

Kinematics and Statics Analysis of Dexterous Hand

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Abstract: This study is based on the “Shadow” dexterous hand, whose mechanical structure are studied, and the displacement, velocity, acceleration and static force of dexterous hand joints are researched. Then the kinematics model is established by using D-H parameter method, and the forward and inverse kinematics equations and Jacobian matrix are derived. Finally the parameters of Differential kinematic and static force are analyzed and solved.

Introduction

The traditional Chinese massage technique is to generate external force by “massage manipulation”, which act on specific parts or acupoints in the body, so that the body's physiological and pathological conditions can be regulated and to achieve the therapeutic effect [1,2]. Therefore the design and research of the dexterous hand is the key to reach the practical level for the traditional Chinese medicine massage robot, and the hand should acquire sufficient abilities of dexterity and personification. Robot dexterous hand technology has been continuously improved and the most typical representative is the Shadow dexterous hand produced by Shadow Company.

This paper is based on the study of the structure characteristics of Shadow dexterous hand and the kinematic model is established. The data and conclusions of dexterous hand can provide theoretical support for the application of dexterous hand in Chinese massage.

Shadow Dexterous Hand

The Shadow dexterous hand in this paper is an advanced humanoid robot hand system that provides 24 movements to imitate the dexterity of the human hand as closely as possible. The Shadow Dexterous Hand is a self-contained system – all actuation and sensing required is built into the Hand. The hand has been devised to be similar to a typical human hand, but the fingers are all the same length as shown in Figure 1.

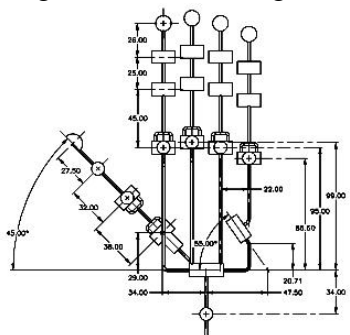


Fig. 1 Size of Shadow dexterous hand

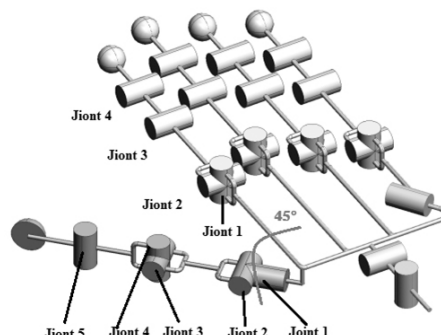


Fig. 2 Structure of Shadow dexterous hand

The Shadow dexterous hand consists of 24 joints altogether and a total of 20 degrees of freedom as shown in Figure 2. The thumb has 5 joints and 5 degrees of freedom and each of the other hands has 4 joints and 3 degrees of freedom, and the little finger has an extra joint in the palm. The distal joints of the fingers are coupled in a manner similar to a human finger.

Kinematic Research of the Finger

In order to realize the control of the motion trajectory of the robot hand, kinematic and statics research of the hand is needed including the forward and inverse kinematic, differential kinematics and statics [3-5].

Establishment of the coordinate system of thumb. Due to the similar structure of the five fingers, the thumb is taken as an example to elaborate the kinematic mechanism of single finger. Figure 3 illustrates the coordinate system of thumb, in which the coordinate $O_0 - x_0y_0z_0$ indicates base coordinate system and the coordinate $O_t - x_ty_t z_t$ indicates fingertip coordinate system.

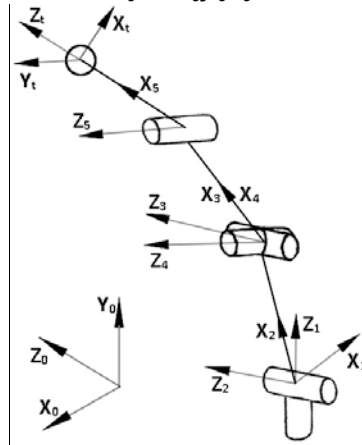


Fig. 3 Thumb link coordinates of Shadow dexterous hand

The forward kinematics of thumb. Table 1 shows the D-H parameters of thumb.

