Three-Dimensional Target Tracking Based on Velocity Pursuit

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***Abstract*—This paper deals with a method for pursuer to track a moving target in the three-dimensional space. The method is based on the use of the geometrical rules combined with the kinematics equations of the pursuer and the target. The maneuvers of the target are not a priori known to the pursuer. In this paper, the velocity pursuit is proposed to implement tracking the moving target. Simulations are conducted to demonstrate the effectiveness and reliability of the proposed control strategy.**

Keywords-tracking; three-dimensional space; velocity pursuit

I. INTRODUCTION

The problem of tracking has been an increasingly hot issue in the research field. Various methods and algorithms have been suggested and applied for the tracking problem.

Tracking and interception of moving target using mobile robots is a vital field [1-7]. In [1], the control of a wheeled mobile robot to track a moving target with limited control inputs is studied. A tracking control method is proposed in [2] for differential-drive wheeled mobile robots with nonholonomic constraints by using a backstepping-like feedback linearization. In [3], the authors consider the tracking control problem for a group of nonholonomic wheeled mobile robots with limited information of a desired trajectory. In [4], a kinematics model for the tracking problem is derived and two strategies are suggested for robot navigation, namely the velocity pursuit guidance law and the deviated pursuit guidance law. The theoretical framework of controlling a convoy of wheeled mobile robots is considered in [5], where the control strategy is derived on guidance laws based on geometrical rules. Combining geometrical rules with the kinematics equations of the robot, a new approach is designed in [6] for robot navigation using the proportional navigation law. By using cubic navigation functions, a method is presented in [7] for robot tracking a target moving in a two-dimensional working space.

In this paper, under the velocity pursuit, the aim is to implement the solution of tracking in three dimensions. Different from the study of [4] and [5], the three-dimensional scalar kinematic equations given in [8] are applied to design the control strategy of tracking. Based on the study in [7], a surveillance problem of intercepting and maintaining the target at constant distance from the moving pursuer will be further considered in the three-dimensional space. In the absence of interference, the pursuer tracks a moving target with constant distance. By altering the control method, the pursuer can track a moving target with catching it. In this paper, we focus the attention on the simulation results for tracking problem only.

The remainder of this paper is organized as follows. Section II formulates the problem. The kinematics equations of the pursuer and the target are derived in Section III. In Section IV, the guidance law of the velocity pursuit is discussed. Section V proposes the control strategy of tracking under velocity pursuit. The simulation results are given in Section VI. Section VII is devoted to conclusion.

II. PROBLEM FORMULATION

The pursuer and the target are seen as the controllable particle movement in the three-dimensional space. Let $H_T(t) = (x_T(t), y_T(t), z_T(t))$ represents the path of the target at time $t \geq t_0 \geq 0$, and let $H_P(t) = (x_P(t), y_P(t), z_P(t))$ represents the path of the pursuer at time $t \geq t_0 \geq 0$, t_0 is the initial time. $H_T(t)$ and $H_P(t)$ are measured in the three-dimensional coordinate system. It is assumed that $H_T(t)$ is a smooth function. The target moves autonomously in the three-dimensional workspace with constant linear velocity and variable Euler angles. The maneuvers of target are not a priori known to the pursuer, which means that on-line strategies are necessary. It is assumed that the pursuer has a sensory system which can obtain pose information of the target. The mathematical formulation for the problem of tracking is on the basis of the geometrical rules combined with the kinematics equations of the pursuer and the target.

