Decision model for project bidding based on information diffusion

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ABSTRACT: A decision model for construction enterprises is proposed in view of lack of data and uncertainty of quantitative relation existing in bidding decision. The ratio of bidding price of an opponent to the cost estimation of decision maker is regarded as random variable, and historical data of the ratio are diffused to monitoring points through the normal diffusion function. The probabilities and exceeding probabilities of monitoring points are estimated using the information diffusion results. Furthermore, the probability distributions of decision maker's bidding price being lower than opponent's are acquired, and profit expectations with different bidding prices of decision maker are figured out as the basis of bidding decision. The model is able to deal with bidding decision of single or multiple opponents. The case study illustrates the feasibility of this model. Finally, a new bidding decision method utilizing small sample data is presented in the paper.

INTRODUCTION

Under the condition of market economy, the construction enterprises have to undertake the task of project construction through bidding. However, with the fierce competition in the market, how to adopt a reasonable method to make scientific decision for bidding is a problem that decision makers encounter. The bidding price plays an important role in winning the bid.

Friedman(1956) proposed a bidding decision model based on probability distribution, this model regard the ratio of the bidding price of opponent to the cost estimation of decision maker as random variable, and then calculate the probability of the decision maker's bidding price being lower than the competitor's. The model provides an important reference for the later research. Gates(1967) presented another bidding decision model based on probability theory. Models of Friedman and Gates adopt probability analysis to determine the possibility of winning bid on the basis of the record of historical data, and made bidding decision. However, on the basis of same preconditions, the two models can draw different conclusions. Researchers had a long discussion about which of the two models being more reasonable , and no final conclusion has yet been reached on this matte (Crowley 2000).

The bidding decision models based on statistics and probability theory need to make use of a certain amount of data for decision analysis, and the amount of data has a great influence on the accuracy of the calculation results. However, it is difficult to obtain large amounts of data.

According to the principle of information diffusion，in the case of incomplete samples, reasonable diffusion functions that can improve the non - diffusion estimation exist(Huang 1997). The data used for bidding decision are often small sample, which are incomplete sample, and the traditional histogram method that is used to estimate the probability distribution function of random variables will lead to larger error. In this paper, information diffusion method is used to establish the information matrix, which can make full use of the information provided by the data. The information diffusion method that can improve the decision making process will provide a more scientific basis for decision making.

INFORMATION DIFFUSION FUNCTION

Definition 1(Huang 2005a) Let X with universe U be a sample and V be a subset of U. A mapping from $X \times V$ to $[0,1]$

$$m: X \times V \rightarrow [0,1]$$

$(x,v) \rightarrow m(x,v)$, $\forall (x,v) \in X \times V$

is called an information diffusion of X on V if it is decreasing, i.e.,

$\forall x \in X$, $\forall v', v'' \in V$, if $\|v' - x\| \leq \|v'' - x\|$, then $m(x, v') \geq m(x, v'')$,

m is called a diffusion function and V is called a monitoring space. When $V = U$, we say that $m(x, v)$ is sufficient.

The information diffusion function can be used to diffuse the information from the sample to the monitoring point.

The normal information diffusion function is as follows (Huang 2005a).

$$m(x, u) = \frac{1}{h\sqrt{2p}} \exp\left[-\frac{(x-u)^2}{2h^2} \right] \qquad (1)$$

where, h is diffusion coefficient according to the value of Table 1.

Table 1. Values of diffusion coefficient

Sample size	5	6	7	8
Calculation formula of h	0.8146 l	0.5690l	0.456 0l	0.3860 l
Sample size	9	10	≥ 11	
Calculation formula of h	0.3362 l	0.2986l	2.6851l/(n-1)	

note：$l = \max_{1 \leq i \leq n}\{x_i\} - \min_{1 \leq i \leq n}\{x_i\}$, x_i is one dimensional data，n is sample size

BIDDING DECISION MODEL

Calculation of winning bid probability
Situation with an opponent
Let

$$R^c = \frac{x}{C_e} \qquad (2)$$

where, x is bidding price of opponent; C_e is the estimation of cost made by decision maker.

R^c is a random variable, and the probability of bidding price of decision maker being lower than that of the opponent is as follows(Friedman 1956)

$$P(R^c \geq \frac{y_0}{C_e}) = \int_{y_0/C_e}^{\infty} f_{R^c}(r)dr \qquad (3)$$

where, $f_{R^c}(r)$ is the probability distribution density function of R^c.

Let

$$r^b = \frac{y_b}{C_e} \qquad (4)$$

where, y_b is the bidding price of decision maker.

