure with low load-carrying capacity cannot maintain the initial equilibrium shape which definitely results in instability of the whole structure. For statically indeterminate structures, the individual single limb instability does not necessarily cause instability of the whole structure, but the internal force bore by above unstable single limb will be transferred to other single limb which will buckle when another new limit state is reached, similar processes continue to evolve and the number of unstable single limb accumulates continuously until enough to induce the overall instability. For crane lattice boom system, a higher statically indeterminate structure which consists of many delicate beams and columns and the overall instability can be seen as constant accumulation of the single limb instability, so it is considerable to implement the single limb stability calculation. The computation formula is as follow [3]:

$$\frac{1}{A_x \varphi} \left[\frac{N}{4} + \frac{M_x(z)}{2(1-\gamma_m N/N_{Ex})B} + \frac{M_y(z)}{2(1-\gamma_m N/N_{Ey})H} \right] \leq \lim \sigma \quad (3)$$

Where, A_x is the single limb cross-sectional area, B and H are the width and height of calculation section respectively.

Parametric realization and calculation process

With Microsoft Visual Studio 2012 as development platform, calculation software based on multi-theory and multi-method for the crane lattice boom stability was developed by using C# language, as shown in Fig. 1. The calculation process is as follows:

- 1) Build the mechanical analytical model of lattice boom, then on the basis of the working characteristics of lattice boom, do load analysis and load combinations, and determine the corresponding safety coefficient and resistance coefficient.
- 2) Program calculation software based on four design theories and methods (the allowable stress method based on linear theory, the allowable stress method based on nonlinear theory, the limit state method based on linear theory and the limit state method based on nonlinear theory).
- 3) Input the working condition (elevation angle, lifting weight and other basic parameters), and select suitable calculation method for analytical calculation.
- 4) Output the stability calculation report about the dangerous section in the form of text.
- 5) Obtain the optimum theory and method by comparing the generalized margin of various methods.

Engineering example

Taking 290t lattice boom crane (28 kinds of working conditions) provided by an enterprise load combinations B1 as an engineering example, as shown in Fig. 2. Load and load combinations B [4] are shown in Table 1.

Table 1. Load and load combinations B of lattice boom crane based on limit state method

List of Loads			Load combinations B					
			Partial load factors	B1	B2	B3	B4	B5
Gravity acceleration and impacts	Mass of boom		$\gamma_{PB1} = 1.16$	φ_1	φ_1	1	—	—
	Mass of rated load		$\gamma_{PB2} = 1.28$	φ_2	φ_3	1	—	—
Acceleration from drives	Mass of boom and rated load	Hoist drives excluded	$\gamma_{PB3} = 1.48$	φ_5	φ_5	—	φ_5	—
		Hoist drives included	$\gamma_{PB4} = 1.48$	—	—	φ_5	—	—
In-service wind			$\gamma_{PB5} = 1.16$	1	1	1	1	1
Resistance coefficient			$\gamma_m = 1.1$					
High risk coefficient			$\gamma_n = 1.1$					

Three dangerous sections (the head section, the center of gravity section, the root section) are selected, and the key parameters of each dangerous section are listed in Table 2. Based on two theories and two methods, the stress of the overall stability and single limb stability of each dangerous section are rapidly and accurately calculated by using the compiled software. Due to space limitations, there will only be the stability stress calculation results of 6 kinds of typical working conditions (as shown in Table 3) given, listed in Table 4.

Table 2. Parameter of dangerous section

Location of dangerous section	Cross-sectional area/mm ²	Height/mm	Width/mm	Equivalent slenderness Ratio λ_{hx}	Equivalent slenderness Ratio λ_{hy}
head	59208	2900	1776	43.7	48
center of gravity	59208	2900	3477		
root	59208	967	5000		

Table 3. Typical working conditions table

Condition No.	Rated capacity/t	Radius/m	Elevation angle / °
1	265	38.88	74.0840
2	265	55.00	66.0006
3	265	63.50	61.5611
4	250	66.00	60.2242
5	240	69.00	58.5982
6	130	99.89	39.6627