III. PREPARE YOUR PAPER BEFORE STYLING

In the three-dimensional space, the spatial representation of pursuer is shown in Figure I. The kinematics equations of motion for the pursuer are denoted by

$$\begin{cases} \dot{x}_P = v_P \cos\theta_P \cos\phi_P \\ \dot{y}_P = v_P \cos\theta_P \sin\phi_P \\ \dot{z}_P = v_P \sin\theta_P, \end{cases} \tag{1}$$

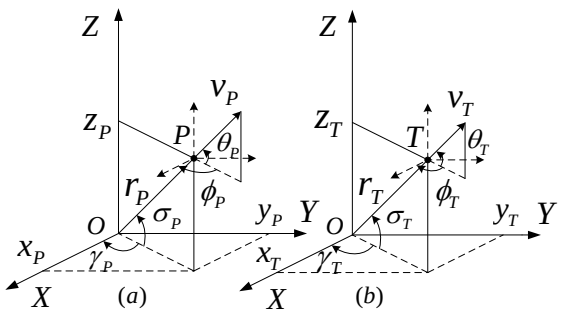

FIGURE I. (A) THREE-DIMENSIONAL REPRESENT OF PURSUER. (B) THREE-DIMENSIONAL REPRESENT OF TARGET

where (x_P, y_P, z_P) are the coordinates of the pursuer in the three-dimensional coordinate system, v_P is the linear velocity, θ_P and ϕ_P are the flight path and heading angles. The line of sight between the pursuer and the origin is denoted by OP. σ_P is the pitch angle of OP, and γ_P is the yaw angle of OP.

In this paper, the polar representation is applied to derive the kinematics equations for the pursuer and the target. In polar coordinates, the kinematics equations of the pursuer are denoted by

$$\begin{cases} v_{Pr} = \dot{r}_P = v_P \cos(\sigma_P - \theta_P)\cos(\gamma_P - \phi_P) \\ v_{P\sigma} = r_P \dot{\sigma}_P = -v_P \sin(\sigma_P - \theta_P)\cos(\gamma_P - \phi_P) \\ v_{P\gamma} = r_P \dot{\gamma}_P \cos\sigma_P = -v_P \cos\theta_P \sin(\gamma_P - \phi_P). \end{cases} \tag{2}$$

The spatial representation of target is also shown in Figure I, where (x_T, y_T, z_T) are the coordinates of the target in the three-dimensional coordinate system, v_T is the linear velocity, θ_T and ϕ_T are the flight path and heading angles. The line of sight between the target and the origin is denoted by OT. σ_T is the pitch angle of OT, and γ_T is the yaw angle of OT. In polar coordinates, the kinematics equations of the target are denoted by

$$\begin{cases} v_{Tr} = \dot{r}_T = v_T \cos(\sigma_T - \theta_T)\cos(\gamma_T - \phi_T) \\ v_{T\sigma} = r_T \dot{\sigma}_T = -v_T \sin(\sigma_T - \theta_T)\cos(\gamma_T - \phi_T) \\ v_{T\gamma} = r_T \dot{\gamma}_T \cos\sigma_T = -v_T \cos\theta_T \sin(\gamma_T - \phi_T). \end{cases} \tag{3}$$

The geometry of three-dimensional tracking problem is illustrated in Figure II. The line of sight between the pursuer and the target is denoted by PT. σ is the pitch angle of PT, and γ is the yaw angle of PT. The relative distance between the pursuer and the target is given by

