Reliability Optimum Design for Bevel Gear Driven Systems Based on Genetics Algorithm

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Abstract. Genetics algorithm is a global group search optimum calculation method based on the natural selections and the genetic variations. Mechanic reliability design makes sure that the reliability indexes can be realized. However, it cannot guarantee that the products have the best working performances and the optimum design parameters. In this paper, genetics algorithm is introduced into the reliability optimum design for bevel gear driven systems, and then the optimum design parameters of driven systems are obtained under the allowable reliability degree. This method can quantificational make optimum analysis for bevel gear driven systems, and is very useful for the engineering application.

Keywords: Bevel gear driven systems; reliability optimum design; random parameters; genetics algorithm.

1. Introduction

With the development of modern industry and machinery science technology, the bevel gear driven system is required to have the characteristics of light-duty, high-speed, large carrying power, long life and high reliability [1-3]. In the era of rapid development computer, compared with the actual situation, the accuracy of calculation parameters is far behind the precise analysis of engineering structures. Therefore, considering the random factors of actual situation have a very significant impact on the reliability optimum design for bevel gear driven systems based on genetics algorithm.

Genetics algorithm describes problems by a single string, makes optimum calculation by using the fitness function without regard to the gradient information such as functional derivatives, etc. Therefore, it is good at solving the complex nonlinear problems that other subjects don’t solve or are difficult to solve. Reliability optimum design for bevel gear driven systems based on genetics algorithm can guarantee that the driven systems have the best working performances and the optimum design parameters.

2. Establishing Reliability Optimum Mathematical Model for Bevel Gear Driven Systems

2.1 Establishing Optimum Objective Function.

The least all-in cost of bevel gear driven system design is set for the objective function. It can be written as:

\[ \min_{C_T} = C_1(\mu_d) + C_2(\sigma_d) + C_3(\mu_s) + C_4(\sigma_s) \]

Where, \( C_T \) is the all-in cost, \( C_1(\mu_d) \) and \( C_2(\sigma_d) \) are respectively the cost functions of strength equalizing value and strength standard deviation, \( C_3(\mu_s) \) and \( C_4(\sigma_s) \) are respectively the cost functions of stress equalizing value and stress standard deviation.

In many factors, the cost of bevel gear material is an important factor for the all-in cost. Therefore, the cost of material is mainly taken into consideration. \( C_T \) is set for:

\[ C_T = (V_1 + V_2) \rho \]

Where, \( \rho \) is the material density of bevel gear, \( V_1 \) and \( V_2 \) are respectively the volumes of driving gear and driven gear. The bevel gear drive mechanism is shown in Fig.1. The gear mesh volumes of driving gear and driven gear are respectively obtained, as shown in Eqs.1 and 2.
\[ V_1 = \frac{\pi}{3} b \cos \delta_1 \times \left( \frac{m_{z_1}}{2} \right)^2 + \frac{m_{z_2}}{2} \left( \frac{R - b \times m_{z_1}}{R} \right) + \left( \frac{R - b \times m_{z_2}}{R} \right)^2 \]  

(1)

\[ V_2 = \frac{\pi}{3} b \cos \delta_2 \times \left( \frac{m_{z_2}}{2} \right)^2 + \frac{m_{z_1}}{2} \left( \frac{R - b \times m_{z_2}}{R} \right) + \left( \frac{R - b \times m_{z_1}}{R} \right)^2 \]  

(2)

Fig. 1 Driven structure of orthogonal bevel gears

Where, \( z_1 \) and \( z_2 \) are respectively the tooth numbers of driving gear and driven gear; \( m \) is the big end module; \( R \) is the cone distance; \( b \) is the tooth width; \( \delta_1 \) and \( \delta_2 \) are respectively the reference cone angles of driving gear and driven gear.

2.2 Determining Reliability Optimum Constraints.

The contact stresses of driving gear and driven gear are calculated respectively. We selected the smaller value. The contact stress of bevel gear is obtained by Eq. 3.

\[ \sigma_H = Z_H Z_u Z_p Z_\varepsilon Z_\beta \frac{K_A K_v K_{\alpha H} F_{tm}}{d_{ml} \delta_H} \sqrt{u^2 + 1} \]  

(3)

Where, \( u \) is the gear ratio; \( Z_H \) is the node region coefficient; \( Z_E \) is the elastic coefficient and we set \( Z_E = 189.8 \text{ N/mm}^2 \); \( Z_\varepsilon \) is the overlap ratio coefficient and we set \( Z_\varepsilon = \frac{4}{\alpha - \varepsilon} \); \( Z_\beta \) is the helix angle coefficient of contact strength calculation; we set \( Z_\beta = 0.85 \); \( K_A \) is the utilization coefficient; \( K_V \) is the dynamic load factor; \( K_{\alpha H} \) is the distribution coefficient of longitudinal form load; \( K_{\alpha v} \) is the partition ratio of longitudinal form load; \( F_{tm} \) is the nominal shear stress on pitch circle of tooth width midpoint; \( d_{ml} \) is the pitch circle diameter of virtual cylindrical gear of bevel pinion; \( b_{eH} \) is the effective tooth width and we set \( b_{eH} = 0.85 b \); \( u \) is the gear ratio.

