Calculation of inductance variation for onboard magnetic flux compression generator

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Abstract. Onboard magnetic flux compression generator (MFCG) would be used widely in new types of projectiles that need huge electric energy, while the inductance variation is significant for onboard MFCG design. In order to achieve the inductance variation, image current model was applied to the circuit parameters calculation of an onboard MFCG. And the inductance variation expression was derived. Subsequently, by simulation software MATLAB, a numerical computation program of onboard MFCG inductance variation was written within deriving process. Curve of inductance change was achieved after substituting design parameters and computing, and next optimum design and experiments benefit from it.

Keywords: pulsed power; MFCG; inductance variation; onboard; image current.

1 Introduction

MFCG is equipment for conductor motion and compressing seed flux through the release of chemical explosives, pulse power generating means to achieve instant power. As a small size, light weight pulse power generating means MFCG is seen in electromagnetic pulse bombs and other new ammunition onboard power supply. MFCG equivalent circuit diagram shown in Figure 1, it can be regarded as substantially generator inductance $L_g(t)$, the resistance $R_g(t)$ and the load inductance $L_d$, resistance $R_d$ of a series circuit\textsuperscript{1,2}. When the switch $K_1$ is closed, the seed current source $C$ charges generator formed seed magnetic flux, while loop current is $i(t)$; if $i(t)$ up to a certain magnitude, the switch $K_2$ is closed and $L_g(t)$ fast reduce. According to the principle of conservation of magnetic flux, the loop will have a strong current\textsuperscript{3}. The entire process of inductance variation $L_g(t)$ is a critical parameter difficult to control and measure, thus MFCG inductance variation calculations will directly affect the effectiveness estimate of the design.

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2 Onboard MFCG model

According to a certain type of 100mm onboard platform requirements, design a multi-segment coil magnetic flux compression generator. Its structure shown in Figure 2, where the left is MFCG, connected directly to the right of warhead.

Fig 1. Equivalent circuit diagram of MFCG

1-The initial force of magnetic source; 2-Detonators, Booster Charge; 3-Explosive; 4-Pry off Switch of Copper; 5-Front insulation cover; 6-Coil; 7-Coil Package; 8-Back insulation cover; 9-Aluminum Armature

Fig 2. Onboard MFCG structure chart

Tubular armature material is aluminum, while coil material is copper. Wherein aluminum armature’s inner diameter is 45mm, length of 130mm, wall thickness 3.8mm. Armature built RDX explosives, which detonation velocity upon diameter of 45mm is about 6400m/s; Generator coil’s outer diameter is 100mm, total length 100mm, divided into two segments (N1 and N2), whose respective lengths l1=50mm and l2=50mm. The first segment is made by a conductive cross-section of 4mm2 single wire around the system, the second by the same two parallel wire around the system, the number of turns, respectively N1=11 and N2=6, which each coil segment parameters in Table 1. Also, according to the requirements of warhead, coiled copper wire is as a load, which the load inductance Ld is 0.443μH. Because of the conductivity of copper σCu=5.755×107(Ω·m)-1, load resistance Rd is 0.695mΩ.

Table 1. Each segment data of coil

<table>
<thead>
<tr>
<th>No.</th>
<th>Pitch (mm)</th>
<th>Parallel multi-core conductor strand number</th>
<th>Turns</th>
<th>Equivalent cross-sectional area/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.545</td>
<td>1</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8.333</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

At the appropriate time, via explosive detonators, explosive charges on the one hand push the armature radially expanding deformation, on the other hand delivering a detonation wave in the axial...
direction. Thus, tubular armature formed the cone angle of 12° cone under explosive action moves to the right, and constant contact, short circuit coil, so that the generator inductance is reduced. Inductance variation calculation of theory will be established according to this procedure.

