On Switched Control of Networked System with Limited Communication and Handoff Delay

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Abstract. In this paper, the issues of stability analysis and controller design of networked switched control system are discussed by considering the communication limitation and handoff delay simultaneously. Based on Lyapunov stability theory and linear matrix inequality, the minimum average dwell time and the controller design which can guarantee the system stability are presented. The feasibility of the proposed approach is verified by numerical simulation.

Introduction

Compared in the study of networked control systems, people have a longer time to study of hybrid system especially switched system. 'Switch' as a kind of control thoughts was applied to the control system very early. Now people generally accepted that switched system is a kind of hybrid systems with hierarchy type characteristics [1-3]. As a kind of important industrial system, switched system is widely used in the actual production. Relevant examples were visible in the literature [4][5]. In order to simplify the model, the research of networked switched control systems usually ignores handoff delay and only considers the transmission delay caused by the network, but due to the control system scale is more and more complex and precision require higher in actual industrial production, handoff delay caused by the subsystem switch will have to become one of the factors we consider that ensure the stability of the system [6][7].

Although communications technology has rapidly developed in recent years, but due to the amount of data transmission increased limited communications has become an important problem in network system. Limited communication restrictions feedback signal and control signal real-time transmission and reduce the performance even cause system instability, therefore need find an appropriate method to solve the problems of networked control system modeling, analysis and design with limited communication [8-15]. In this paper, on the basis of previous scholars' research and according to practical engineering, research design of switch controller for networked switched control system with limited communication and handoff delay, it has important significance for the actual industrial production.

Modeling of Switched Control System with Limited Communication

The switched system can be described as follow:

$$\begin{aligned} \dot{x}(t) &= A_{\Delta}x(t) + B_{\Delta}u(t) \\ y(t) &= C_{\Delta}x(t) \end{aligned} \quad (1)$$

Where $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $u = [u_1, u_2, \dots, u_m]^T \in R^m$, $y = [y_1, y_2, \dots, y_p]^T \in R^p$ are state, input and output of the system, respectively. A_{Δ} , B_{Δ} and C_{Δ} are matrixes with appropriate dimensions. Δ is

switch signal, satisfy set $\eta = \{1, \dots, N\}$. Assuming that system all n state can be collected, but only σ ($1 \leq \sigma < p$) sensor nodes and ρ ($1 \leq \rho < m$) actuator node can communicate with the controller node every moment because of limited communication.

Introduce two valued function $\sigma_i(t): \mathbb{R} \rightarrow \{0,1\}$. Represents communication between i ($1 \leq i \leq p$) sensor nodes and controller at time t , $\sigma_i(t)=0$ means not communication $\sigma_i(t)=1$ means communication. Communication between sensor nodes and controller can be expressed by the sequence $\sigma(t) = [\sigma_1(t) \sigma_2(t) \dots \sigma_p(t)]^T$. Similarly, introduce two valued function $\rho_j(t): \mathbb{R} \rightarrow \{0,1\}$. Represents communication between j ($1 \leq j \leq m$) actuator nodes and controller at time t . $\rho_j(t)=0$ means not communication $\rho_j(t)=1$ means communication. Communication between actuator nodes and the controller can be expressed by the sequence $\rho(t) = [\rho_1(t) \rho_2(t) \dots \rho_m(t)]^T$. Controller receives output of the sensor node i and the actuator node j receives output of controller can expressed as $\bar{y}_i(t) = \sigma_i(t) y_i(t)$, $\bar{u}_j(t) = \rho_j(t) u_j(t)$ respectively. In order to establish the communication model of networked control system with communication limitation definition $\bar{y}(t) = [\bar{y}_1, \bar{y}_2, \dots, \bar{y}_p]^T \in \mathbb{R}^p$, $\bar{u}(t) = [\bar{u}_1, \bar{u}_2, \dots, \bar{u}_m]^T \in \mathbb{R}^m$,

$M_\sigma(t) = \begin{pmatrix} \sigma_1(t) & & \\ & \ddots & \\ & & \sigma_p(t) \end{pmatrix}$, $M_\rho(t) = \begin{pmatrix} \rho_1(t) & & \\ & \ddots & \\ & & \rho_m(t) \end{pmatrix}$. Utilize matrix form of communication

sequence the original system model can be rewritten the following form, where $\bar{B}_\Delta(t) = B_\Delta M_\rho(t)$, $\bar{C}_\Delta(t) = M_\sigma(t) C_\Delta$.

$$\begin{aligned} \dot{x}(t) &= A_\Delta x(t) + \bar{B}_\Delta(t) \bar{u}(t) \\ \bar{y}(t) &= \bar{C}_\Delta(t) x(t) \end{aligned} \quad (2)$$

The proposed system diagram is shown in Fig.1.

