Analytical solution on transient aerodynamic heating of hypersonic vehicle hot structure

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Abstract. New type hypersonic vehicles become more and more important due to its high performance, safety and low price. Thermal protection system(TPS), Which are correlated to the safety of hypersonic vehicles, are one of the most critical techniques in design and manufacture of hypersonic vehicles. The reusable ceramic tiles, such as C/C, C/SiC and insulator tiles would be susceptible to excessive recession during reentry and fatal weakness of UHTCs is prone to thermal fragmentation. An analytical solution of a simplified transient thermal problem was derived for thermal response on non-ablation materials subjected to a simplified heat plus. The solution is introduced two non-dimensional parameters. Analysis of influencing factors on transient thermal response has been performed. Results and data obtained from calculation will give guidance to thermal protection design.

Keywords: hot structure; thermal response; high temperature; analytical solution.

1 Introduction

New type hypersonic vehicles become more and more important due to its high performance, safety and low price. Thermal protection system (TPS), Which are correlated to the safety of hypersonic vehicles, are one of the most critical techniques in design and manufacture of hypersonic vehicles.

New-type hypersonic vehicle will encounter significant challenges due to severe thermal heating environments during its long-time hypersonic flying in the atmosphere. Traditional hypersonic vehicles are of high enthalpy, high heat flux and short time reentry, while new type hypersonic vehicles with high enthalpy, non-high heat flux and long time reentry. Four main requirements for TPS have been put forwards for new hypersonic vehicles: high temperature capability, high efficient insulator, lightweight and low cost, etc. However, traditional ablation TPS doesn't meet the new requirements. There are some problems on non-ablation thermal protection materials: The reusable ceramic tiles, such as C/C, C/SiC and insulator tiles would be susceptible to excessive recession during re-entry. Therefore, it is necessary to perform analysis and sizing for transient aerodynamic heating of hot structure in order to give guidance to thermal protection design.

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2 Impact factor analysis on transient aerodynamic heating

The ultimate aim of TPS is to ensure the safety and operation smoothly of whole vehicle. Temperature, thermal stress, weight and so on are some important parameters of TPS design which are affected by thermal environment and physical parameters.

Nowadays, many researches focusing thermal protection materials have been done. Numerical computations have been used for transient aerodynamic heating, while few researches focusing on analytical solution have been done. A simplified transient thermal problem was investigated in an attempt to gain basic insight into thermal response of non-ablation materials subjected to a transient aerodynamic heating plus. An analytical solution of the transient thermal problem was derived for thermal response on non-ablation materials subjected to a simplified heat plus. The solution is introduced two non-dimensional parameters. These two variables account for the thermal material properties as well as for the geometric parameters of structure and combine thermal material properties with thermal environment parameters. Analysis of influencing factors on transient thermal response has been performed.

3 Non-dimension transient heat conduction equations

When the flow velocity across the vehicles is not fast, and temperature is not enough high, the aerodynamic heating can be written as:

$$q_w = h(T_r - T_w)$$

Where, q_w is wall heat flux, T_r is adiabatic temperature, T_w is wall temperature and h is convection coefficient.

In above case, for an infinite plate with thickness δ , one dimension governing differential equation for heat conduction through the plate can be written as:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial y^2} \quad (0 < y < \delta, \tau > 0) \quad (1)$$

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial y^2} \quad (0 < y < \delta, \tau > 0) \quad (2)$$

Where $a = k/\rho C_p$

Initial condition is a uniform temperature

$$T(y, 0) = T_0 \quad (0 \leq y \leq \delta) \quad (3)$$

Boundary condition

$$\frac{\partial T(y, t)}{\partial y} \Big|_{y=0} = 0 \quad (4)$$

$$k \frac{\partial T(y, t)}{\partial y} \Big|_{y=\delta} = h[T_w(\delta, t) - T_r] \quad (5)$$

