Robust Model for Multimodal Location of the Hazmat under Uncertainty

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Abstract—In reality, the demand of O-D pair can be affected by various factors and then lead to uncertain demand. Finally, it will impact the location of transfer yards. A bi-objective nonlinear robust model is established for locating multimodal transfer yards with hazardous materials in this paper. To solve easier by Cplex, a linear program is adopted. Finally, the developed model is applied to a numerical experiment, and the results are analyzed.

Keywords—hazmat; multimodal; uncertain demand; robust optimization; transfer yard location

I. INTRODUCTION

With the rapid development of the current industry, social chemical products constantly updated and the expansion of enterprise operation scope, the demand for hazmat increased year by year, so more and more companies began to engage in the transportation of hazmat. However, the transportation of hazmat will cause its movement. Meanwhile, the uncertain traffic and other road conditions will make the occurrence of the risk further increases, so the transportation of hazmat should be paid more attention. In 1982, there are about 1.5 billion tons of transportation of hazardous substance in US, accumulated mileage reached 784 billion miles (List and Abkowitz, 1986) [1]. In Virginia, hazmat transportation account for about 13% of the total transportation task (Schmidt and Price, 1979) [2].

As the trade volume of the world and transportation distance increased, single mode of transportation cannot meet the needs of all parts of the country, so in the past 20 years, more and more companies tend to choose multi-mode to complete the transportation task (i.e., multimodal transportation). However, hazmat transportation has the characteristics of low accident rate and great consequence. Therefore, for the characteristics of hazmat transportation, which can make full use of the advantages of multimodal to reduce the risk and cost of transportation. Multimodal transportation of hazmat requires the transfer of hazmat containers or tanks between different modes in transfer yard, the transfer process will further increase the transportation costs and risks. Due to the region and the surrounding population differences, the different location of transfer yard will cause different transfer risk and cost.

II. LITERATURE REVIEW

The research on multimodal transportation still focus on terminal or hub location and routing problem. Sörensen et al. (2012) studied the terminal location problem and established a model to minimize the total cost. The authors proposed two different metaheuristic procedures GRASP and ABHC to solve the problem, and the effectiveness of the two algorithms is compared [3]. On the research of hub location problem, Alumur et al. (2012) studied the hub location problem from the perspective of network design with different possible transportation modes, and jointly consider transportation costs and travel times [4]. Ishfaq and Sox (2012) investigated the effect of limited hub resources on the design of intermodal logistics networks under service time requirements by integrating the hub operation queuing model and the hub location-allocation model [5]. Elhedhli and Wu (2010) also considered the congestion and queues in hub due to its capacity [6], the difference is the former considered the time delays due to the congestion and queues in hub, and then affected the service time and decision. The latter considered the congestion cost, and as part of the total cost, eventually influence the decision.

However, the problem of multimodal transportation for hazmat is different from the regular’s. Transportation risk should be also considered. Most existing literatures about hazmat focus on the problem of routing optimization. Verma et al. (2012) studied the problem for rail-truck intermodal transportation of hazmat. A bi-objective model with minimize risk and cost is proposed, and an algorithm is developed to solve the problem [7]. Assadipour et al. (2015) further considered congestion at intermodal yards and equipment capacity, and it will cause more risk and costs. They established a non-linear MIP model and solved by a multi-objective genetic algorithm [8]. All above the literatures were deeply researched, but under deterministic scenario.

Most existing research of multimodal transportation is about multimodal location for regular goods or hazmat under deterministic case. In reality, the hazmat demand of OD pair is uncertain and its distribution is difficult to estimate. Therefore, the multimodal location of the hazmat under uncertainty is researched in this paper.

III. MODELING

In reality, the exact relation between each uncertainly source and the uncertain demand in terms of the probability distribution is often difficult to estimate. However, the robust optimization strategy is to immunize the transfer yard location decisions against the worst case demand uncertainty realization in the ellipsoidal uncertainty set (Shahabi and Unnikrishnan 2014), so we don’t need to know its specific distribution [9].
Therefore, we apply the robust optimization solve the problem in this paper.