Let us suppose that decision maker has encountered the opponent several times, and the decision maker holds record of R^c that is $\{r_1^c, r_2^c, \mathbf{L}, r_N^c\}$.

Let monitoring points of R^c be $U = \{u_1, u_2, \ldots, u_m\}$, then the information provided by r_i^c can be diffused to the monitoring point u_j. According to formula (1), the diffusion formula is as below

$$f_i(u_j) = \frac{1}{h\sqrt{2p}} \exp\left[-\frac{(r_i^c - u_j)^2}{2h^2} \right] \qquad (5)$$

According to Huang (2005b),there are formulas as follows

$$C_i = \sum_{j=1}^{m} f_i(u_j), \qquad (6)$$

$$q(u_j) = \sum_{i=1}^{N} \frac{f_i(u_j)}{C_i} \qquad (7)$$

$$Q = \sum_{j=1}^{m} q(u_j) \qquad (8)$$

$$p(u_j) = \frac{q(u_j)}{Q} \qquad (9)$$

where, $p(u_j)$ is the probability estimation of monitoring point u_j.

Exceeding probability of u_j can be figured out as follows

$$P(u_j) = \sum_{k=j}^{m} p(u_j) \qquad (10)$$

If

$$r_j^b = u_j - \Delta \qquad (11)$$

where, $\Delta = \dfrac{u_i - u_{i-1}}{2}$.

Then the bidding price of decision maker is y_j.

$$y_j = r_j^b \cdot C_e^n \qquad (12)$$

where, C_e^n is the cost estimation made by decision maker for the intended bidding project.

Evidently, when y_j is adopted as the bidding price by decision maker, the probability of y_j being lower than bidding price of opponent is $P(u_j)$.

Situation with multiple opponents

In the situation with multiple opponents, it is necessary for the decision maker winning the bidding that the decision maker's bidding price is lower than all opponents'. Bidding of each opponent is independent event, and if there are C opponents, probability of decision maker winning all opponents can be calculated, then the probability of decision maker's bidding price being lower than all opponents' is as follows.

$$P(u_j) = \prod_{k=1}^{C} P_k(u_j) \qquad (13)$$

where, $P_k(u_j)$ is the probability of bidding price y_j of decision maker being lower than that of the opponent k.

Bidding price decision making

If decision maker's bidding price is y_j, profit expectation value of decision maker is

$$M(y_j) = (y_j - C_e^n) \cdot P(u_j) - C_b[1 - P(u_j)] \qquad (14)$$

where, C_b is the decision maker's cost on bidding.

Then, the optimal bidding of decision maker is y_p

$$s.t. \qquad M(y_p) = \max_{j}(M(y_j)) \qquad (15)$$

CASE STUDY

Background

A construction enterprise was going to bid for a project. It was known that there were three opponents A, B and C intending to bid for the project. The decision maker had met the enterprises several times, and the bidding records are follows.

Table 2. Data of bidder A

No.	1	2	3	4	5
r^c	1.030	1.100	1.070	1.050	1.130
No.	6	7	8	9	10
r^c	1.120	1.100	1.140	1.140	1.150

Table 3. Data of bidder B

No.	1	2	3	4
r^c	1.050	1.060	1.030	1.140
No.	5	6	7	8
r^c	1.030	1.130	1.130	1.120

Table 4. Data of bidder C

No.	1	2	3	4
r^c	1.100	1.020	1.050	1.020
No.	5	6	7	8
r^c	1.030	1.050	1.150	1.040

In the above tables, r^c is the observed value of R^c, R^c is defined by formula (2).

The decision maker estimated the cost of this project to be 53.62 million yuan.

Probability of winning the bid

The monitoring space r^c was chosen as $U =\{u_1, u_2, u_3 , u_4, u_5 , u_6, u_7, u_8, u_9, u_{10}\}= \{ 1.020, 1.034, 1.049, 1.063, 1.078, 1.092, 1.107, 1.121, 1.136, 1.150\}$. According to formula (5), the information diffusion value matrix of A's data to the monitoring points was obtained. Probabilities of monitoring points were figured out on the basis of formula (9), exceeding probabilities of monitoring points were calculated according to formula (10), they are listed below.

Table 5. Estimation of monitoring points probabilities and exceeding probabilities of opponent A

Monitoring point	u_1	u_2	u_3	u_4	u_5
Probability	0.046	0.060	0.073	0.085	0.099
Exceeding probability	1.000	0.954	0.894	0.822	0.736
Monitoring point	u_6	u_7	u_8	u_9	u_{10}
Probability	0.115	0.130	0.138	0.135	0.119
Exceeding probability	0.637	0.522	0.392	0.254	0.119

According to formula (12), probability distributions of decision maker's bidding price being lower than A's were figured out.