The mathematical problem defined by Eqs.1 through 4 can be completely nondimensionalized. Non-dimensional variables can be written as follows:

$$\eta = \frac{y}{\delta} \quad (6)$$

$$\tau = \frac{t}{t_D} \quad (7)$$

$$\theta = \frac{T - T_r}{T_0 - T_r} \quad (8)$$

Two non-dimensional parameters, Fourier number F_0 and Biot number Bi are introduced here, Two parameters can be defined as

$$F_0 = \frac{at_D}{\delta^2} \quad (9)$$

$$Bi = \frac{h\delta}{k} \quad (10)$$

In terms of these non-dimensional variables, the simplified thermal problem can be written in the following non-dimensional form

$$\frac{\partial \theta}{\partial \tau} = F_0 \left(\frac{\partial^2 \theta}{\partial \eta^2} \right) \quad (11)$$

The boundary condition at $\eta = 1$ becomes

$$\frac{\partial \theta}{\partial \eta} = Bi \quad (12)$$

The boundary condition at $\eta = 0$ becomes

$$\frac{\partial \theta}{\partial \eta} = 0 \quad (13)$$

The initial condition becomes

$$\theta = 1 \quad (14)$$

An analytical solution can be written as follows

$$\begin{aligned} \theta(\eta, \tau) &= \sum_{n=0}^{\infty} C_n \exp(-\mu_n^2 F_0 \tau) \cos(\mu_n \eta) \\ C_n &= \frac{2 \sin \mu_n}{\mu_n + \cos \mu_n \sin \mu_n} \\ \tan \mu_n &= \frac{Bi}{\mu_n} \end{aligned} \quad (15)$$

The analytical solution can be simplified as

$$\theta(\tau, \eta) = \theta(F_0, Bi) \quad (16)$$

From Eqs, 15, it has been shown that non-dimensional thermal response is completely governed by two non-dimensional parameters, F_0 and Bi .

However, the value of physical temperature depends on seven physical parameters, physical parameters of materials, material size, heating time, convection coefficient and adiabatic temperature, physical temperature can be written as

$$T(y,t) = T(\rho,k,C_p,t_p,\delta,h,T_r) \tag{17}$$

Comparing (15) with (16), it has been shown that the dependence of non-dimensional thermal response on relevant material parameters has been reduced from 7 to only 2 after non-dimensionalization.

In our paper, two non-dimensional parameters have been introduced to decide the non-dimensional thermal response. These two variables account for the thermal material properties (ρ , k , C_p) as well as for the geometric parameters of structure as shown in (8) and (9). In addition, Biot number also combines thermal material properties with thermal environment parameters.

If we change parameters in (8) and (9) according to table 1, it can be found that F_0, Bi still maintain unchangeable, the temperature distribution remains unchanged. Therefore, through the adjustment of physical parameters, thermal response can be designed according to our requirements.

Table 1. Value of physical parameters

state	1	2	3
Convection efficient	h	$2h$	$2h$
conductivity	k	$2k$	$\sqrt{2}k$
density	ρ	$\sqrt{2}\rho$	2ρ
Heat capacity	C_p	$\sqrt{2}C_p$	$\sqrt{2}C_p$
Material size	δ	δ	$1/\sqrt{2}\delta$
F_0	F_0	F_0	F_0
Bi	Bi	Bi	Bi
Temperature	T	T	T

When Mach number reaches 25 during hypersonic vehicles re-entry, the temperature will be up to 1600K, and chemical energy and wall radiation must be considered. The net heat flux of heating surface can be written as

$$\begin{aligned} q_n &= q_{or} \left(1 - \frac{h_w}{h_r} \right) - q_{rad} \\ q_{rad} &= \varepsilon \sigma T_w^4 \\ h_w &= f(P, T_w) \end{aligned} \tag{18}$$

Where q_{or} denotes cold wall heat flux, h_w is wall enthalpy, as function of pressure and wall temperature, h_r is adiabatic wall enthalpy.

It should be noted that wall radiation heat flux in Eqs.17 is proportional to temperature to the fourth power. It is difficult to obtain analytical solution to the heat conduction equation. Therefore, numerical calculation of thermal response has been performed. Moreover, comparison between analytical results and numerical results of thermal response has been done.