Table 1. D-H parameters of thumb

Joint	d_i	a_{i-1}	α_{i-1}	θ_i
1	0	0	0	θ_1
2	0	0	-90°	θ_2
3	0	b_2	0	θ_3
4	0	0	90°	θ_4
5	0	b_3	0	θ_5

Equation 1 indicates the coordinate transformation matrix of adjacent link coordinate so that the transition matrix which shows the position from fingertip coordinate to base coordinate can be deduced by Equation 2.

$$\begin{aligned}
 {}^0_1T &= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & b_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 {}^3_4T &= \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^4_5T = \begin{bmatrix} c_5 & -s_5 & 0 & b_3 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^5_tT = \begin{bmatrix} 0 & 0 & 1 & b_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 {}^0_tT &= {}^0_1T {}^1_2T {}^2_3T {}^3_4T {}^4_5T {}^5_tT = \begin{bmatrix} n_x & o_x & a_x & x_t \\ n_y & o_y & a_y & y_t \\ n_z & o_z & a_z & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\
 &= \begin{bmatrix} -c_1c_{23}s_{45} - s_1c_{45} & c_1s_{23} & c_1c_{23}c_{45} - s_1s_{45} & b_4(c_1c_{23}c_{45} - s_1s_{45}) + b_3(c_1c_4c_{23} - s_1s_4) - b_2c_1c_2 \\ -s_1c_{23}s_{45} + c_1c_{45} & s_1s_{23} & s_1c_{23}c_{45} + c_1s_{45} & b_4(s_1c_{23}c_{45} + c_1s_{45}) + b_3(s_1c_4c_{23} + c_1s_4) + b_2s_1c_2 \\ s_{23}s_{45} & c_{23} & -s_{23}c_{45} & -b_4s_{23}c_{45} - b_3s_{23}c_4 - b_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)
 \end{aligned}$$

The motion ranges of thumb's joints are shown in Table 2. Given the numerical values of θ_1 , θ_2 , θ_3 , θ_4 and θ_5 , the fingertip's posture in base coordinate system can be obtained according to Equation 3 to 11, and the fingertip's position can be obtained according to Equation 12 to 14.

Table 2. D-H parameters of thumb

Joint		θ_1	θ_2	θ_3	θ_4	θ_5
Motion range	Max.	60°	75°	15°	30°	90°
	Min.	-60°	0°	-15°	-30°	-10°

The inverse kinematics of thumb. According to Equation 1, $({}^4_5T)^{-1}$, $({}^5_tT)^{-1}$, 0_3T and 0_4T can be derived as shown below.

$$\begin{aligned}
 ({}^4_5T)^{-1} &= \begin{bmatrix} c_5 & s_5 & 0 & -b_3c_5 \\ -s_5 & c_5 & 0 & b_3c_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ({}^5_tT)^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -b_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 {}^0_4T &= \begin{bmatrix} c_1c_4c_{23} - s_1s_4 & -c_1s_4c_{23} - s_1c_4 & c_1s_{23} & b_2c_1c_2 \\ s_1c_4c_{23} + c_1s_4 & -s_1s_4c_{23} + c_1c_4 & s_1s_{23} & b_2s_1c_2 \\ -s_{23}c_4 & s_4s_{23} & c_{23} & -b_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3)
 \end{aligned}$$

Due to ${}^0_4T({}^5_tT)^{-1}({}^4_5T)^{-1} = {}^0_4T$, Equation 4 can be obtained by put Equation 2 and 3 into it.

$$\begin{aligned}
 &\begin{bmatrix} c_5a_x - s_5n_x & s_5a_x + c_5n_x & o_x & x_t - b_4a_x + b_3(s_5n_x - c_5a_x) \\ c_5a_y - s_5n_y & s_5a_y + c_5n_y & o_y & y_t - b_4a_y + b_3(s_5n_y - c_5a_y) \\ c_5a_z - s_5n_z & s_5a_z + c_5n_z & o_z & z_t - b_4a_z + b_3(s_5n_z - c_5a_z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} c_1c_4c_{23} - s_1s_4 & -c_1s_4c_{23} - s_1c_4 & c_1c_{23} & b_2c_1c_2 \\ s_1c_4c_{23} + c_1s_4 & -s_1s_4c_{23} + c_1c_4 & s_1s_{23} & b_2s_1c_2 \\ -s_{23}c_4 & s_4s_{23} & c_{23} & -b_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4)
 \end{aligned}$$