Table 6. Probability distributions of decision maker's bidding price being lower than A's

Decision maker's bidding price(ten thousand yuan)	5430	5508	5585	5662	5740
Probability of being lower than A	1.000	0.954	0.894	0.822	0.736
Decision maker's bidding price(ten thousand yuan)	5817	5895	5972	6050	6127
Probability of being lower than A	0.637	0.522	0.392	0.254	0.119

Similarly, probabilities and exceeding probabilities of monitoring points according to data of bidder B are listed below.

Table 7. Estimation of monitoring points probabilities and exceeding probabilities of opponent B

Monitoring point	u_1	u_2	u_3	u_4	u_5
Probability	0.066	0.077	0.083	0.088	0.094
Exceeding probability	1.000	0.934	0.857	0.774	0.686
Monitoring point	u_6	u_7	u_8	u_9	u_{10}
Probability	0.103	0.115	0.126	0.129	0.12
Exceeding probability	0.593	0.490	0.375	0.249	0.120

The probability distributions of decision maker's bidding price being lower than B's were obtained.

Table 8. Probability distributions of decision maker's bidding price being lower than B's

Decision maker's bidding price(ten thousand yuan)	5430	5508	5585	5662	5740
Probability of being lower than B	1.000	0.934	0.857	0.774	0.686
Decision maker's bidding price(ten thousand yuan)	5817	5895	5972	6050	6127
Probability of being lower than B	0.593	0.490	0.375	0.249	0.120

Probabilities and exceeding probabilities of monitoring points according to data of bidder C are listed below.

Table 9. Estimation of monitoring points probabilities and exceeding probabilities of opponent C

Monitoring point	u_1	u_2	u_3	u_4	u_5
Probability	0.105	0.111	0.111	0.106	0.100
Exceeding probability	1.000	0.895	0.784	0.673	0.567
Monitoring point	u_6	u_7	u_8	u_9	u_{10}
Probability	0.096	0.094	0.095	0.094	0.088
Exceeding probability	0.467	0.371	0.277	0.182	0.088

The probability distributions of decision maker's bidding price being lower than C's were obtained.

Table 10 Probability distributions of decision maker's bidding price being lower than C's

Decision maker's bidding price(ten thousand yuan)	5430	5508	5585	5662	5740
Probability of being lower than C	1.000	0.895	0.784	0.673	0.567
Decision maker's bidding price(ten thousand yuan)	5817	5895	5972	6050	6127
Probability of being lower than C	0.467	0.371	0.277	0.182	0.088

The probability distributions of decision maker's bidding price being lower than all opponents' were worked out on the basis of formula (13).

Table 11. Probability distributions of decision maker's bidding price being lower than all opponents

Decision maker's bidding price(ten thousand yuan)	5430	5508	5585	5662	5740
Probability of being lower than all opponents	1.000	0.798	0.601	0.428	0.286
Decision maker's bidding price(ten thousand yuan)	5817	5895	5972	6050	6127
Probability of being lower than all opponents	0.176	0.095	0.041	0.012	0.001

Decision making of bidding price

Bidding cost being ignored, bidding profit expectation values of decision maker were obtained in accordance with formula (14).

Table 12. Bidding profit expectation values of decision maker

Decision maker's bidding price(ten thousand yuan)	5430	5508	5585	5662	5740
Profit expectation values(ten thousand yuan）	68.5	116.4	134.3	128.8	108.3
Decision maker's bidding price(ten thousand yuan)	5817	5895	5972	6050	6127
Profit expectation values(ten thousand yuan)	80.3	50.6	24.9	7.9	0.9

Obviously, the maximum bidding profit expectation value is 134.3 ten thousand yuan with the bidding price 5585 ten thousand yuan, and then the optimal bidding price is 5585 ten thousand yuan.

CONCLUSIONS

As a result of business competition, it is very difficult to get the data of competitors, the decision maker has to make full use of less data to make decision analysis. Because of the lack of data, the bidding decision models based on probability theory whose basis is the law of large numbers are of obvious limitation. Information diffusion theory has been proved to be a good method for small sample data analysis (Huang 2005a).In this paper, the information diffusion theory is applied to the bidding decision making, and the decision model is put forward to make full use of the information provided by the data, and the decision result depends on objective data. The model mentioned in this paper has some practical value.

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