Research and Simulation of satellite orbit modeling
Wenle Yuan, Xuanmin Lu, Jinjie Cao, Wensheng Luo
(School of Electronics and Information, Northwestern Polytechnical University, 710072 Xi'an)

Keywords: Beidou satellite navigation system; Beidou satellite orbit modeling; observation data simulation; trajectory simulation; geometric accuracy factor

Abstract. China's Beidou satellite navigation system has completed the regional network in Asia Pacific, and successfully applied in surveying and mapping, telecommunications, transportation, disaster relief and public safety and other areas, resulting in significant economic and social benefits. With the wide application of Beidou navigation system, Beidou navigation products are more and more, how to simulate the satellite orbit on the ground, has become a major problem of Beidou navigation product performance testing needs to be solved. In order to solve this problem, through the analysis of the Beidou satellite orbit simulation and modeling, and using the Chebyshev polynomial fitting algorithm of satellite orbit. Finally, the operation results of simulation module is analyzed in detail, the simulation results are consistent with the Beidou satellite related research data, verify the validity of the designed system. Therefore, the conclusion: Beidou satellite orbit modeling successfully.

1. Introduction

In the simulation of the Beidou satellite constellation, need to know the instantaneous satellite position and velocity. Typically, the instantaneous satellite position and velocity is through the orbit satellite navigation message issued the parameters according to the orbit model solution obtained, so the modeling and Simulation of satellite orbit satellite constellation simulation is a key link. This section will introduce the orbit model, satellite orbit simulation, Chebyshev polynomial fitting algorithm of satellite orbit.

2. Track model analysis

Similar to the shape of the earth rotation ellipsoid, strictly speaking, is an ellipsoid of irregular shape, and the earth's mass distribution is not uniform, therefore, to the earth satellite gravity is not always toward the earth. The motion of the satellite in space, except by the earth's gravitational field perturbation, but also by the the other celestial gravitation, especially the perturbation and the solar radiation pressure and other factors, the orbit is very complicated, it is difficult to make accurate mathematical methods by satellite. In all of the perturbation effect of the earth's gravitational field, the maximum perturbation, so the simulation of satellite orbit parameters considering the gravitational field of the earth perturbation. On the basis of Kepler's Law: the motion of the satellite orbit by an elliptical geocentric plane, the ellipse with a focus on the coincidence. Ideal elliptical orbit can use the following 6 orbit parameters (Kepler Track parameter):

- Track long axis $a$;
- orbital eccentricity $e$;
- inclination of orbit $i$: Track the dihedral angle and the equatorial plane;
- right ascension of ascending node $\Omega$: The earth's equatorial plane, the angle between the vernal equinox and the ascending node;
- argument of perigee $\omega$: The center of the earth's orbit plane, near the intersection point between the location and the center of the earth;
- The orbital inclination, RAAN, near point distance of three parameters to determine the
relative position and direction of satellite orbital plane and the earth.

- true anomaly \( f \) : The angle between the earth's orbit and near the earth's orbit.

The position of the satellite in the orbit plane is determined by the true near point angle.

The schematic diagram of Kepler's orbital parameters is shown in Figure 1:

![Fig. 1 Schematic diagram of Kepler track parameters](image)

Figure 2-2, the instantaneous position of the satellite in the orbital plane is described by the near point angle, in addition to the real point of the near point, there are commonly used near point angle and near the point of peace.

3. Beidou satellite orbit simulation

The motion orbit satellite by satellite ephemeris in orbit long radius, eccentricity, orbit inclination, RAAN, near point distance determined. The simulation on the orbit of the satellite, according to the 5 parameters, calculate the space position of the satellite orbit. The satellite orbit simulation, through the following 5 steps.

1. to establish the coordinate system

   The earth is the origin, establishment of space right in Cartesian coordinates, the X axis is semi axis pointing longitude \(-180^\circ\) direction, The axis of the Y axis is pointing to the north pole, and the Z axis is determined by the X axis and the Y axis.

2. Create Ellipse

   In the equatorial plane of the earth, the semi major axis of the ellipse, which is an ellipse, is created \( a \); Heart rate \( e \). Place the earth in one of the focal points of the ellipse.

3. setting the inclination angle of the track

   The rotation matrix is obtained according to the definition of the inclination of the orbit \( R_i \), Lift the ellipse around the X axis \( i \)

   \[
   R_i = \begin{bmatrix}
   1 & 0 & 0 \\
   0 & \cos i & \sin i \\
   0 & -\sin i & \cos i
   \end{bmatrix}
   \]

4. set up the ascension point of ascension

   According to the definition of the ascending node, the rotation matrix is obtained

   \[
   R_\Omega = \begin{bmatrix}
   \cos \Omega & 0 & -\sin \Omega \\
   0 & 1 & 0 \\
   \sin \Omega & 0 & \cos \Omega
   \end{bmatrix}
   \]