$$r = \sqrt{(x_T - x_P)^2 + (y_T - y_P)^2 + (z_T - z_P)^2}. \tag{4}$$

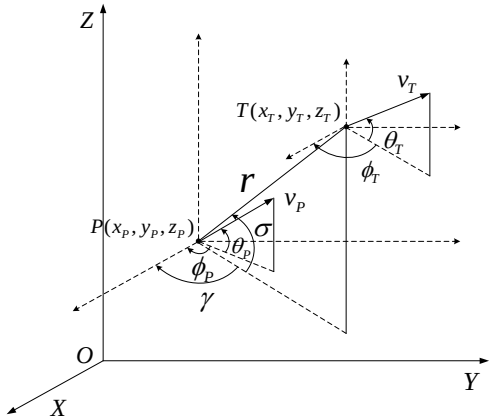

FIGURE II. GEOMETRY OF THREE-DIMENSIONAL TRACKING PROBLEM

The pitch angle of PT and the yaw angle of PT are given by

$$\tan\sigma = \frac{z_T - z_P}{\sqrt{(x_T - x_P)^2 + (y_T - y_P)^2}}, \tag{5}$$

$$\tan\gamma = \frac{y_T - y_P}{z_T - z_P}. \tag{6}$$

In this paper, the polar representation is applied to derive the kinematics equations between the pursuer and the target in the three-dimensional space. Based on [8], the differential equations for the range, the pitch angle of PT and the yaw angle of PT are

$$\begin{cases} \dot{r} = v_T \cos(\sigma - \theta_T)\cos(\gamma - \phi_T) - v_P \cos(\sigma - \theta_P)\cos(\gamma - \phi_P) \\ r\dot{\sigma} = v_P \sin(\sigma - \theta_P)\cos(\gamma - \phi_P) - v_T \sin(\sigma - \theta_T)\cos(\gamma - \phi_T) \\ r\cos\sigma\dot{\gamma} = v_P \cos\theta_P \sin(\gamma - \phi_P) - v_T \cos\theta_T \sin(\gamma - \phi_T). \end{cases} \tag{7}$$

Equation (7) provides a kinematics model for the tracking problem, which represents the relative velocities of the target with respect to the pursuer.

IV. VELOCITY PURSUIT GUIDANCE LAW

In this paper, the velocity pursuit will be discussed in more details when elaborating the corresponding control law for the tracking problem in the three-dimensional space.

In the velocity pursuit, the velocity vector of the pursuer lies in the line of sight joining the pursuer and the target. Thus, the flight path angle and the heading angle of the pursuer are given by

$$\begin{cases} \theta_P = \sigma \\ \phi_P = \gamma. \end{cases} \tag{8}$$

Combining (7) with (8), the differential equations for the range, the pitch angle of PT and the yaw angle of PT are

$$\begin{cases} \dot{r} = v_T \cos(\theta_P - \theta_T)\cos(\phi_P - \phi_T) - v_P \\ r\dot{\theta}_P = -v_T \sin(\theta_P - \theta_T)\cos(\phi_P - \phi_T) \\ r\cos\theta_P \dot{\gamma} = -v_T \cos\theta_T \sin(\phi_P - \phi_T). \end{cases} \tag{9}$$

V. Control Strategy of Tracking

In this section, the aim is to design control strategy to implement tracking in the three-dimensional space. Next an analysis of the control strategy being used is elaborated, and some important results concerning the tracking are proved. In this section, the negative influence of the interference is ignored. In the first case, the pursuer tracks a moving target without catching it, and the pursuer keeps a constant distance from the target. Thus, it is easy to obtain

$$\dot{r} = 0. \tag{10}$$

The second one is that the pursuer tracks a moving target with catching it.

In the velocity pursuit, the control strategy of tracking will be discussed under the proposed reasonable assumption.

Assumption 1: Under the velocity pursuit, the following constraints are satisfied, that is $\theta_P, \theta_T \in (-\pi/2, \pi/2)$.

Firstly, the problem of tracking with constant distance will be discussed under the velocity pursuit. Inserting (10) into (9), the linear velocity of the pursuer can be obtained that

$$v_P = v_T \cos(\theta_P - \theta_T)\cos(\phi_P - \phi_T). \tag{11}$$

Equation (11) describes the control strategy for the linear velocity of the pursuer to keep constant distance from the target.

With regard to tracking with constant distance, the aim of the pursuer is to imitate the target in the motion. This is formulated mathematically as follows.

Theorem 1: Under the velocity pursuit and Assumption 1, the flight path angle and the heading angle of the pursuer track the flight path angle and the heading angle of the target, respectively. i.e., $\theta_P(t) \to \theta_T(t), \phi_P(t) \to \phi_T(t)$.