Fig. 1. Structure of networked switched control system with limited communication

Controller Design of Limited Communication System with Handoff Delay

Assumed that the maximum delay of switch is τ when the system switched between N controllers, consider state feedback controller (3) exists in limited communication switched control system.

$$\begin{aligned} \dot{x}_c(t) &= A_{c,\Delta(t')} x_c(t) + B_{c,\Delta(t')} x(t) \\ u(t) &= C_{c,\Delta(t')} x_c(t) + D_{c,\Delta(t')} x(t) \end{aligned} \quad (3)$$

Where $t' = t + \tau$, set up $X = [x \ x_c]^T$, then closed-loop networked switched control system with limited communication and handoff delay can be described as:

$$\dot{X}(t) = (\tilde{A}_\Delta + \tilde{B}_\Delta \tilde{K}_{\Delta(t')})X(t) \quad (4)$$

$$Y(t) = \tilde{C}_\Delta(t)X(t)$$

$$\text{Where } \tilde{A}_\Delta = \begin{pmatrix} A_\Delta & 0 \\ 0 & 0 \end{pmatrix}, \tilde{B}_\Delta = \begin{pmatrix} 0 & \bar{B}_\Delta \\ E & 0 \end{pmatrix} = \begin{pmatrix} 0 & B_\Delta M_\rho(t) \\ E & 0 \end{pmatrix}, \tilde{C}_\Delta = [\bar{C}_\Delta \quad 0] = [M_\sigma(t)C_\Delta \quad 0], \tilde{K}_{\Delta(t')} = \begin{pmatrix} B_{c,\Delta(t')} & A_{c,\Delta(t')} \\ D_{c,\Delta(t')} & C_{c,\Delta(t')} \end{pmatrix}.$$

Theorem: Considering system (4), for arbitrary $(i, j) \in \eta \times \eta$, $i \neq j$, let $\bar{Q}_{1,i} = R^{1/2}Q_{1,i}R^{1/2}$, $\bar{Q}_{2,i} = R^{1/2}Q_{2,i}R^{1/2}$ if there are matrixes $Q_{1,i} > 0$, $Q_{2,i} > 0$, M_i and constant $b \geq a > 0$ let the following condition were established:

$$\tilde{A}_i Q_i + Q_i \tilde{A}_i^T + M_i^T \tilde{B}_i^T + \tilde{B}_i M_i - a Q_i < 0 \quad (5)$$

$$\tilde{A}_i Q_j + Q_j \tilde{A}_i^T + M_j^T \tilde{B}_i^T + \tilde{B}_i M_j - b Q_j < 0 \quad (6)$$

$$\frac{c_1}{\lambda_1} < \frac{c_2}{\lambda_2} e^{-aT_f} \quad (7)$$

Where $Q_i = \begin{bmatrix} Q_{1,i} & 0 \\ 0 & Q_{2,i} \end{bmatrix}$ is positive definite matrix, switch signal Δ meet the average dwell time condition:

$$\tau_\alpha > \tau_\alpha^* = \frac{T_f (\bar{\alpha} + \ln \mu)}{\ln(\frac{c_2}{\lambda_2}) - \ln(\frac{c_1}{\lambda_1}) - aT_f} \quad (8)$$

For system (4), there are a series of state feedback controllers that make the closed-loop system about $(c_1, c_2, T_f, R, \Delta)$ finite time stable. Where $\mu = \lambda_2 / \lambda_1$, $\lambda_1 = \min_{i \in \eta} (\lambda_{\min}(\bar{Q}_i))$, $\lambda_2 = \max_{i \in \eta} (\lambda_{\max}(\bar{Q}_i))$, $c_1 \geq 1$, $\bar{\alpha} = (b-a)\tau$ and controller gain $\tilde{K}_i = M_i Q_i^{-1}$.