A. Parameter and Variable Definition

- \( M \): Set of OD pair, indexed by \( m \).
- \( N \): Set of nodes in network, indexed by \( i \) and \( j \).
- \( K \): Set of transportation modes, indexed by \( k \) and \( l \).
- \( n^m \): The number of shipments for the \( m \)th OD pair.
- \( r_{ik} \): Risk due to transfer one hazmat container from \( i \) to \( j \) use \( k \)th mode.
- \( r_{il} \): Risk due to moving one hazmat container from \( i \) to \( j \) use \( l \)th mode.
- \( C_{ik} \): Cost due to transfer one hazmat container from \( i \) to \( j \) use \( k \)th mode.
- \( C_{il} \): Cost due to moving one hazmat container from \( i \) to \( j \) use \( l \)th mode.
- \( f_i \): Fixed costs of establishment and operating a transfer yard.

\[
\begin{align*}
X_{ik}^m = \begin{cases} 1, & \text{if the } m \text{th OD pair transport from } i \text{ to } j \text{ use mode } k \\ 0, & \text{otherwise} \end{cases} \\
Y_i = \begin{cases} 1, & \text{if the transfer yard is established at node } i \\ 0, & \text{otherwise} \end{cases} \\
Z_{il}^m = \begin{cases} 1, & \text{if the } m \text{th OD pair transfer from mode } k \text{ to mode } l \text{ at node } i \\ 0, & \text{otherwise} \end{cases}
\end{align*}
\]

B. A Bi-Objective Model under Deterministic Demand

We first established a deterministic model. Because of the toxic, explosive characteristics of hazmat and the serious consequences of the hazmat transportation accidents to the public, it is necessary to consider the minimization of the total cost and risk of transportation. So the objective of the model is to minimize the total cost and risk, and establish a bi-objective model for multimodal location of the hazmat.

**Obj1:** \( R = \min \sum_{i,j} \sum_{m} \sum_{k} C_{ik} X_{ik}^m + \sum_{i,j} \sum_{m} \sum_{k} r_{ik} n^m X_{ik}^m \)

**Obj2:** \( C = \min \sum_{i,j} \sum_{m} \sum_{k} C_{il} Y_i + \sum_{i,j} \sum_{m} \sum_{k} C_{il} X_{il}^m Z_{il}^m + \sum_{i,j} \sum_{m} \sum_{k} r_{il} n^m Z_{il}^m \)

s.t.

\[
\sum_{j} X_{ik}^m - \sum_{j} X_{ijkl}^m - 1 \quad & \text{if } i = \text{org}(m) \quad \forall m \in M, i \in N \quad \text{(1)} \\
\sum_{i} X_{ijkl}^m - \sum_{i} X_{ik}^m - 1 \quad & \text{if } i = \text{dest}(m) \quad \forall m \in M, i \in N \\
\sum_{j} X_{ik}^m - \sum_{j} X_{ijkl}^m \quad & \text{other} \quad \forall m \in M, i \in N \\
\sum_{j} X_{ik}^m \leq 1 \quad & \forall m \in M, i \in N \quad \text{(2)}
\]

The **Obj1** and **Obj2** represent the minimization of total risk and cost respectively. The **Obj1** contains the risk of transportation and transshipment. The **Obj2** include the transportation cost, transshipment cost, and the fixed cost to establish and operate the transfer yards.

Constraint (1) is to ensure flow conservation; Constraint (2) states that any OD pair only can choose one transportation mode and one road at most; Constraint (3) represents that any OD pair only can transfer once at most in a node; Constraint (4) and constraint (5) ensure that the carrier transferred at the node \( j \) only they choose different transportation modes when arrive node \( j \) and then depart from it; Constraint (6) ensure that carrier transfer at a node \( i \) is possible only if that transfer yard is installed. Constraint (7), (8) and (9) enforce the binary constraints on the decision variables.

C. Robust Model for Multimodal Location of the Hazmat under Uncertainty

The real demand can be described by mean value plus the perturbation value of disturbance factor. As follows:

\[
\bar{n}^m = \bar{n}^m + \sum_{g \in g} b_g^m \tilde{\mu}_g^m \quad \forall m \in M \quad \text{(10)}
\]

where \( \bar{n}^m \) is the mean demand of \( m \)th OD pair; \( \tilde{\mu}_g^m \) represents the \( g \)th source of uncertainty which is an independent random variable, and \( |\tilde{\mu}_g^m| \leq 1 \); \( b_g^m \) is the levels of demand uncertainty, which is weights associated with the \( g \)th variable \( \tilde{\mu}_g^m \).