5. set the argument of perigee

   According to the definition of the AP, get the rotation matrix \( R_\omega \), the earth moves to a focal point of the ellipse, and through the center of the earth and perpendicular to the orbital plane trajectory as
the rotation axis, the rotation angle around the axis orbit \( \omega \)

\[
R_\omega = \left[ \begin{array}{ccc}
\cos \omega + B_1^2 (1 - \cos \omega) & B_1 B_2 (1 - \cos \omega) - B_2 \sin \omega & B_1 B_3 (1 - \cos \omega) + B_3 \sin \omega \\
B_1 B_2 (1 - \cos \omega) + B_2 \sin \omega & \cos \omega + B_2^2 (1 - \cos \omega) & B_2 B_3 (1 - \cos \omega) - B_3 \sin \omega \\
B_2 B_3 (1 - \cos \omega) - B_3 \sin \omega & B_3 B_2 (1 - \cos \omega) + B_2 \sin \omega & \cos \omega + B_3^2 (1 - \cos \omega)
\end{array} \right]
\]  

(3)

Among:

\[
\begin{bmatrix}
B = AR, R_{i_\Omega} \\
A = \begin{bmatrix}
0 & 1 & 0
\end{bmatrix}
\end{bmatrix}
\]  

(4)

4. The Chebyshev fitting algorithm of satellite orbit

The update cycle of Beidou satellite ephemeris for an hour, so there will be some ephemeris error, thus calculated the Beidou satellite instantaneous position and instantaneous speed will have a certain error. So, the need for simulation of satellite orbit approximation through a certain algorithm, commonly used algorithm Lagrange difference algorithm and Chebyshev fitting algorithm. In this paper, using the Chebyshev fitting algorithm.

Assumed in time interval \([t_0, t_0 + \Delta t]\), using the Chebyshev polynomial approximation \(n\), among \(t_0\) Is the starting time, \(\Delta t\) is Interval time.First, the conversion of time interval:

\[
\tau = \frac{2}{\Delta t} (t - t_0) - 1, t \in [t_0, t_0 + \Delta t]
\]  

(5)

The instantaneous position and instantaneous velocity of the satellite can be expressed as the component of the X axis:

\[
x(t) = \sum_{i=0}^{n} C_{x_i} T_i(\tau)
\]  

(6)

Among:

- \(n\)——Chebyshev polynomial order;
- \(C_{x_i}\) ——The Chebyshev polynomial coefficients.

The calculation method of the component along the Y direction and the Z direction is similar to the X direction \(T_i(\tau)\) and \(T'_i(\tau)\) the expressions are as follows:

\[
\begin{align*}
T_0(\tau) &= 1 \\
T_1(\tau) &= \tau \\
T_n(\tau) &= 2T_{n-1}(\tau) - T_{n-2}(\tau) \quad |\tau| \leq 1, n \geq 2
\end{align*}
\]  

(7)

\[
\begin{align*}
T'_0(\tau) &= \tau' \\
T'_1(\tau) &= 4\tau \tau' \\
T'_n(\tau) &= \frac{2n}{n-1} T'_{n-1}(\tau) - \frac{n}{n-2} T'_{n-2}(\tau) \quad n \geq 3
\end{align*}
\]  

(8)

From equation (5) coefficients by least squares to Chebyshev polynomials:

\[
C_x = (B^T B)^{-1} B^T X
\]  

(9)

among (8) :
\[ B = \begin{bmatrix}
T_0(\tau_1) & T_1(\tau_1) & T_2(\tau_1) & \cdots & T_n(\tau_1) \\
T_0(\tau_2) & T_1(\tau_2) & T_2(\tau_2) & \cdots & T_n(\tau_2) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
T_0(\tau_m) & T_1(\tau_m) & T_2(\tau_m) & \cdots & T_n(\tau_m)
\end{bmatrix} \quad (10) \\
X = [x_1 \ x_2 \ x_3 \ \cdots \ x_n]^T \quad (11)
\]

Obtained coefficient \( C \), we can use type (6) to calculate the corresponding time interval at any time the position and velocity of the satellite. The simulation using 13 order Chebyshev polynomial fitting.

5. The results of simulation analysis are demonstrated and analyzed

The parameters of the simulation analysis function include the position of the observation station, the observation time, the calculation time interval, the height of the cut-off angle, and the interface is shown in Figure 2:

\[ T_n(\tau) = \sum_{m=0}^{n} C_m T_m(\tau) \]

Figure 2 Configuration Interface of Simulation analysis

The initial value setting of the simulation analysis module: the location of the observation station \((110^\circ E \ 40^\circ N \ 0)\), observation time period (September 20, 2016 00 minutes 0 seconds to 0 minutes, 0 minutes, 5 minutes, seconds), the time interval of 60 seconds, the height of the cut-off point for the following simulation results are based on the above set conditions.

Visible satellite number curve drawing and result analysis:

Can be seen from Figure 3, the number of visible satellites are available in 6, only a very short period of time was 6, the rest are in 8 or so, can be up to 11. The simulation results are consistent with the Beidou satellite research data show that the accuracy of the simulation system.
6. Summary

The key technology of the Beidou satellite orbit calculation model and Chebyshev polynomial fitting algorithm. Using VC6.0 and MATLAB as a combination of architecture, design and completion of the two regional Beidou satellite constellation simulation and visible satellite constellation simulation, GUI design is completed by MFC, and with MATLAB star Zuo Jianmo, matlabin will eventually be compiled into C files so as to achieve the combination of the two.

Through the research work in this paper, we have made some achievements in the research and implementation of the test system of the Beidou navigation products, and it has important reference value for the construction of the Beidou satellite navigation system.

References