We selected the equalizing values of all parameters in Eq. 3. Therefore, the contact stress equalizing value of bevel gear \( \bar{\sigma}_H \) is determined. Meanwhile, according to the literature [2], the variation coefficient of contact stress can be written as [4-6]:

\[ C_{\sigma_H} = \left[ C_{\sigma_H} + C_{\varepsilon} + C_{\delta} + C_{\rho} + \frac{1}{4} \left( C_{\varepsilon} + C_{\rho} + C_{\delta} + C_{\alpha H} + C_{\varepsilon} + C_{\rho} + C_{\delta} + C_{\alpha H} \right) \right]^{1/2} \]  

(4)

The standard deviation of contact stress is:

\[ s_{\sigma_H} = C_{\sigma_H} \times \bar{\sigma}_H \]  

(5)

The allowable contact stress is:

\[ \sigma_{H_{lim}} = \frac{s_{\sigma_H}}{Z_{\varepsilon} Z_{\rho} Z_{\delta} Z_{\chi}} \]  

(6)

Where, \( \sigma_{H\text{lim}} \) is the contact fatigue strength limit of experimental gear; \( Z_{\varepsilon} \) is the least safety factor of contact strength calculation; \( Z_L \) is the lubricant coefficient; \( Z_V \) is the velocity coefficient; \( Z_R \) is the tooth width coefficient; \( Z_{\chi} \) is the environmental condition coefficient.
is the coefficient of rugosity; $Z_X$ is the dimension coefficient of contact strength calculation. The above parameters can be determined by the corresponding forms of literature [1].

We selected the equalizing values of all parameters in Eq. 6. Therefore, the allowable contact stress equalizing value of bevel gear $\bar{\sigma}_{hp}$ is determined. Meanwhile, according to the literature [2], the variation coefficient of allowable contact stress can be written as:

$$C_{\sigma_{hp}} = \left[ C_{\sigma_{\sigma_{hp}}} + C_{\sigma_{Z_X}} + C_{\sigma_{Z_m}} + C_{\sigma_{Z_v}} + C_{\sigma_{Z_S}} \right]^{1/2}$$

(7)

The standard deviation of allowable contact stress is:

$$s_{\sigma_{hp}} = C_{\sigma_{hp}} \times \bar{\sigma}_{hp}$$

(8)

According to the above conditions, the reliability optimum constraint of contact stress can be determined as:

$$G_i(x) = R_h - [R_h] \geq 0$$

(9)

Suppose stress and strength all conform to the normal distribution, the constraint can be written as:

$$z_{R_h} = \frac{\mu_{\sigma_{hp}} - \mu_{\sigma_{hp}}}{\sqrt{\sigma_{\sigma_{hp}}^2 + \sigma_{\sigma_{hp}}^2}} \geq z_{R_h}$$

In the same way, the dedendum flexural stress of bevel gear can be written as:

$$\sigma_f = \frac{K_A K_V K_{\beta_f} F_{tm} Y_{Fa} Y_{\delta} Y_\beta}{b_{pm} m_{pm}}$$

(10)

Where, $K_A$ is the utilization coefficient; $K_V$ is the dynamic load factor; $K_{\beta_f}$ is the distribution coefficient of longitudinal form load of flexural strength calculation; $K_{Fa}$ is the partition ratio of tooth space load; $F_{tm}$ is the nominal shear stress on pitch circle of tooth width midpoint; we set $b_{ef} = 0.85b$; $m_{pm}$ is the normal module of tooth width midpoint; $Y_{Fa}$ is the stress correction factor; $Y_\delta$ is the overlap ratio coefficient; $Y_\beta$ is the helix angle coefficient; we set $Y_k = 1$.

We selected the equalizing values of all parameters in Eq. 10. Therefore, the dedendum flexural stress equalizing value of bevel gear $\bar{\sigma}_f$ is determined. Meanwhile, according to the literature [2], the variation coefficient of allowable dedendum flexural stress can be written as:

$$C_{\sigma_f} = \left[ C_{\sigma_{\sigma_{F_{tm}}} + C_{\sigma_{K_A}} + C_{\sigma_{K_V}} + C_{\sigma_{K_{\beta_f}}} + C_{\sigma_{F_{tm}}} + C_{\sigma_{Y_{Fa}}} + C_{\sigma_{Y_{\delta}}} + C_{\sigma_{Y_\beta}} + C_{\sigma_{Y_\delta}} \right]^{1/2}$$

(11)

The standard deviation of dedendum flexural stress is:

$$s_{\sigma_f} = C_{\sigma_f} \times \bar{\sigma}_f$$

The allowable dedendum flexural stress is:

$$\bar{\sigma}_{fp} = \frac{\sigma_{F_{lim}} Y_{ST} Y_{\delta_{relT}} Y_{relT} Y_X}{S_{F_{min}}}$$

(12)

Where, $\sigma_{F_{lim}}$ is the bending fatigue limit of experimental gear; $Y_{ST}$ is the stress correction factor of experimental gear and we set $Y_{ST} = 2.0$; $S_{F_{min}}$ is the least safety factor of flexural strength calculation; $Y_{\delta_{relT}}$ is the sensitivity coefficient of relative tooth root fillet; $Y_{relT}$ is the condition coefficient of relative heel of tooth; $Y_X$ is the dimension coefficient of flexural strength calculation; The above parameters can be determined by the corresponding forms of literature [1].