Prove:

The stability analysis of complex switch system is usually based on Lyapunov stability theory, which mainly includes single Lyapunov function and multiple Lyapunov function. Multi Lyapunov function method is that each subsystem has its own Lyapunov function analysis method, which is more general than single Lyapunov function method [13][14].

Select multiple Lyapunov function $V_i(X) = X^T Q_i^{-1} X = x^T Q_{1,i}^{-1} x + x_c^T Q_{2,i}^{-1} x_c$, controller gain in the time of $[t_k, t_k + \tau]$ is $\tilde{K}_j$, in $[t_k + \tau, t_{k+1}]$ is $\tilde{K}_i$.

In $[t_k, t_k + \tau]$: $\dot{V}_i(t) - bV_i(t) = X^T(t)(\tilde{A}_i^T Q_i^{-1} + \tilde{K}_j^T \tilde{B}_i^T Q_i^{-1} + Q_i^{-1} \tilde{A}_i + Q_i^{-1} \tilde{B}_i \tilde{K}_j - bQ_i^{-1})X(t)$,

in $[t_k + \tau, t_{k+1}]$: $\dot{V}_i(t) - aV_i(t) = X^T(t)(\tilde{A}_i^T Q_i^{-1} + \tilde{K}_i^T \tilde{B}_i^T Q_i^{-1} + Q_i^{-1} \tilde{A}_i + Q_i^{-1} \tilde{B}_i \tilde{K}_i - aQ_i^{-1})X(t)$.

Consider transformation as follow:

$$\begin{aligned} & Q_i^{-1}(\tilde{A}_i Q_i + Q_i \tilde{A}_i^T + M_i^T \tilde{B}_i^T + \tilde{B}_i M_i - aQ_i)Q_i^{-1} \\ &= Q_i^{-1} \tilde{A}_i + \tilde{A}_i^T Q_i^{-1} + Q_i^{-1} M_i^T \tilde{B}_i^T Q_i^{-1} + Q_i^{-1} \tilde{B}_i^T M_i Q_i^{-1} - Q_i^{-1} a \\ &= Q_i^{-1} \tilde{A}_i + \tilde{A}_i^T Q_i^{-1} + Q_i^{-1} \tilde{B}_i^T \tilde{K}_i + \tilde{K}_i^T \tilde{B}_i^T Q_i^{-1} - aQ_i^{-1} < 0 \end{aligned} \quad (9)$$

we can get $\dot{V}_i(t) - aV_i(t) < 0$ $t \in [t_k + \tau, t_{k+1}]$. Similarly $\dot{V}_i(t) - bV_i(t) < 0$ $t \in [t_k, t_k + \tau]$, it means that $V_i(t)$ drop along the trajectory for each subsystem, taking into account $b \geq a > 0$, can get satisfy the entire time period ($t \in [t_k, t_{k+1}]$): $V_{\Delta(t)}(X_t) \leq e^{a(t-t_k)} e^{(b-a)\tau} V_{\Delta(t_k)}(X_{t_k})$.

Based on the definition of λ_1 and λ_2 given before, can be known for any $i, j \in \eta, i \neq j$ the following inequality were established:

$$X^T(t) \bar{Q}_i^{-1} X(t) \leq X^T(t) \lambda_{\max}(\bar{Q}_i^{-1}) X(t) = \frac{1}{\lambda_{\min}(\bar{Q}_i)} X^T(t) X(t) \leq \frac{1}{\lambda_1} (X^T(t) X(t)) \quad (10)$$

$$X^T(t)\bar{Q}_j^{-1}X(t) \geq X^T(t)\lambda_{\min}(\bar{Q}_j^{-1})X(t) = \frac{1}{\lambda_{\max}(\bar{Q}_j)}X^T(t)X(t) \geq \frac{1}{\lambda_2}(X^T(t)X(t)) \quad (11)$$