Proof: Combining Assumption 1 with the equation (9), one can derive

$$\begin{cases} \dot{\theta}_P = -\dfrac{v_T}{r}\sin(\theta_P - \theta_T)\cos(\phi_P - \phi_T) = f_v(\theta_P, \phi_P) \\ \dot{\phi}_P = -\dfrac{v_T}{r}\sec\theta_P \sin(\phi_P - \phi_T)\cos\theta_T = g_v(\theta_P, \phi_P) \end{cases} \tag{12}$$

This system has two equilibrium solutions, namely $(\theta_{P1}^* = \theta_T, \phi_{P1}^* = \phi_T)$ and $(\theta_{P2}^* = \theta_T, \phi_{P2}^* = \phi_T + \pi)$. After partial deviation, it can be obtained that

$$\begin{cases} \dfrac{\partial f_v}{\partial \theta_P} = -\dfrac{v_T}{r}\cos(\theta_P - \theta_T)\cos(\phi_P - \phi_T) \\ \dfrac{\partial f_v}{\partial \phi_P} = \dfrac{v_T}{r}\sin(\theta_P - \theta_T)\sin(\phi_P - \phi_T) \\ \dfrac{\partial g_v}{\partial \theta_P} = -\dfrac{v_T}{r}\sec\theta_P \tan\theta_P \sin(\phi_P - \phi_T)\cos\theta_T \\ \dfrac{\partial g_v}{\partial \phi_P} = -\dfrac{v_T}{r}\sec\theta_P \cos(\phi_P - \phi_T)\cos\theta_T. \end{cases} \tag{13}$$

Linearizing near each equilibrium solution, it yields that

$$T_1 = \begin{bmatrix} \left.\dfrac{\partial f_v}{\partial \theta_P}\right|_{(\theta_{P1}^* = \theta_T, \phi_{P1}^* = \phi_T)} & \left.\dfrac{\partial f_v}{\partial \phi_P}\right|_{(\theta_{P1}^* = \theta_T, \phi_{P1}^* = \phi_T)} \\ \left.\dfrac{\partial g_v}{\partial \theta_P}\right|_{(\theta_{P1}^* = \theta_T, \phi_{P1}^* = \phi_T)} & \left.\dfrac{\partial g_v}{\partial \phi_P}\right|_{(\theta_{P1}^* = \theta_T, \phi_{P1}^* = \phi_T)} \end{bmatrix} = \begin{bmatrix} -\dfrac{v_T}{r} & 0 \\ 0 & -\dfrac{v_T}{r} \end{bmatrix}, \tag{14}$$

$$T_2 = \begin{bmatrix} \left.\dfrac{\partial f_v}{\partial \theta_P}\right|_{(\theta_{P2}^* = \theta_T, \phi_{P2}^* = \phi_T + \pi)} & \left.\dfrac{\partial f_v}{\partial \phi_P}\right|_{(\theta_{P2}^* = \theta_T, \phi_{P2}^* = \phi_T + \pi)} \\ \left.\dfrac{\partial g_v}{\partial \theta_P}\right|_{(\theta_{P2}^* = \theta_T, \phi_{P2}^* = \phi_T + \pi)} & \left.\dfrac{\partial g_v}{\partial \phi_P}\right|_{(\theta_{P2}^* = \theta_T, \phi_{P2}^* = \phi_T + \pi)} \end{bmatrix} = \begin{bmatrix} \dfrac{v_T}{r} & 0 \\ 0 & \dfrac{v_T}{r} \end{bmatrix}. \tag{15}$$

From (14) and (15), the determinants of T_1 and T_2 are

$$\det(T_1) = (\frac{v_T}{r})^2 > 0, \tag{16}$$

$$\det(T_2) = (\frac{v_T}{r})^2 > 0. \tag{17}$$

According to Hartman and Grobman theorem [9-10], there exists a topological equivalence between the nonlinear system and its linearized systems. Therefore, the following equivalent linearized systems can be obtained

$$\begin{cases} \dot{\theta}_P = \left.\dfrac{\partial f_v}{\partial \theta_P}\right|_{(\theta_{P1}^*=\theta_T,\phi_{P1}^*=\phi_T)} \bullet(\theta_P-\theta_T) + \left.\dfrac{\partial f_v}{\partial \phi_P}\right|_{(\theta_{P1}^*=\theta_T,\phi_{P1}^*=\phi_T)} \bullet(\phi_P-\phi_T) \\ \dot{\phi}_P = \left.\dfrac{\partial g_v}{\partial \theta_P}\right|_{(\theta_{P1}^*=\theta_T,\phi_{P1}^*=\phi_T)} \bullet(\theta_P-\theta_T) + \left.\dfrac{\partial g_v}{\partial \phi_P}\right|_{(\theta_{P1}^*=\theta_T,\phi_{P1}^*=\phi_T)} \bullet(\phi_P-\phi_T), \end{cases} \quad (18)$$

$$\begin{cases} \dot{\theta}_P = \left.\dfrac{\partial f_v}{\partial \theta_P}\right|_{(\theta_{P2}^*=\theta_T,\phi_{P2}^*=\phi_T+\pi)} \bullet(\theta_P-\theta_T) + \left.\dfrac{\partial f_v}{\partial \phi_P}\right|_{(\theta_{P2}^*=\theta_T,\phi_{P2}^*=\phi_T+\pi)} \bullet(\phi_P-\phi_T) \\ \dot{\phi}_P = \left.\dfrac{\partial g_v}{\partial \theta_P}\right|_{(\theta_{P2}^*=\theta_T,\phi_{P2}^*=\phi_T+\pi)} \bullet(\theta_P-\theta_T) + \left.\dfrac{\partial g_v}{\partial \phi_P}\right|_{(\theta_{P2}^*=\theta_T,\phi_{P2}^*=\phi_T+\pi)} \bullet(\phi_P-\phi_T). \end{cases} \quad (19)$$

The characteristic roots of T_1 are denoted by λ_{11} and λ_{12}, and the characteristic roots of T_2 are λ_{21} and λ_{22}. Based on (14) and (15), it is obvious to obtain $\lambda_{11} = \lambda_{12} = -v_T / r < 0$ and $\lambda_{21} = \lambda_{22} = v_T / r > 0$. Thus, the first equilibrium solution is asymptotically stable and the second equilibrium solution is unstable.

Combining the result of Theorem 1 with the equation (11), the pursuer tracks a moving target with a constant distance.

In the sequel, the following theorem relates that the pursuer tracks a moving target with catching it.

Theorem 2: Under the velocity pursuit, the pursuer reaches its moving target when $v_P > v_T$.

Proof: Based on (9), it is easy to obtain

$$\dot{r} = v_T \cos(\theta_P - \theta_T)\cos(\phi_P - \phi_T) - v_P \le v_T - v_P < 0. \quad (20)$$

Since $\dot{r} < 0$, the range is decreasing and the pursuer reaches its target when $v_P > v_T$.

VI. Simulation Results

The simulations are conducted in this section, where the method for tracking is implemented in the three-dimensional space. In the absence of interference, simulations are conducted to implement tracking of the moving target under the velocity pursuit. For simplicity, it is assumed that the velocities, the distances and the time are without units.

Example 1: In the absence of interference, the pursuer tracks a moving target with keeping a constant distance. Under the velocity pursuit, simulation result is illustrated in Figure III.

Example 2: In the absence of interference, the pursuer tracks a moving target with catching it. Under the velocity pursuit, simulation result is illustrated in Figure IV.