3 Onboard MFCG inductance variation calculation theory

Based on magnetic flux compression generator equivalent circuit theory, estimating the effect of running the generator can be equivalent inductance $L_g(t)$ variation. Seen by the MFCG work process, both the coil magnetic flux compression generator equivalent inductance $L_g(t)$ and the equivalent DC resistance $R_g(t)$ are time-varying functions. Calculation can be used mirror current model 4, and we make the following assumptions here:

The induced current direction of the armature is opposite on the coil current direction, and they are the same size and have the same conductive cross section;

When the armature expansion, the radius of the equivalent armature current loop keep expanding until contacting with the coil, merged into a conductive ring;

The armature is a good conductor, does not consider the armature flux loss;

Calculating the resistance that the temperature is maintained at room temperature.

When calculating inductance variation, each of the conductive wire on the coil and each corresponding conductive paths of mirror current on the armature are simplified to a conductive ring, which ring inductance is calculated with

$$L = \mu_0 R \left( \ln \frac{8R}{a} - 2 \right)$$

(1)

Where $a$ is the radius of conducting cross-section of the ring, $R$ is the radius of the ring, $\mu_0$ is the vacuum permeability.

Provided the total inductance of the generator $L_g(t)$, the inductance of the coil is $L_s$, the resistance of the coil is $R_s$, the current flowing the coil is $I_s$; armature inductance $L_a$, armature resistance $R_a$, the current flowing armature is $I_a$, the mutual inductance between coil and armature is $M$. Easy to obtain the relationship between them like $L_g(t)=L_s+L_a+M$ and $I_s=I_a=i(t)$.

Because the armature and coil current direction is opposite, when the coil and armature are treat as two separate circuits, the following equation for loop equations of the armature and coil are available

$$\frac{d(LI)}{dt} - \frac{d(MI)}{dt} + RI = 0$$

(2)

According to the assumption (3), we have $R_a=0$. Since the initial flux of armature is zero, so there

$$I_a = \frac{M}{L_a} I_s$$

(3)

Coupled with the energy conservation equation, together into equation (2) order to give

$$L_g(t) = L_s - \frac{M^2}{L_a}$$

(4)

Wherein the coil inductance, the armature inductance and mutual inductance between them is calculated by the following equations 5

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\[ L = \sum_{i=n+1}^{N} L_i + 2 \sum_{i=n+1}^{N} \sum_{j=i+1}^{N} M_{ij} \]  \hspace{1cm} (5)

\[ M = 2 \sum_{i=n+1}^{N} \sum_{j=i+1}^{N} M'_{ij} \]  \hspace{1cm} (6)

Where: \( N \) is the number of turns; \( L_i \) for the ring inductance; \( M_{ij} \) is the mutual inductance between each rings in the coil, each rings in the armature; \( M'_{ij} \) is the mutual inductance between each rings of the coil and armature; \( n \) is dynamic energy equivalent number, related to operation state of the generator (\( n \geq 1 \)), which is an integer of a time-varying. Where \( M_{ij}, M'_{ij} \) calculated by the following equation

\[ M_{ij} = \mu_0 \sqrt{R_i R_j} \left[ \left( \frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right] \]  \hspace{1cm} (7)

Where in

\[ k = \frac{2 \sqrt{R_i R_j}}{\sqrt{(R_i + R_j)^2 + X_j^2}} \]  \hspace{1cm} (8)

\( R_i, R_j \) is the radius of the two rings which seeking mutual inductance; \( X_j \) is axial distance between the two rings seeking mutual inductance; \( K(k) \) and \( E(k) \) is the first and the second kind total ellipse integral within the modulus \( k \).

Dynamic energy equivalent number \( n \) and an armature ring radius \( r_j \) may occur following two situations:

- After detonated \((t>0)\) and before the armature touching coil\((t<t_0)\)
- In Figure 3, \( k \) represents the \( k \)-th equivalent ring just detonated, in which case \( n=k \). Detonating at \( t=0 \), provided the radial velocity of detonation \( V_y = D \tan \alpha \), axial detonation velocity of \( V_x = D \). The contact time between the armature and coil from start is \( t_0 = (R-r)/V_y \), the time that detonation wave sweep the first segment is \( t_1 = l_1/V_x \), where \( R \) is the inner radius of the coil, \( r \) is the outside radius of the armature. Where, \( t_0 > t_1 \).

