3 types of vehicle-bridge coupling vibration analysis of model comparison

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Abstract: Compares two 1/4 car models, two 1/4 model (changes the spring stiffness) and 1/2 car models three vehicles at the same speed in the displacement effect. Comparison: when the vehicle speed is less than 130km/h, and displacement response curves in the three models are calculated when a highway bridge coupling vibration analysis of vehicle is two one-fourth model can be used to simulate.

Introduction

Vehicle-bridge coupled vibration attracts many researchers’ attention as early as 100 years ago. The current study has simplified the vehicle and bridge model in it to make for a simpler solution.

In recent years, many Chinese researchers are trying to use numerical method to solve coupling problems. The document[4] builds overall 3D model to make the interaction between vehicle and bridge as a whole, thus establishing differential equations, with the kelp of numerical methods, the dynamic response is obtained with respect to simple-supported girder and continuous beam in highway. The document[5-6] makes a model of vehicle bridge coupling of simple-supported beam bridge employing d’alembert’s principle and uses numerical methods to solve the problem. Step-by-step integral is used to calculate the coupling system consisting of mass with uniform variable speed and a simple supported beam. The document[8] acquires the power series solution to Willis equation. The document[9] derives the vibration equation by ignoring moving constant loads of mass vehicle and thus gets the exact solution. The document[10] uses concentrated mass as the simplified bridge model and adds 2D multi-axis trailer load.

To investigate the influence of vehicle model on vehicle-bridge coupling, three kinds of models: two quarter-car models (spring stiffness changed) and half-car models with different velocity, are used to analyze displacement response. The overall laws are analyze in the first place which followed the analysis of difference of models.

Model

one-fourth car models

Figure 1 is vehicle model, the spring-damper-mass system is for simulating vehicles. Where

\( m_v \) is vehicle mass. \( m_i \) is the frame and wheel mass; \( k_a \) is vertical stiffness; \( c_a \) is vertical damping;

\( k_b \) is vertical stiffness; \( c_b \) is vertical damping; \( v \) is the speed when vehicles go cross the bridge.
The vibration equation of vehicle and bridge are as follows:

\[
\begin{align*}
\frac{d^2y_1}{dt^2} + m_a \frac{d^2y_a}{dt^2} + c_a \left( \frac{dy_1}{dt} + \frac{dy_a}{dt} \right) + k_a (y_1 + w) &= 0, \\
\frac{d^2y_b}{dt^2} + c_b \left( \frac{dy_b}{dt} + \frac{dy_a}{dt} \right) + k_b (y_b - y_1) &= 0.
\end{align*}
\]

Supposing \( w(x,t) = \sum_n X_n(x)T_n(t) \) by using variable separation method. Using simple supported beam as boundary condition and presuming that \( X_n(x) = \sin(n\pi x/l) \). According the orthogonality of principal mode, the vibration differential equation under loads is as follows:

\[
\ddot{T}_n(t) + w_n^2 T_n = \frac{2(m_l + m_b)g}{ml} \sin \frac{n\pi vt}{l} + \\
\frac{2m_i}{ml} \frac{d^2y_i}{dt^2} \sin \frac{n\pi vt}{l} + \frac{2m_b}{ml} \frac{d^2y_b}{dt^2} \sin \frac{n\pi vt}{l}
\]

With the help of variable separation method, suppose \( w(x,t) = \sum_n X_n(x)T_n(t) \). By simply-supported beams with boundary condition, let \( X_n(x) = \sin(n\pi x/l) \). Again according to the principal mode of orthogonality, get the vibration differential equation of the bridge under the action of load as follows:

\[
\ddot{T}_n(t) + w_n^2 T_n = \frac{2(m_l + m_b)g}{ml} \sin \frac{n\pi vt}{l} + \frac{2m_i}{ml} \frac{d^2y_i}{dt^2} \sin \frac{n\pi vt}{l} + \frac{2m_b}{ml} \frac{d^2y_b}{dt^2} \sin \frac{n\pi vt}{l},
\]

Two quarter-car models will be used. The vibration equations for vehicle and bridge are taken into consideration separately, he the results are in superposition. The other one is the same.

**Half-car models**

Figure 2 is vehicle model, the spring-damper-mass system is for simulation of vehicles. Where \( m_v \) is vehicle mass; \( I_b \) is the rigidity of vehicle body; \( m_l \) is the mass of wheels and structures; \( k_a \) is vertical stiffness; \( c_a \) is vertical damping; \( k_b \) is vertical stiffness; \( c_b \) is vertical damping; \( v \) is the speed when vehicles go cross the bridge(constant value). The vibration equations are as follows:\(^6\):

![Figure 1. One-fourth model under the effect of simply supported beam](image-url)
\[
\frac{dT(t)}{dt^2} + w_n^2 T_n(t) = 2p_1(t) \sin \frac{n\pi(vt-a)}{l} \delta_1(t) + 2p_2(t) \sin \frac{n\pi vt}{l} \delta_2(t)
\]

Which:

\[
\delta_1(t) = \begin{cases} 
1, & \frac{a}{v} \leq t \leq \frac{l+a}{v}; \\
0, & \text{else},
\end{cases}
\]

\[
\delta_2(t) = \begin{cases} 
1, & 0 \leq t \leq \frac{l}{v}; \\
0, & \text{else},
\end{cases}
\]