Equation 5 can be derived from Equation 4 to get θ_2 , θ_3 and θ_5 shown in Equation 6, 7 and 8.

$$\begin{cases} o_z = c_{23} \\ z_t - b_4a_z + b_3(s_5n_z - c_5a_z) = -b_2s_2. \\ \Delta s_5 - \nabla c_5 = * \end{cases} \quad (5)$$

In Equation 5, $\Delta = 2b_3(x_t n_x + y_t n_y + z_t n_z)$, $\nabla = -2b_3(x_t a_x + y_t a_y + z_t a_z - b_4)$, $* = b_2^2 - b_4^2 - b_3^2 - x_t^2 - y_t^2 - z_t^2 + 2b_4(x_t a_x + y_t a_y + z_t a_z)$.

$$\theta_2 = \text{atan2}\left(\frac{b_4a_z - z_t - b_3(s_5n_z - c_5a_z)}{b_2}, \pm \sqrt{1 - \left[\frac{b_4a_z - z_t - b_3(s_5n_z - c_5a_z)}{b_2}\right]^2}\right). \quad (6)$$

$$\theta_5 = \text{atan2}(*, \pm \sqrt{\Delta^2 + \nabla^2 - *^2}) - \text{atan2}(\nabla, \Delta). \quad (7)$$

$$\theta_3 = \text{atan2}(\pm \sqrt{1 - o_z^2}, o_z) - \theta_2. \quad (8)$$

According to Equation 9, $({}^3_4T)^{-1}$ and 0_3T can be derived as shown below.

$$({}^3_4T)^{-1} = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ -s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^0_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & -s_1 & b_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & c_1 & b_2s_1c_2 \\ -s_{23} & -c_{23} & 0 & -b_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (9)$$

Due to ${}^0T({}^5T)^{-1}({}^4T)^{-1}({}^3T)^{-1} = {}^0T$, Equation 10 can be obtained.

$$\begin{bmatrix} c_{45}a_x - s_{45}n_x & -0_x & s_{45}a_x + c_{45}n_x & x_t - b_4a_x + b_3(s_5n_x - c_5a_x) \\ c_{45}a_y - s_{45}n_y & -0_y & s_{45}a_y + c_{45}n_y & y_t - b_4a_y + b_3(s_5n_y - c_5a_y) \\ c_{45}a_z - s_{45}n_z & -0_z & s_{45}a_z + c_{45}n_z & z_t - b_4a_z + b_3(s_5n_z - c_5a_z) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & -s_1 & b_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & c_1 & b_2s_1c_2 \\ -s_{23} & -c_{23} & 0 & -b_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (10)$$

Equation 11 can be derived from Equation 10 to get θ_1 and θ_4 shown in Equation 12 and 13.

$$\begin{cases} s_{45}a_x + c_{45}n_x = -s_1 \\ s_{45}a_y + c_{45}n_y = c_1 \\ s_{45}a_z + c_{45}n_z = 0 \end{cases}. \quad (11)$$

$$\theta_4 = \text{atan2}(n_z, -a_z) - \theta_5. \quad (12)$$

$$\theta_1 = \text{atan2}(-s_{45}a_x - c_{45}n_x, s_{45}a_y + c_{45}n_y). \quad (13)$$

Equation 6, 7, 8, 12 and 13 are the inverse kinematics equations of thumb. Given the numerical value of (x_t, y_t, z_t) , each joint's angular displacement can be obtained according to these equations.

The differential kinematics of thumb. The differential kinematics is to establish the relationship between the linear velocity of the fingertip in fingertip coordinate [6,7] and the angular velocity of the joints, that is, $V = J(\theta)\dot{\theta}$. $J(\theta)$ is the Jacobian matrix, which builds connection between v_t and $\dot{\theta}$.

For rotating joint, V can be expressed as $V = \begin{bmatrix} v \\ \omega \end{bmatrix}$ and $J(\theta)$ as $J(\theta) = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$, what means $v = J_v \cdot \dot{\theta}$ and $\omega = J_\omega \cdot \dot{\theta}$. v and ω are the linear velocity and angular velocity of the fingertip.