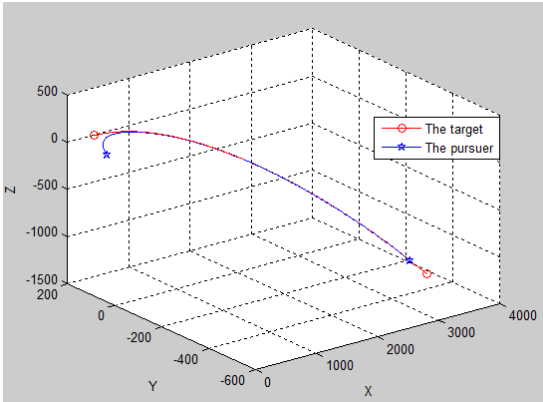

FIGURE III. TRACKING WITH CONSTANT DISTANCE UNDER THE VELOCITY PURSUIT

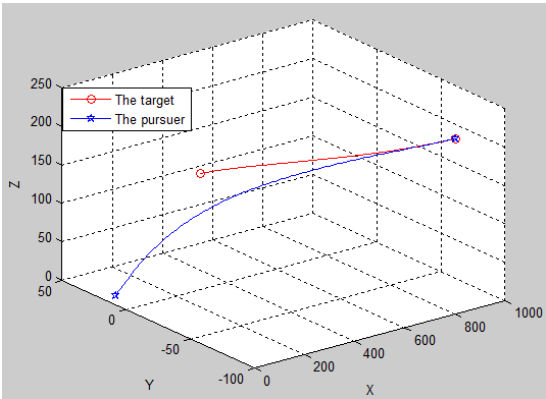

FIGURE IV. TRACKING UNDER THE VELOCITY PURSUIT

VII. Conclusion

In this paper, the method for tracking is implemented in the three-dimensional space. The control strategy is based on the guidance law of velocity pursuit. In the absence of interference, the pursuer tracks a moving target with keeping a constant distance, and pursuer tracks a moving target with catching it. Simulation results demonstrate the validity of the proposed control strategy.

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References

[1] L. Huang, "Control approach for tracking a moving target by a wheeled mobile robot with limited velocities," IET Control Theory Appl, vol. 3, no. 12, pp. 1565-1577, 2010.

[2] D. K. Chwa, "Tracking control of differential-drive wheeled mobile robots using a backstepping-like feedback linearization," IEEE Trans. on Systems, Man, and Cybernetics-Part A: Systems and Humans, vol. 40, no. 6, pp.1285-1295, 2010.

[3] W. J. Dong, "Tracking control of multiple-wheeled mobile robots with limited information of a desired trajectory," IEEE Trans. on Robotics, vol. 28,no.1, pp. 262-268, 2012.

[4] F. Belkhouche and B. Belkhouche, “A method for robot navigation towards a moving goal with unknown maneuvers,” Robotica, vol. 23, pp. 709-720,2005.

[5] F. Belkhouche and B. Belkhouche, “Modeling and controlling a robotic convoy using guidance laws strategies,” IEEE Trans. on Systems, Man, and Cybernetics–Part B: Cybernetics, vol. 35, no. 4, pp. 813-825, 2005.

[6] F. Belkhouche and B. Belkhouche, “Wheeled mobile robot navigation using proportional navigation,” Advanced Robotics, vol. 21, pp. 395-420, 2007.

[7] J. Vargas, S. Mendez, and F. Belkhouche, “Tracking under the nonholonomic constraint using cubic navigation laws,” Proc. of IEEE Conf. on Systems, Man, and Cybernetics, 2009, pp. 2788-2793.

[8] S. A. Elgamel and J. J. Soraghan, “Target tracking enhancement using a Kalman filter in the presence of interference,” IEEE International Geoscience and Remote Sensing Symposium, 2009, pp. 681-684.

[9] S. Wiggins, “Introduction to Applied nonlinear Dynamical Systems and Chaos,” Springer-Verlag, New York, 1990.

[10] J.K. Hale and H. Kocak, “Dynamics and Bifurcation,” Springer-Verlag, New York, 1991.