Table 4. Results of stability calculation

Condition No.	Method	Theory	Location of dangerous section						limσ/admσ /MPa
			Head		Center of gravity		Root		
			S_{Z1}/MPa	S_{D1}/MPa	S_{Z2}/MPa	S_{D2}/MPa	S_{Z3}/MPa	S_{D3}/MPa	
1	Allowable stress method	Linear	197.2671	162.4446	193.3098	152.3100	201.4380	149.3214	395.5/324.63
		Nonlinear	200.1164	166.3779	195.3126	154.8618	204.0518	151.6932	
	Limit state method	Linear	252.1013	208.8594	247.4435	197.1220	251.6446	189.8646	
		Nonlinear	257.8817	216.8399	252.0288	203.0394	256.3881	195.4619	
2	Allowable stress method	Linear	209.3195	169.1356	227.6734	186.3518	233.1346	183.4296	
		Nonlinear	211.1730	171.0638	232.1390	190.8606	237.8755	188.0251	
	Limit state method	Linear	262.3811	211.4011	282.8945	230.5137	288.8277	226.0235	
		Nonlinear	265.2989	214.4220	290.3315	237.9500	296.8114	233.6149	
3	Allowable stress method	Linear	236.1253	197.8025	250.3849	210.7879	249.3099	200.7553	
		Nonlinear	241.0002	202.9207	257.3730	217.9471	255.3989	206.7688	
	Limit state method	Linear	296.0332	247.4274	311.0258	260.7892	308.6850	247.2452	
		Nonlinear	304.5973	256.4090	323.0246	273.0119	318.9849	257.2565	
4	Allowable stress method	Linear	233.7863	197.7499	247.1595	209.8661	244.5926	198.5405	
		Nonlinear	239.2158	203.4569	254.4753	217.3904	250.7701	204.6916	
	Limit state method	Linear	292.8448	247.1627	306.6054	259.2998	302.3741	244.1088	
		Nonlinear	302.4326	257.2320	319.1557	272.1453	312.7620	254.3042	
5	Allowable stress method	Linear	235.7607	201.5008	248.1338	212.6059	243.7414	199.5090	
		Nonlinear	241.9770	208.0404	256.0456	220.7673	250.2120	205.9920	
	Limit state method	Linear	295.1391	251.7269	307.4972	262.4309	300.9492	244.9800	
		Nonlinear	306.1841	263.3386	321.0928	276.3969	311.7983	255.7084	
6	Allowable stress method	Linear	219.4839	199.8268	228.8654	208.3326	213.5931	186.2087	
		Nonlinear	228.2847	209.1134	238.6048	218.5327	220.9264	193.8650	
	Limit state method	Linear	271.9461	247.2312	279.2012	253.1865	258.9324	224.2816	
		Nonlinear	287.3912	263.5259	295.5309	270.2648	270.7484	236.5552	

Note: S_{Zi} ($i=1, 2, 3$) represents the stress of the overall stability, S_{Di} ($i=1, 2, 3$) represents the stress of the single limb stability, $\text{adm}\sigma$ is the allowable stress.

In order to compare the material utilization, strength reserve and dangerous condition under different design methods, the concept of generalized margin is introduced.

For allowable stress method, the definition of generalized margin is as follow:

$$n_{\sigma} = 1 - \sigma / \text{adm}\sigma \quad (4)$$

For limit stress method, the definition of generalized margin is as follow:

$$n_{\sigma} = 1 - \sigma / \lim \sigma \quad (5)$$

According to above definition, the generalized margin of each method is gotten from Table 4, listed in Table 5.

Table 5. Generalized margin of each method

Condition No.	Method	Theory	Location of dangerous section					
			Head		Center of gravity		Root	
			n_{z1}	n_{D1}	n_{z2}	n_{D2}	n_{z3}	n_{D3}
1	Allowable stress method	Linear	0.3923	0.4996	0.4045	0.5308	0.3795	0.5400
		Nonlinear	0.3836	0.4875	0.3984	0.5230	0.3714	0.5327
	Limit state method	Linear	0.3626	0.4719	0.3744	0.5016	0.3637	0.5199
		Nonlinear	0.3480	0.4517	0.3628	0.4866	0.3517	0.5058
2	Allowable stress method	Linear	0.3552	0.4790	0.2987	0.4260	0.2818	0.4350
		Nonlinear	0.3495	0.4730	0.2849	0.4121	0.2672	0.4208
	Limit state method	Linear	0.3366	0.4655	0.2847	0.4172	0.2697	0.4285
		Nonlinear	0.3292	0.4578	0.2659	0.3984	0.2495	0.4093
3	Allowable stress method	Linear	0.2726	0.3907	0.2287	0.3507	0.2320	0.3816
		Nonlinear	0.2576	0.3749	0.2072	0.3286	0.2133	0.3631
	Limit state method	Linear	0.2515	0.3744	0.2136	0.3406	0.2195	0.3749
		Nonlinear	0.2298	0.3517	0.1833	0.3097	0.1935	0.3495
4	Allowable stress method	Linear	0.2798	0.3908	0.2386	0.3535	0.2465	0.3884
		Nonlinear	0.2631	0.3733	0.2161	0.3303	0.2275	0.3695
	Limit state method	Linear	0.2596	0.3751	0.2248	0.3444	0.2355	0.3828
		Nonlinear	0.2353	0.3496	0.1930	0.3119	0.2092	0.3570
5	Allowable stress method	Linear	0.2738	0.3793	0.2356	0.3451	0.2492	0.3854
		Nonlinear	0.2546	0.3591	0.2113	0.3199	0.2292	0.3655
	Limit state method	Linear	0.2538	0.3635	0.2225	0.3365	0.2391	0.3806
		Nonlinear	0.2258	0.3342	0.1881	0.3011	0.2116	0.3535
6	Allowable stress method	Linear	0.3239	0.3844	0.2950	0.3582	0.3420	0.4264
		Nonlinear	0.2968	0.3558	0.2650	0.3268	0.3195	0.4028
	Limit state method	Linear	0.3124	0.3749	0.2941	0.3598	0.3453	0.4329
		Nonlinear	0.2733	0.3337	0.2528	0.3167	0.3154	0.4019

Note: n_{zi} ($i=1, 2, 3$) represents the generalized margin of the overall stability, n_{Di} ($i=1, 2, 3$) represents the generalized margin of the single limb stability.

To visually compare and analyze the difference among the generalized margin of each method, the data-changing tendency of Table 5 is shown in Fig. 3.

From Fig. 3, it can be seen that the generalized margin values based on 4 calculation methods, 6 calculation conditions, 3 calculation cross sections, and 2 calculation items are all less than 1, which indicates that the stability of boom can meet the design specification requirements. The limit state method based on nonlinear theory has the minimum generalized margin, which proves that the geometrical deformation of lattice boom structure is nonlinear, when this method is used to calculate the stability of boom, it can not only give full play to the bearing capacity of material, but also has a reliable generalized margin. Structural nonlinearity decreases as the elevation angle reduces, as a result, the gap between the generalized margin of limit state method based on nonlinear theory and allowable stress method based on nonlinear theory is narrowing. To fully explain the reasons for above phenomenon, an analysis of overall stability stress of the limit state method based on nonlinear theory and allowable stress method based on nonlinear theory is implemented, each stress is shown in Table 6 and Table 7.