The thumb consists of 5 degrees of freedom and so $J(\theta) = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v1} & J_{v2} & J_{v3} & J_{v4} & J_{v5} \\ J_{\omega1} & J_{\omega2} & J_{\omega3} & J_{\omega4} & J_{\omega5} \end{bmatrix}$.

$J_{\omega i}$ is the representation of the Z_i axis of each coordinate in the base coordinate system, so $J_{\omega i}$ can be derived according to Equation 1, as shown below in Equation 14.

$$J_{\omega1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, J_{\omega2} = J_{\omega3} = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, J_{\omega4} = J_{\omega5} = \begin{bmatrix} c_1s_{23} \\ s_1s_{23} \\ c_{23} \end{bmatrix}. \quad (14)$$

J_{vi} can be derived via the formula - $J_{vi} = J_{\omega i} \times ({}^0R \ i p_n)$ and furthermore 0R and $i p_n$ can be obtained from Equation 1, as shown in Equation 15 and 16, to get J_{vi} in Equation 17.

$$\begin{aligned} {}^0R &= \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^2R &= \begin{bmatrix} c_1c_2 & -c_1s_2 & -s_1 \\ s_1c_2 & -s_1s_2 & c_1 \\ -s_2 & -c_2 & 0 \end{bmatrix}, {}^3R &= \begin{bmatrix} c_1c_{23} & -c_1s_{23} & -s_1 \\ s_1c_{23} & -s_1s_{23} & c_1 \\ -s_{23} & -c_{23} & 0 \end{bmatrix}, \\ {}^4R &= \begin{bmatrix} c_1c_{23}c_4 - s_1s_4 & -c_1c_{23}c_4 - s_1c_4 & c_1s_{23} \\ s_1c_{23}c_4 + c_1s_4 & -s_1c_{23}c_4 + c_1c_4 & s_1s_{23} \\ -s_{23}c_4 & s_{23}s_4 & c_{23} \end{bmatrix}, {}^5R &= \begin{bmatrix} c_1c_{23}c_{45} - s_1s_{45} & -c_1c_{23}c_{45} - s_1c_{45} & c_1s_{23} \\ s_1c_{23}c_{45} + c_1s_{45} & -s_1c_{23}c_{45} - c_1c_{45} & s_1s_{23} \\ -s_{23}c_{45} & s_{23}s_{45} & c_{23} \end{bmatrix}. \end{aligned} \quad (15)$$

$$\begin{aligned} {}^1p_t &= \begin{bmatrix} b_4c_{23}c_{45} + b_3c_{23}c_4 + a_3c_{23} + b_2c_2 \\ b_4s_{45} + b_3s_4 \\ -b_4s_{23}c_{45} - b_3s_{23}c_4 - a_3s_{23} - b_2s_2 \end{bmatrix}, {}^2p_t &= \begin{bmatrix} b_4c_2c_{45} + b_3c_3c_4 + a_3c_3 + b_2 \\ b_4s_3c_{45} + b_3s_3c_4 + a_3s_3 \\ b_4s_{45} + b_3s_4 \end{bmatrix}, \\ {}^3p_t &= \begin{bmatrix} b_4c_{45} + b_3c_4 + a_3 \\ 0 \\ b_4s_{45} + b_3s_4 \end{bmatrix}, {}^4p_t &= \begin{bmatrix} b_4c_5 + b_3 \\ b_4s_5 \\ 0 \end{bmatrix}, {}^5p_t &= \begin{bmatrix} b_4 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (16)$$