We selected the equalizing values of all parameters in Eq. 12. Therefore, the allowable dedendum flexural stress equalizing value $\bar{\sigma}_{fp}$ is determined. Meanwhile, according to the literature [2], the variation coefficient of allowable dedendum flexural stress can be written as:

$$C_{\sigma_{fp}} = \left[ C_{\sigma_{\sigma_{F_{lim}}} + C_{\sigma_{Y_{ST}}} + C_{\sigma_{Y_{\delta_{relT}}} + C_{\sigma_{Y_{relT}}} + C_{\sigma_{Y_X}} \right]^{1/2}$$

The standard deviation of allowable dedendum flexural stress is:

$$s_{\sigma_{fp}} = C_{\sigma_{fp}} \times \bar{\sigma}_{fp}$$

According to the above conditions, the reliability optimum constraint of dedendum flexural stress can be determined as:
\[ G_2(x) = R_F - [R_F] \geq 0 \]  

Suppose stress and strength all conform to the normal distribution, the constraint can be written as:

\[ z_{R_F} = \frac{\mu_{\sigma_{R_{F}}}}{\sqrt{\sigma_{\sigma_{R_{F}}}^2 + \sigma_{\mu_{R_{F}}}^2}} \geq z_{[R_F]} \]

2.3 Design Variables and Edge-restraint Conditions.

The tooth number of driving bevel gear \( z_{\text{max}} \geq z_1 \geq 17\cos \delta \); the module of gear pair \( m \geq 2 \); the tooth width coefficient of gear pair \( 0.35 \geq \psi, \geq 0.2 \); the density of gear material \( \rho \geq 0 \).

3. Making Fitness Function with Punishment Function Method

According to the exterior point punishment function method, we made the fitness function for the constraint nonlinear programming problem, as follows [7-9]:

\[ \text{Val}(x) = f(x) + L(x) \]

Where, \( x \) represents the chromosome; \( f(x) \) is the objective function; \( L(x) \) is the penalty term. For the minimum value, we have:

\[
\begin{cases}
L(x) = 0, & \text{the feasible } x \smallskip
L(x) = -s_1[G_1(x)] - s_2[G_2(x)] < 0, & \text{the infeasible } x
\end{cases}
\]

Where, \( s_1 \) and \( s_2 \) are respectively the penalty factors of two inequality constraint functions. According to the design criteria of encased gear, we set \( s_1 = 1 \) and \( s_2 = 0.5 \), respectively. We regarded Eqs. 9 and 11 as the penalty terms in the fitness function. Meanwhile, we regarded the boundary conditions of design variables as the matrix of variables top and bottom limitation in the genetics algorithm.

4. Model Solution of Reliability Optimum Design for Bevel Gears Based on Genetics Algorithm

In calculation, we selected the orthogonal 90\(^\circ\) encased straight bevel gear driven system as the optimum design object. Meanwhile, we set that the gear-driven useful horsepower is 9.8kW; the speed of driving gear is 1000r/min; the gear ratio is 3; the working condition coefficient \( K \) is 1.8; the material of small bevel gear is 40Cr, modified treatment, the rigidity is 250HBS; the material of big bevel gear is 35SiMnA, modified treatment, the rigidity is 230HBS; the densities of two bevel gears are set according to their materials; the reliability of allowable contact stress \([R_H]\) and dedendum flexural stress \([R_F]\) are respectively 0.9999.

According to the above calculation conditions, we wrote the programs by MATLAB. By calling the genetics search function and iterative computations, the least optimum result of all-in cost of bevel gear driven system design is obtained.

Table 1 shows the data of search path of population optimum. Fig. 2 shows the trend curve of average changes of each generation optimal sufficiency in genetics algorithm.
The calculation shows that breeding to the 89th generation, the optimal solution of reliability optimum design for bevel gear driven system is obtained. The optimal solution is:

$$x^* = \left(x_1^*, x_2^*, x_3^*\right)^T = (18.728, 2.0, 0.25)^T$$

$$C_T = 0.375 \text{kg}$$
The optimum parameters of bevel gear driven system are obtained under meeting reliability constraints of Eqs.10 and 11 more than 99.99%. Then, the optimum parameters are changed into integers that are final parameters of optimum design.

5. Summary

1) The reliability optimum design model of bevel gear driven system is established. The design parameters of system reliability optimum are obtained based on genetics algorithm.
2) The method is fit for the programming operation. The theory of reliability is introduced into the optimum design of bevel gear to provide the theoretical basis for quantification design in the gear manufacturing process.

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