![Figure 3](image.png)

**Figure 3.** Calculation model of dynamic energy equivalent number schematic A

So
\[ n = k = \text{int} \left( t \frac{D}{P} \right) \]  

(9)

In this case, the \( k \)-th radius \( r_k = r \), and arbitrary \( j \)-th ring radius

\[ r_j = r + P \tan \alpha |n - j| \]

(10)

After pry off switch is turned on \((t > t_1)\)

Figure 4. Calculation model of dynamic energy equivalent number schematic B

As shown in Figure 4, where \( n \) represents the \( n \)-th armature ring just contacting with the coil, in the same token there

\[ n = \text{int} \left[ \left( t - t_0 \right) \frac{D}{P} \right] + 1 \]

(11)

At this time, the radius of the \( n \)-th ring is \( r_n = R \), the radius of the \( k \)-th ring is \( r_k = r \), and the radius of an arbitrary \( j \)-th ring is

\[ r_j = R - P \left( j - n \right) \tan \alpha \]

(12)

Because of the different pitches between each segment, axial distance \( X_{ij} \) between \( i \)-th and \( j \)-th ring is particularity.

i. When \( i, j \leq N_1 \), \( X_{ij} = |i - j| P_1 \);

ii. When \( i, j > N_1 \), \( X_{ij} = |i - j| P_2 \);

iii. When \( i(j) \leq N_1 \) and \( j(i) > N_1 \), \( X_{ij} = P_1 \left( N_1 - i(j) \right) + P_2 \left( j(i) - N_1 \right) \).

4 Numeral calculations

According to the aforementioned calculation method, write computing program code using the Matlab, its flow chart shown in Figure 5.
Above onboard MFCG inductance variation numerical calculation, amount of data 100 to obtain inductance variation curve shown in Figure 6, in which the entire curve can be divided into three segments. a segment from 0μs to 16μs, is the accelerating falling segment; b segment from 16μs to 23μs, is the rapid falling segment; c segment from 23μs to finish, is the steady falling segment.

![Inductance Variation Curve](image)

**Figure 6. Curve of Inductance Variation**

Corresponding analysis schematic diagram is shown in Figure 7, where A is the point in time of the armature contacting the first segment coil about 16μs, B is the point in time of the armature contacting the second segment coil about 23μs. In the a segment armature not yet started short-circuit coil ring, only limited compression flux in space and the compression space increases, which can be observed in the slope of the curve is increasing. In the b segment armature under the action of detonation energy fast shorted the each rings of first segment coil, resulting in a rapid decline in the inductance, had the largest slope in the entire curve. And in the c segment armature moved to the
second segment coil which conductive ring becomes the twice width, shorting circuit speed is halved, with the slope of the curve changed.

![Figure 7. Analysis schematic diagram](Image)

The last available initial inductance $L_g(0)$ is about $15.558\mu\text{H}$, at the end of the inductance $L_g(tf)$ is about $0.244\mu\text{H}$, running time $T_f$ is about $31.388\mu\text{s}$, that is, the generator has nearly 64 times the magnitude of the inductance change in the ideal state.

### 5 Conclusion

According to a certain type of shaped charge designed onboard platform multi-stage coil type MFCG has its own peculiarities in structure, and its inductance variation calculations also has its own characteristics. Mirror current law is an important method of calculation MFCG circuit parameters, can effectively simplify the process of magnetic flux compression generator operating parameters calculation. The mirror current law is applied to design verification calculation of onboard MFCG, the obtained initial and terminal inductance 15.558$\mu$H and 0.244$\mu$H succinctly shows the expected impact of the initial design. It provides the basis to adjusting parameters of onboard magnetic flux compression generator such as structure parameters, and improves the efficiency of the design optimization.

### References