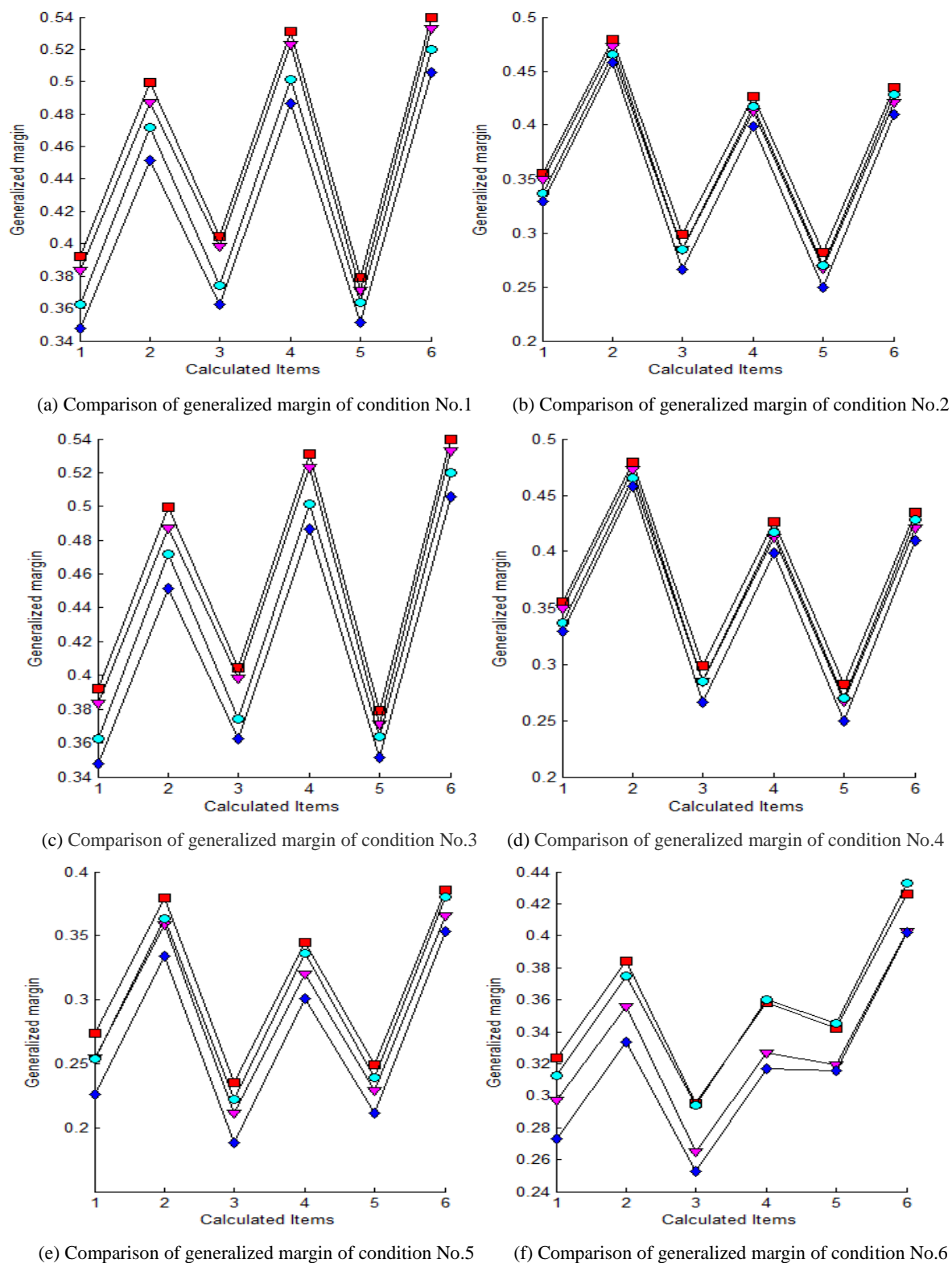


Figure 3. Comparison of generalized margin of each working condition

Table 6. Analysis of the overall stability stress of limit state method based on nonlinear theory

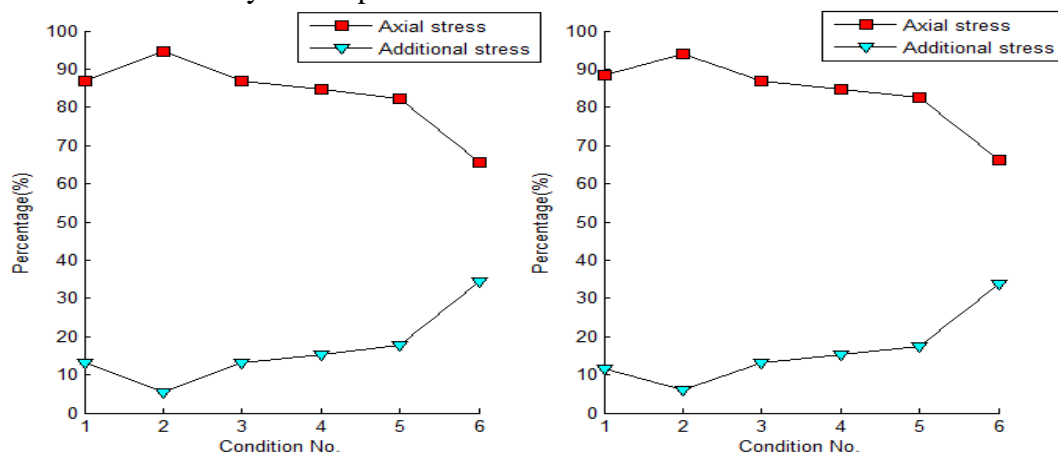
Condition No.	1	2	3	4	5	6
The head dangerous section	Total stress	257.8817	265.2989	304.5973	302.4326	306.1841
	Axial stress	224.1320	251.0002	264.3469	255.9979	252.0449
	Stress in rotary plane	1.5842	1.9344	2.1168	2.0206	2.0118
	Stress in amplitude plane	32.1655	12.3643	38.1336	44.4141	52.1274
The center of gravity dangerous section	Total stress	252.0288	290.3315	323.0246	319.1557	321.0928
	Axial stress	230.6570	256.9178	269.9370	261.4883	257.4131
	Stress in rotary plane	10.1652	12.1605	13.1972	12.6237	12.5583
	Stress in amplitude plane	11.2066	21.2533	39.8904	45.0437	51.1214
The root dangerous section	Total stress	256.3881	296.8114	318.9849	312.7620	311.7983
	Axial stress	238.4737	264.0067	276.6336	268.0656	263.8438
	Stress in rotary plane	16.8547	19.7450	21.2426	20.3625	20.2380
	Stress in amplitude plane	1.0597	13.0598	21.1086	24.3339	27.7165

Table 7. Analysis of the overall stability stress of allowable stress method based on nonlinear theory

Condition No.	1	2	3	4	5	6
The head dangerous section	Total stress	200.1164	211.1730	241.0002	239.2158	241.9770
	Axial stress	176.9447	198.4161	209.0856	202.6328	199.6274
	Stress in rotary plane	1.1030	1.3190	1.4319	1.3700	1.3684
	Stress in amplitude plane	22.0686	11.4379	30.4827	35.2130	40.9812
The center of gravity dangerous section	Total stress	195.3126	232.1390	257.3730	254.4753	256.0456
	Axial stress	182.8766	203.7958	214.1675	207.6241	204.5075
	Stress in rotary plane	7.1541	8.3838	9.0254	8.6602	8.6435
	Stress in amplitude plane	5.2819	19.9595	34.1802	38.1910	42.8946
The root dangerous section	Total stress	204.0518	237.8755	255.3989	250.7701	250.2120
	Axial stress	189.9826	210.2402	220.2554	213.6034	210.3537
	Stress in rotary plane	11.9787	13.7592	14.6864	14.1325	14.0935
	Stress in amplitude plane	2.0905	13.8761	20.4571	23.0342	25.7648

Note: Total stress= Axial stress+ Stress in rotary plane+ Stress in amplitude plane, the unit of stress is MPa.

For an intuitive analysis of the cause for above phenomenon, the percentage of each stress is gotten from Table 6 and Table 7, as shown in Fig. 4~Fig. 6. Simultaneously, the sum of stress in rotary plane and stress in amplitude plane caused by bending moment is considered as an additional stress, which can make it easy to compare.



(a) Limit state method based on nonlinear theory (b) Allowable stress method based on nonlinear theory

Figure 4. Proportion of overall stability stress of the head dangerous section

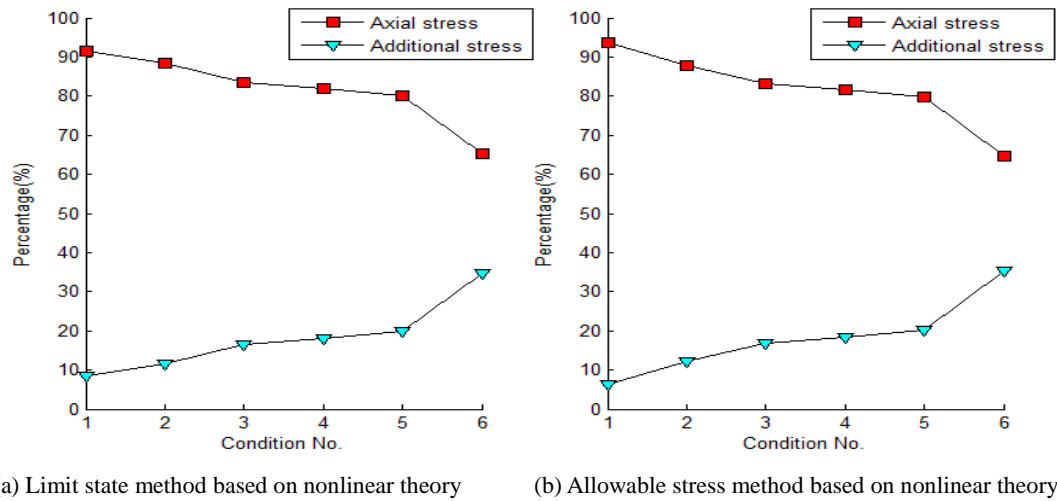


Figure 5. Proportion of overall stability stress of the center of gravity dangerous section

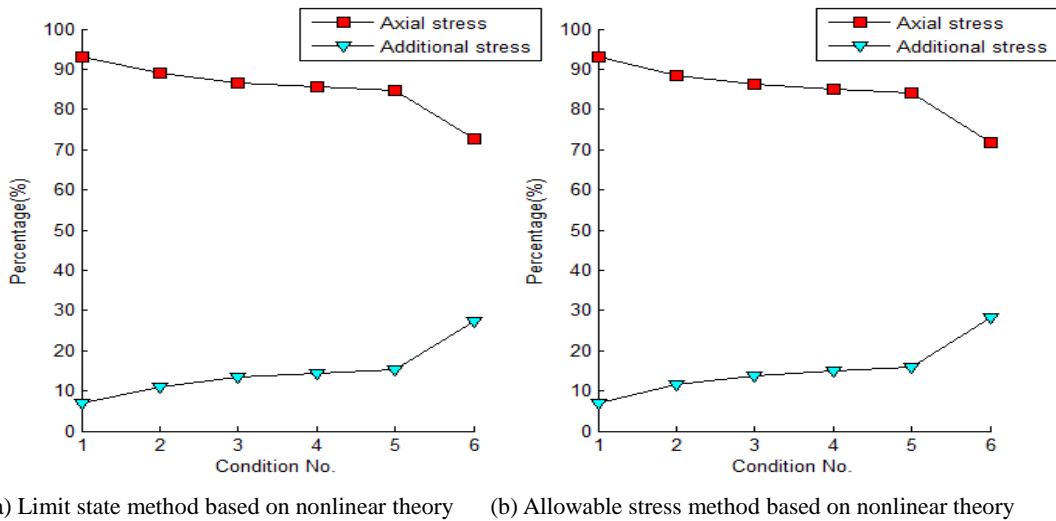


Figure 6. Proportion of overall stability stress of the root dangerous section

From the data gotten by comparative analysis of the data in Fig. 4~Fig. 6, with the reduction of boom elevation angle (the increase of condition No.), the percentage of axial stress $\sigma_N = N/\varphi A$ in total stress decreases, on the contrary, the percentage of additional stress $\sigma_M = M/((1 - N/N_E) W)$ in total stress rises, that is to say, structural nonlinearity lowers with the reduction of elevation angle. For condition No.1~condition No.3, the elevation angle is relatively larger and structural nonlinearity is much higher, as a result, the percentage of axial stress $\sigma_N = N/\varphi A$ in total stress is larger while the percentage of additional stress $\sigma_M = M/((1 - N/N_E) W)$ in total stress is smaller. Nevertheless, the data in condition No.4~condition No.6 have the opposite characteristics compared with condition No.1~condition No.3. Therefore, to some degree, the limit state method based on nonlinear theory and allowable stress method based on nonlinear theory are proved to be scientific and adaptive.