4 Results and discussion

Consider one dimensional transient heat analysis of infinite plate,

Thickness of plate: $\delta = 0.04cm$

Physical properties of high conductivity C/C materials:

$$\lambda_1 = 70.0W/m \cdot K$$

$$C_{p_1} = 1230J/Kg \cdot K$$

$$\rho_1 = 1800Kg/m^3$$

Initial temperature: $T_0 = 300K$

Boundary conditions:

$$\frac{\partial t(y,\tau)}{\partial y} \Big|_{y=0} = 0$$
$$\lambda \frac{\partial t(y,\tau)}{\partial y} \Big|_{y=\delta} = q_{or} \left(1 - \frac{h_w}{h_r}\right) - \varepsilon \sigma T_w^4$$

Thermal environment parameters, material properties and results are listed in table 2.

Table 2. Value of thermal environment parameters, material properties and results

state	H_r (KJ/K g)	q_{or} (KW/m ²)	q_r	K_2/K_1	ρ_2/ρ_1	C_{p2}/C_{p1}	δ_2/δ_1	$T_w(K)$	$T_b(K)$	Heating time(s)
1	4000	4000	Yes	1	1	1	1	2330.0	2329.9	400s
2	4000	4000	Yes	2	$\sqrt{2}$	$\sqrt{2}$	1	2323.9	2320.3	400s
3	4000	4000	Yes	$\sqrt{2}$	2	$\sqrt{2}$	$1/\sqrt{2}$	2323.9	2320.3	400s
4	4000	2000	No	1	1	1	1	2971.9	2955.5	400s
5	4000	4000	No	2	$\sqrt{2}$	$\sqrt{2}$	1	2971.9	2955.6	400s
6	4000	4000	No	$\sqrt{2}$	2	$\sqrt{2}$	$1/\sqrt{2}$	2971.9	2955.5	400s
7	4000	2000	No	1	1	1	1	2064.7	2063.4	400s

Effects of thermal environment parameters (H_r, q_{or}), radiation, physical parameters of materials (ρ, k, C_p) and plate size(δ) on thermal response prediction of surface temperature and back temperature are compared. It should be noticed that F_0, Bi maintain invariance.

Effects of physical parameters of materials and plate size (state 1, 2, 3 listed in table 2) on thermal response are shown in Fig.1. Under same thermal environment, just change physical parameters of materials and plate size, finally radiation equilibrium temperature reach to the same value, but thermal response are different due to difference of parameters concerned. Therefore, it can be concluded that it is necessary to delay radiation equilibrium time, increase the materials temperature capability during the thermal protection design.

Effects of thermal environment, physical parameters and plate size (state 4, 5, 6 shown in table 2) on thermal response are shown in Fig.2. If wall radiation heat flux is not neglect and total heat flux is doubled, physical parameters of materials and plate size may have many mapping modes while keeping F_0, Bi equal, thermal response of surface and back will remain unchanged.

Figure 1. Effects of physical parameters and plate size on thermal response

Figure 2. Effects of thermal environment, physical parameters and plate size on thermal response

Fig.3 illustrates the effects of wall radiation (state 4 and 7 listed in table 2) on the surface. In the figure, it has been shown that effects of wall radiation may be neglect when surface temperature is less than 1200K. Wall radiation leads to a sharp decrease of surface temperature and have shorten radiation equilibrium time when surface temperature reaches up to 1200K. Surface temperature has been decreased by 900K after 400s heating.

Effects of physical parameters of materials on thermal response (state 1, 4, 5) are shown in Fig.4. During the early heating time, before wall temperature reaches 1200K, if heat flux is doubled, through adjusting physical parameters of materials, temperature distributions could remain unchangeable. But when surface temperature reaches up to about 1200K (back temperature to about 1300K), variations of physical parameter of materials take little effect on thermal response, Effects of thermal environment parameters (H_r , q_{or}) become more and more important.

Figure 3. Effects of radiation on thermal response

Figure 4. Effects of physical parameters on thermal response of surface

5 Conclusion

A simplified transient thermal problem was investigated in an attempt to gain basic insight into thermal response of non-ablation materials subjected to a transient aerodynamic heating plus.

An analytical solution was derived for the transient response of this simplified transient problem. The analytical solution introduces two non-dimensional parameters. A numerical study was also performed to obtain thermal response of materials. Furthermore, Analysis of influencing factors on transient thermal response has been performed.

Results and data obtained from calculation will give guidance to thermal protection design.

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