Then can get: $X^T(t)\bar{R}^{1/2}Q_i^{-1}\bar{R}^{1/2}X(t) \leq \mu X^T(t)\bar{R}^{1/2}Q_j^{-1}\bar{R}^{1/2}X(t)$ where $\bar{R}^{1/2} = \begin{pmatrix} R^{1/2} & 0 \\ 0 & R^{1/2} \end{pmatrix}$. It can be rewritten

$V_i(X_{t_k}(t)) \leq \mu V_j(X_{t_k}(t))$, because the system status does not jumping for any time $t \in (0, T_f)$, then $V_{\Delta(t)}(X_{t_k}(t)) \leq \mu V_{\Delta(t_k)}(X_{t_k}(t))$. In consideration of $\mu \geq 1$ and $N_{\Delta}(0, t) \leq N_0 + t/\tau_{\alpha} \leq T_f/\tau_{\alpha}$ can get:

$$\begin{aligned} V_{\Delta(t)}(X_t(t)) &\leq e^{a(t-t_k)}e^{(b-a)\tau}V_{\Delta(t_k)}(X_{t_k}) \\ &\leq \mu e^{a(t-t_k)}e^{(b-a)\tau}V_{\Delta(t_k)}(X_{t_k}) \\ &= \mu e^{a(t-t_k)}e^{(b-a)\tau}V_{\Delta(t_{k-1})}(X_{t_k}) \\ &< \mu e^{a(t-t_k)}e^{(b-a)\tau}e^{a(t_k-t_{k-1})}e^{(b-a)\tau}\mu V_{\Delta(t_{k-1})}(X_{t_{k-1}}) \\ &\vdots \\ &< [\mu e^{\bar{\alpha}}]^{N_{\Delta}(0,t)}e^{a(t-t_k)}e^{a(t_k-t_{k-1})}\dots e^{a(t_1-t_0)}V_{\Delta(0)}(X_0(t)) \\ &= e^{at}[\mu e^{\bar{\alpha}}]^{N_{\Delta}(0,t)}V_{\Delta(0)}(X_0(t)) \\ &< e^{at}[\mu e^{\bar{\alpha}}]^{T_f/\tau_{\alpha}}V_{\Delta(0)}(X_0(t)) \\ &\leq e^{(\bar{\alpha}+\ln \mu)(T_f/\tau_{\alpha})}[e^{aT_f}V_{\Delta(0)}X_0(t)] \end{aligned} \quad (12)$$

On account of:

$$V_{\Delta(t)}(X(t)) = x^T(t)Q_{1,\Delta(t)}^{-1}x(t) + x_c^T(t)Q_{2,\Delta(t)}^{-1}x_c(t) \geq \lambda_{\min}(\bar{Q}_{1,\Delta(t)}^{-1})x^T(t)Rx(t) \geq \frac{1}{\lambda_2}x^T(t)Rx(t) \quad (13)$$

$$\begin{aligned} V_{\Delta(0)}(X_0(0)) &= x^T(0)Q_{1,\Delta(0)}^{-1}x(0) + x_c^T(0)Q_{2,\Delta(0)}^{-1}x_c(0) \\ &= x^T(0)Q_{1,\Delta(0)}^{-1}x(0) \\ &\leq \lambda_{\max}(\bar{Q}_{1,\Delta(0)}^{-1})x^T(0)Rx(0) \\ &\leq \frac{1}{\lambda_1}x^T(0)Rx(0) \end{aligned} \quad (14)$$

therefore $x^T(t)Rx(t) \leq \lambda_2 V_{\Delta(t)}(X(t)) < \frac{\lambda_2 c_1}{\lambda_1} e^{(\bar{\alpha}+\ln \mu)(T_f/\tau_{\alpha})+aT_f}$, when the conditions $\frac{c_1}{\lambda_1} < \frac{c_2}{\lambda_2} e^{-aT_f}$ and

$\ln(\frac{c_2}{\lambda_2}) - \ln(\frac{c_1}{\lambda_1}) - aT_f > 0$ are met, can be obtained $\frac{T_f}{\tau_{\alpha}} \leq \frac{\ln(\frac{c_2}{\lambda_2}) - \ln(\frac{c_1}{\lambda_1}) - aT_f}{\bar{\alpha} + \ln \mu}$ by average dwell

time formula, then $e^{(\bar{\alpha}+\ln \mu)(T_f/\tau_{\alpha})} \leq \frac{\lambda_1 c_2}{\lambda_2 c_1 e^{aT_f}}$ was derived. Substitutes the formula to

$$x^T(t)Rx(t) \leq \lambda_2 V_{\Delta(t)}(X(t)) < \frac{\lambda_2 c_1}{\lambda_1} e^{(\bar{\alpha}+\ln \mu)(T_f/\tau_{\alpha})+aT_f} \text{ then } x^T(t)Rx(t) < c_2.$$

If there are three positive real numbers $c_1, c_2 (c_1 < c_2), T_f$, a positive definite matrix R and switch signal Δ in a networked control system, when the input and disturbance of the networked switched control system are zero and the system satisfies the following inequality: $x_0^T(t)Rx_0(t) < c_1 \Rightarrow x^T(t)Rx(t) < c_2, \forall t \in [0, T_f]$ the networked switched control system is stability about $(c_1, c_2, T_f, R, \Delta)$. In summary, closed-loop networked switched control system (4) is finite time stability.

Simulation Verification and Analysis

Consider networked control system (2) with limited communication and handoff delays, system switch before and after matrix are as follows:

$$A_1 = \begin{bmatrix} -2 & 1.2 \\ -0.3 & -1.5 \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0.9 \\ 0.5 & -3 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0.5 & -0.2 \\ 1.3 & 2 \end{bmatrix}$$

Select parameters $a = 0.02$, $b = 1.1$, $R = I$, $c_1 = 1$, $c_2 = 25$, $T_f = 5$ handoff delay $\tau = 0.05s$, by the linear matrix inequalities in the theorem, using the LMI toolbox in MATLAB can get the following matrix to prove the method is feasible:

$$Q_1 = \begin{bmatrix} 38.5304 & 5.5027 \\ 5.5027 & 34.1334 \end{bmatrix}, Q_2 = \begin{bmatrix} 30.8979 & 8.7130 \\ 8.7130 & 39.8057 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1.1077 & 25.0513 \\ 30.2814 & -6.9043 \end{bmatrix}, M_2 = \begin{bmatrix} 14.7620 & -39.3663 \\ 4.1789 & 71.9879 \end{bmatrix}$$

For any switch signal Δ , get average dwell time $\tau_a > \tau_a^* = 1.1951s$. It means when the average dwell time of the switch signal is larger than the minimum average dwell time 1.1951s system is finite time stable about $(1, 25, 5, I, \Delta)$, we obtained the controller parameter matrix as following:

$$\tilde{K}_1 = \begin{pmatrix} -0.0783 & 0.7468 \\ 0.8352 & -0.3373 \end{pmatrix}, \tilde{K}_2 = \begin{pmatrix} 0.8085 & -1.1751 \\ -0.4029 & 1.9119 \end{pmatrix}.$$

The simulation results are shown in figure, figure 2 shows for a defined constant c_1 , effects of c_2 changes on average dwell time, figure 3 shows the response curve of close-loop state.

Fig. 2. Average dwell time τ_a^* at different c_2

Fig. 3. System close-loop state response curve

In Fig. 2 the minimum value of the average dwell time decreases with the increase of c_2 when c_1 value is fixed. Fig. 3 is system closed-loop state response curve when the initial value $x = [1 \ -1]^T$, from the curve it can be seen that the state response of the system is stable, it is shown that the controller design method mentioned in this paper is effective in the presence of handoff delay and limited communication.

Conclusion

In this paper, we addressed the controller design problem for networked switched control system with limited communication and handoff delay using Lyapunov theory and LMI methods. Firstly, the model of system is analyzed. On the basis of analyzed, the design method of the controller is given by using the concept of average dwell time. Finally, the feasible solution is obtained by the linear matrix inequality method, which verifies the validity and feasibility of the design method. Future work aims to address performance issues such as how to obtain more general stability criteria and how to choose the controller design parameters to improve transient response. Also, the influence of network transmission performance on the networked control system.

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