$$\begin{aligned} J_1 &= z_1 \times ({}^0R \ {}^1p_t) = \begin{bmatrix} -b_4(s_1c_{23}c_{45} + c_1s_{45}) - b_3(s_1c_4c_{23} + c_1s_4) + b_2s_1c_2 \\ b_4(c_1c_{23}c_{45} - s_1s_{45}) + b_3(c_1c_4c_{23} - s_1s_4) + b_2c_1c_2 \\ 0 \end{bmatrix}, \\ J_2 &= z_2 \times ({}^2R \ {}^2p_t) = \begin{bmatrix} -b_4c_1c_{45}s_{23} - b_3c_1c_4s_{23} + b_2c_1s_2 \\ -b_4s_1s_{23}c_{45} - b_3s_1c_4s_{23} - b_2s_1s_2 \\ -b_4c_{23}c_{45} - b_3c_{23}c_4 - b_2c_2 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
 J_3 &= z_3 \times \begin{pmatrix} {}^0R^3 p_t \end{pmatrix} = \begin{bmatrix} -b_4 c_1 c_{45} s_{23} - b_3 c_1 c_4 s_{23} \\ -b_4 s_1 s_{23} c_{45} - b_3 s_1 c_4 s_{23} \\ -b_4 c_{23} c_{45} - b_3 c_{23} c_4 \end{bmatrix}, \\
 J_4 &= z_4 \times \begin{pmatrix} {}^0R^4 p_t \end{pmatrix} = \begin{bmatrix} -b_4 (c_1 c_{23} s_{45} + s_1 c_{45}) - b_3 (c_1 c_{23} s_4 + s_1 c_4) \\ -b_4 (s_1 s_{45} c_{23} - c_1 c_{45}) - b_3 (s_1 s_4 c_{23} - c_1 c_4) \\ b_4 s_{23} s_{45} + b_3 s_{23} s_4 \end{bmatrix}, \\
 J_5 &= z_t \times \begin{pmatrix} {}^0R^t p_t \end{pmatrix} = \begin{bmatrix} -b_4 (c_1 c_{23} s_{45} + s_1 c_{45}) \\ -b_4 (s_1 s_{45} c_{23} - c_1 c_{45}) \\ b_4 s_{23} s_{45} \end{bmatrix}.
 \end{aligned} \tag{17}$$

The statics of thumb. The statics of thumb is to establish the mapping relationship between the fingertip contact force and the torque of each joint under the static equilibrium condition [8].

According to the robotics theory, a strict correspondence exists between the statics of the fingers and the differential kinematics, so the static model of thumb can be established in the light of the differential kinematics, which is shown below in Equation 18.

$$\tau = J^T \cdot f_{ext}. \tag{18}$$

In Equation 18, τ is output torque of thumb joints, J is the Jacobian matrix of fingertip coordinate system and f_{ext} is the contact force between fingertip and external environment.

Summary

The D-H coordinate method is used in this paper to establish the kinematics model of the Shadow dexterous hand and the forward and inverse kinematics equations and analytic solutions are derived. Then on the base of these, the Jacobian matrix, which expresses the relationship between fingertip linear velocity and joints angular velocity, is deduced. Finally the statics parameters of thumb is analyzed as well to provide reference data for the study of dynamics.

References

- [1] Z.H. Wang, Q.H. Yang, S.M. Q, et al. Output Force Control of Pneumatic Flexible Dexterous Hand. Transactions of the Chinese Society for Agricultural Machinery, 2012(10): 209-214.
- [2] J.J. Mao, Z.J. Sun. Construction and implementation of the master-slave control system of a three-finger dexterous hand. Machinery and Electronics, 2010(10): 55-58.
- [3] Y.L. Zhu, B.J. Guo. Kinematics calculation and analysis of multi-fingered dexterous hand driven by artificial muscles. Machinery Design and Manufacture, 2013(11): 224-227.
- [4] Research on grasping optimization of dexterous robot hand based on directional manipulability Journal of Machine Design, 2012(4): 12-16.
- [5] L.B. Gao, B.J. Guo, K. Wang. Dexterous robot fingers structural design and single-finger control strategy. Chinese Hydraulics and Pneumatics, 2012(2): 11-14.
- [6] L. Jiang, H. Liu. Inverse kinematics of dexterous finger with linkage mechanism. Journal of Harbin Institute of Technology, 2007(3): 394-397.
- [7] P.C. Zhang, T. Zhang. Analysis on solution of 6D of robot Jacobian matrix and singularity configuration based on vector product method. Machinery Design and Manufacture, 2011(8): 152-154.
- [8] H. Wang. Kinematics analysis and test of underwater dexterous hand. Journal of Mechanical Transmission, 2012(3): 67-69.