To further demonstrate the suitability and adaption of limit state method based on nonlinear theory for the stability calculation of lattice boom, a numerical simulation under 6 kinds of working condition (as shown in Table 3) is performed based on limit state method ANSYS. Considering that the overall stability stress in dangerous section cannot be calculated by using ANSYS, it is better to extract the single limb stability stress in each dangerous section to compare with analytic solution of the single limb stability stress calculated by limit state method based on nonlinear theory. The results of comparison are listed in Table 8~ Table 10.

From the data in Table 8~Table 10, it can be observed that the error ε between σ_J and σ_A is less

than 5% which is within the allowed range, the main reasons for error-making are as follows:

1) Analytic solution σ_J based on the second order nonlinear theory fully considers the second order stress caused by the deformation of biaxial bending and the additional stress generated from the geometric features of structural section through the moment amplification coefficient and stability coefficient while the finite element modeling is relatively ideal, it does not take the influence caused by the eccentric load and geometric features of structural section into account, so the calculated value is relatively small.

2) ANSYS solution σ_A uses branch buckling load based on the first class stable, namely the Euler critical force of centrally loaded columns, and fails to apply the second class stable compressive load of eccentrically loaded columns which is being in exploration period for the stability calculation of lattice boom, but finite element method results, the powerful simulation validation of analytical solution and CAE technology, can visually display the stress distribution in each key section and component of the lattice boom.

Table 8. Comparison of head dangerous section

Condition No.	σ_J (MPa)	σ_A (MPa)	ε (%)
1	216.8399	212.211	2.135
2	214.4220	207.132	3.400
3	256.4090	248.564	3.060
4	257.2320	249.443	3.028
5	263.3386	251.732	4.407
6	263.5259	255.327	3.111

Table 9. Comparison of center of gravity dangerous section

Condition No.	σ_J (MPa)	σ_A (MPa)	ε (%)
1	203.0394	196.712	3.116
2	237.9500	231.431	2.740
3	273.0119	265.561	2.729
4	272.1453	263.342	3.235
5	276.3969	266.741	3.493
6	270.2648	260.773	3.512

Table 10. Comparison of root dangerous section

Condition No.	σ_J (MPa)	σ_A (MPa)	ε (%)
1	195.4619	191.273	2.143
2	233.6149	229.780	1.642
3	257.2565	248.912	3.244
4	254.3042	246.221	3.179
5	255.7084	247.823	3.084
6	236.5552	226.881	4.090

Note: $\varepsilon = |\sigma_J - \sigma_A| / \sigma_J \times 100\%$, where, σ_J is analytic solution of single limb stability stress, σ_A is ANSYS solution of single limb stability stress.

Conclusions

1) Compared with traditional allowable stress method, the limit state method on the basis of probability theory is more suitable for the occasion in which the relationship between load and internal force is nonlinear. This method can accurately consider the roles of load, mechanical properties of steel material, performing characteristics of structure and other factors for the lattice boom using partial load coefficient and resistance coefficient instead of the single safety coefficient to calculate the stability. The calculating results are more in line with actual situation.

2) The limit state method based on nonlinear theory can take into full account the second order stress caused by the deformation of biaxial bending and the additional stress generated from geometric features of structural section through the moment amplification coefficient and stability coefficient, and the economical efficiency of structure with enough safety stocks are ensured by adopting this method.

3) As shown in Fig. 4~Fig. 6, with the decline of structural nonlinearity, the fairly perfect results can be given by means of allowable stress method.

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