The robust approach makes the optimal decision at worst case to make the decision has robustness. To avoid the scenario of over conservative, the uncertain random variables \( \tilde{\mu}_g^m \) are assumed to be limited to an ellipsoidal uncertainty set, i.e. \( \tilde{\mu}_g^m, |\mu| \leq \Omega \) (Shahabi and Unnikrishnan, 2014) where \( \Omega \) is uncertainty budget [9]. To ensure the value of \( \Omega \) is valid, the constraint of \( \Omega \leq \sqrt{g} \) should be satisfied. We can build the bi-objective robust model as follows:

\[
\sum_{j,k} X_{ijkl}^m \leq 1 \quad \forall m \in M, i \in N \\
\sum_{j,k} X_{ik}^m + \sum_{j,k} X_{ijkl}^m \leq 1 + Z_{ik}^m \quad \forall m \in M, j \in N, k, l \in K \\
\sum_{j,k} X_{ik}^m - Z_{ik}^m > X_{ijkl}^m \quad \forall m \in M, j \in N, k, l \in K \\
Z_{ijkl}^m \leq Y_i \quad \forall m \in M, i \in N, k, l \in K \\
X_{ik}^m \in \{0,1\} \quad \forall m \in M, i \in N, \quad \text{(11)}
\]

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\sum_{j,k} X_{ik}^m - Z_{ik}^m > X_{ijkl}^m \quad \forall m \in M, j \in N, k, l \in K \\
Z_{ijkl}^m \leq Y_i \quad \forall m \in M, i \in N, k, l \in K \\
X_{ik}^m \in \{0,1\} \quad \forall m \in M, i \in N, \quad \text{(11)}
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Y_i \in \{0,1\} \quad \forall i \in N \\
Z_{ik}^m \in \{0,1\} \quad \forall m \in M, i \in N, k, l \in K \quad \text{(9)}
\]
\[ \text{Obj1} = \text{Risk} : \min \left( \max \sum \sum \sum r_{ij} \left( \sum \phi_{k} + \sum \mu_{k} Z_{m}^{*} \right) \right) \]
\[ \text{Obj2} = \text{Cost} : \min \left( \max \sum \sum \sum C_{ij} \left( \sum \phi_{k} + \sum \mu_{k} Z_{m}^{*} \right) \right) \]
\[ \text{s.t. (1) ~ (9)}. \]

IV. SOLUTION PROCEDURE

Since the maximization problem is based on the random variable \( \tilde{\mu}_{r} \), the objective function \( \text{Obj1} \) and \( \text{Obj2} \) can be rewritten as follows:

\[ \text{Obj1} = \text{Risk} : \min \left( \sum \sum \sum \sum \left( r_{ij} X_{n} + \max \sum \sum b_{i}^{*} \mu_{k} r_{ij} X_{n}^{*} \right) \right) \]
\[ \text{Obj2} = \text{Cost} : \min \left( \sum \sum \sum \sum \left( C_{ij} X_{n} + \max \sum \sum b_{i}^{*} \mu_{k} C_{ij} X_{n}^{*} \right) \right) \]
\[ \text{s.t. (1) ~ (9)}. \]

The min-max formulation is a mixed integer nonlinear program which is difficult to solve, so it can be divided into two steps:

First, the inner maximization problem can be written as S1 and S2 in Eq. (15) and (16).

\[ S1 = \max \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum r_{ij} X_{n} + \sum r_{ij} Z_{m}^{*} \right) \]
\[ S2 = \max \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum C_{ij} X_{n} + \sum C_{ij} Z_{m}^{*} \right) \]

To solve the maximization problem of S1, a lagrangian relaxation scheme is used to determine the optimal solutionexpression for the primitive uncertainty variables \( \tilde{\mu}_{r} \) (Gülpinar et al., 2013) [10], the same to S2.

Considering the Lagrangian multiplier \( \lambda \geq 0 \), and \( \| \phi \| \leq \Omega \), the Lagrangian function can be written as Eq. (17).

\[ L(\tilde{\mu}, \lambda) = \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum r_{ij} X_{n} + \sum r_{ij} Z_{m}^{*} \right) + \lambda(\Omega - \| \phi \|) \]

The first order condition for the Lagrangian function is given as:

\[ \frac{\partial L(\tilde{\mu}, \lambda)}{\partial \tilde{\mu}_{r}} = \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum r_{ij} X_{n} + \sum r_{ij} Z_{m}^{*} \right) - \frac{\mu_{k}}{\lambda} \forall g \in G \] (18)
\[ \frac{\partial L(\tilde{\mu}, \lambda)}{\partial \lambda} = -\Omega \| \phi \| \] (19)

From above conditions, we can get the values of \( \lambda \) and \( \tilde{\mu}_{r} \):

\[ \lambda = \sum \sum \sum \sum \sum r_{ij} X_{n} + \sum \sum r_{ij} Z_{m}^{*} \]
\[ \tilde{\mu}_{r} = \Omega \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum r_{ij} X_{n} + \sum r_{ij} Z_{m}^{*} \right) \forall g \in G \] (20)

Second, plug the optimal value of \( \tilde{\mu}_{r} \) into S1, the maximization problem of S1 can be solved in Eq. (22).

\[ S1 = \Omega \sum \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum r_{ij} X_{n} + \sum r_{ij} Z_{m}^{*} \right) \]

The same to S2 in Eq. (23).

\[ S2 = \Omega \sum \sum \sum \sum \sum b_{i}^{*} \mu_{k} \left( \sum C_{ij} X_{n} + \sum C_{ij} Z_{m}^{*} \right) \]

Incorporating the solution of the inner maximization problems of S1 and S2 into Eq. (15) and (16), the min-max formulation for the robust multimodal location problem can be converted to the following minimization problem.

\[ \text{Obj1} = \text{Risk} : \min \sum \sum \sum \sum \sum \phi_{k} X_{n} + \sum \sum \sum \sum \phi_{k} Z_{m}^{*} + \lambda(\Omega - \| \phi \|) \]
\[ \text{Obj2} = \text{Cost} : \min \sum \sum \sum \sum \sum \phi_{k} C_{ij} X_{n} + \sum \sum \sum \sum \phi_{k} C_{ij} Z_{m}^{*} + \lambda(\Omega - \| \phi \|) \]

Both the objective functions are nonlinear and complicated, to solve easier, \( b_{i}^{*} \) can be written as \( b_{i}^{*} = \phi^{*} b_{i} \), and the \( \text{Obj1} \) and \( \text{Obj2} \) can be reformulated as follows, which can be solved efficiently using CPLEX.

\[ \text{Obj1} = \text{Risk} : \min \sum \sum \sum \sum \sum \phi_{k} X_{n} + \sum \sum \sum \sum \phi_{k} Z_{m}^{*} + \lambda(\Omega - \| \phi \|) \]
\[ \text{Obj2} = \text{Cost} : \min \sum \sum \sum \sum \sum \phi_{k} C_{ij} X_{n} + \sum \sum \sum \sum \phi_{k} C_{ij} Z_{m}^{*} + \lambda(\Omega - \| \phi \|) \]

S.t. (1) ~ (9).

Due to the bi-objective model is difficult to solve because of the incoordination of the target dimension, the two objective functions must be dimensionless unified treatment and transformed into a single objective function:

\[ \text{Obj3} = \alpha \frac{\text{Obj1} - \text{Obj1}^{*}}{\text{Obj1}^{*}} + (1 - \alpha) \frac{\text{Obj2} - \text{Obj2}^{*}}{\text{Obj2}^{*}} \]

where \( \alpha \) is the weight of risk, \( \text{Obj1}^{*} \) and \( \text{Obj2}^{*} \) are the respective optimal value of objective functions \( \text{Obj1} \) and \( \text{Obj2} \).
So the bi-objective model can be transformed into following formulation:

\[
\begin{align*}
\min \quad & \left[ \sum_{m=1}^{mM} \sum_{i=1}^{iN} \sum_{j=1}^{jN} \sum_{k=1}^{kK} r_{ijk} x_{ijk}^* \cdot \sum_{m=1}^{mM} \sum_{i=1}^{iN} \sum_{k=1}^{kK} c_{ijk} x_{ijk}^* \right]^{1/\alpha} \left[ \sum_{m=1}^{mM} \sum_{i=1}^{iN} \sum_{k=1}^{kK} \delta_{ijk} z_{ijk}^* \right]^{1/\alpha} \\
\text{s.t.} \quad & (1) \sim (9).
\end{align*}
\]

The above model is a single objective mixed integer conic quadratic program which can be solved efficiently by CPLEX.

REFERENCES