Analysis of load of the vehicle system:

\[
p_1(t) = m_{a1}g + \frac{m_b}{2} g + m_{a1}y_1 + \frac{m_{b1}}{2} y_b - \frac{l - \theta}{a}
\]

\[
p_2(t) = m_{a2}g + \frac{m_b}{2} g + m_{a2}y_2 + \frac{m_{b2}}{2} y_b - \frac{l - \theta}{a}
\]

Analysis of load of the bogie:

\[
m_{a1} \ddot{y}_1 + c_{a1}(y_1 + w_1) + k_{a1}(y_1 + w_1) + c_{b1}(y_1 - y_b + \frac{a}{2} \theta) + k_{b1}(y_1 - y_b + \frac{a}{2} \theta) = 0
\]

\[
m_{a2} \ddot{y}_2 + c_{a2}(y_2 + w_2) + k_{a2}(y_2 + w_2) + c_{b2}(y_2 - y_b - \frac{a}{2} \theta) + k_{b2}(y_2 - y_b - \frac{a}{2} \theta) = 0
\]

\[
w_1 = \sum_{n} X_n(vt-a)T_n(t); \quad w_2 = \sum_{n} X_n(vt)T_n(t)
\]

Analysis of load of the body:

\[
m_b \ddot{y}_b + c_{b1}(y_b - y_1 - \frac{a}{2} \theta) + k_{b1}(y_b - y_1 - \frac{a}{2} \theta) - \frac{a}{2} \left[ c_{a1} \left( y_b - y_1 - \frac{a}{2} \theta \right) + k_{a1} \left( y_b - y_1 - \frac{a}{2} \theta \right) \right] - \frac{a}{2} \left[ c_{a2} \left( y_b - y_2 - \frac{a}{2} \theta \right) + k_{a2} \left( y_b - y_2 - \frac{a}{2} \theta \right) \right] = 0
\]

Fig. 2. Half-car model under the effect of simply supported beam

Results

Bridge data in document and vehicle data in document are used, the length of simple supported beam is L=32m. Mass per unit length \( m = 5.41 \times 10^3 \text{ kg/m} \), flexural rigidity \( EI = 3.5 \times 10^{10} \text{ N} \cdot \text{m}^2 \). Following are relevant data of quarter-car model: \( m_{a1} = m_{a2} = 4330 \text{ kg} \);

\( m_b = 19250 \text{ kg} \); \( a = 8.4m \); \( k_{a1} = k_{a2} = 4.28 \times 10^6 \text{ N/m} \); \( k_{b1} = k_{b2} = 2.535 \times 10^6 \text{ N/m} \);

\( c_{a1} = c_{a2} = 9.8 \times 10^4 \text{ kg/s} \); \( c_{b1} = c_{b2} = 1.96 \times 10^5 \text{ kg/s} \) 1/2 car models: \( c_{a1} = c_{a2} = 9.8 \times 10^4 \text{ kg/s} \);

\( m_b = 38500 \text{ kg} \); \( m_{a1} = m_{a2} = 4330 \text{ kg} \); \( I_b = 2.446 \times 10^6 \text{ kg} \cdot \text{m}^2 \); \( a = 8.4m \) ;
\[ k_{a_1} = k_{a_2} = 4.28 \times 10^6 \text{N/m} \; ; \; \; k_{b_1} = k_{b_2} = 2.535 \times 10^6 \text{N/m} \; ; \; c_{b_1} = c_{b_2} = 1.96 \times 10^5 \text{kg/s} \]

Figure 3～Figure 6 compare 3 kinds of vehicles vertical displacement at mid-point of beam at same speed. Figure 7 shows the largest displacement of the 3 kinds of models on the bridge under different speed. The comparison of curves in the figure shows that the 3 kinds of models could embody coupling response.
General laws could be arrived at based on above figures:
(1) They all give good expression to the vibration response.
(2) Higher speed will make a slower displacement response of mid-span.
(3) Response Maximum displacement in bridge when vehicle appear in mid-span.
(4) When speed is under 200km/h, response of maximum displacement bridge do not take on a linear relationship.

The difference between vehicle models are as follows:
(1) The difference between 1/4 spring model of rigidity and 1/4 spring model is the change of rigidity. The displacement response in mid-span of bridge fits well with each other. Spring stiffness has no impact on vehicle-bridge coupling.
(2) When speed is under 130km/h, the discrepancy of maximum displacement in 1/4 vehicle model and 1/2 vehicle model is small.
(3) When speed is under 220km/h, the maximum displacement in 1/2 vehicle model is smaller than that 1/2 vehicle model. The vehicle-bridge coupling is more sensitive to moment of inertia.
(4) When speed is more than 220km/h, the maximum displacement in 1/2 vehicle model is greater than that 1/2 vehicle model.
(5) Generally speaking, the vehicle speed is no more than 120/h, The vehicle-bridge coupling can be realized using 1/4 model in highway bridge.

Conclusions
To study the response of factor of vehicle model to the vehicle-bridge coupling, the displacement of mid-span of 3 kinds of models are caculated. Comparison shows that the 3 kinds of models are in good agreement with vibration response of vehicle-bridge coupling. With the add of rotational inertia, the agreement will be better yet make the analysis complicated. When velocity is under 130km/h, the 3 kinds of models are in good agreement with each other. Quarter-car models can be used to analyse vibration of vehicle-bridge coupling in highway bridge.

References: