Research on Hedging Efficiency of SSE 50 Index Futures
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Abstract: Financial assets’ investment income is proportional to the risk, high income corresponds to high risk. As the most important derivatives, stock index futures’ primary function is hedging. Based on the theory of hedging and the method to evaluate its effectiveness, this paper carries out an empirical study on hedging efficiency using the B-VAR model and the ECM-GARCH model. Empirical model based on the Ederington hedge ratio method, using the date of SSE 50 Index Futures from April 16, 2015 to September 30, 2015. The results show that the hedging ratio of investment portfolio is above 85%, and the hedging relationship is highly effective. So it can be proved that the SSE 50 index futures play the role of hedging and avoiding the system risk in cash market.

1. INTRODUCTION

Stock index futures is one of the most important basic product of financial derivatives which is the greatest financial innovation in the past three decades. With the development and improvement of the international stock index futures market, the research and practice of the global financial market has become increasingly mature.

With China's A-share market has experienced ups and downs in 2015, including the SSE Composite Index once rose from 1991.06 up to 5178.19, then fell to 2850 points during two adjacent crash in the next three months. During this disaster, more than two thousand individual stocks hit bottom in several days. If we remove the suspended individual stocks, the stock market almost fell across the board. A number of driving factors give rise to such a phenomenon which involves several complicated reasons. First of all, institutional investors lack of effective coping strategies, when systemic risk has been appeared in the stock market. Secondly, current investors’ structure in our country is irrational that the proportion of ordinary investors to participate in is too high when faced with the systemic risk. However, this type of investor's R & D capabilities and expertise is limited. Domestic investment institutions rarely use stock index futures hedging tools to avoid investment risk, which in some extent, impacts the application and research of the stock index futures hedging and arbitrage strategy.

2. Evaluation and Selection of the Model

The main purpose of our country's stock index futures is to reduce the systemic risk of the stock market. Considering the limitation of the hedge ratio model under the condition of maximizing utility, this paper carries on from the perspective of minimum profit risk.

In this paper we using the B-VAR model for the calculation of the minimum risk hedge ratio can eliminate the residual serial correlation and increase the information volume of the model. Meanwhile, it can be more widely used in all kinds of futures price and spot price model to improve the situation that the traditional model is subject to a number of assumptions. ECM-GARCH model comprehensive considers the stability of the spot price and futures price, long-term equilibrium relationship and short-term dynamic relationship, which is conducive to get a better minimum risk hedging ratio.

Taking into account the advantages of the B-VAR model and ECM-GARCH model, it is difficult to directly distinguish which one is better. So this article will use this two models for empirical testing. If
calculation results of two models tend to be consistent, we can fully demonstrate the hedging efficiency of the SSE 50 index futures.

2.1 Bivariate vector autoregression model (B-VAR). Herbst, Caples (1989), Myers and Thompson (1989) using a new kind of B-VAR model to carry on the empirical analysis found that this method can effectively eliminate the residual of autocorrelation, and can increase the amount of model’s information. There are the following relations in VAR model:

\[ \Delta \ln S_t = C_s + \sum_{i=1}^{s} \alpha_{si} \Delta \ln S_{t-i} + \sum_{i=1}^{s} \beta_{si} \Delta \ln F_{t-i} + \varepsilon_{st} \]  

\[ \Delta \ln F_t = C_f + \sum_{i=1}^{f} \alpha_{fi} \Delta \ln S_{t-i} + \sum_{i=1}^{f} \beta_{fi} \Delta \ln F_{t-i} + \varepsilon_{ft} \]  

Here, Cs and Ct are constant terms of the VAR model; s and f are constants, \( s \neq f \); \( \alpha, \beta, \alpha, \) and \( \beta, \) are regression coefficients; \( \varepsilon \) and \( \varepsilon \) are independent identically distributed random error terms. 

Then order:

\[ Var(\varepsilon_{st}) = \sigma_{ss} \] 

\[ Var(\varepsilon_{ft}) = \sigma_{ff} \] 

\[ \text{cov}(\varepsilon_{ft}, \varepsilon_{st}) = \sigma_{sf} \]

By hedging ratio formula we can know that:

\[ h = \frac{\sigma_{st}}{\sigma_{ff}} \]  

In addition, we can get the same hedge ratio through the regression equation:

\[ \Delta \ln S_t = \alpha + h \Delta \ln F_t + \sum_{i=1}^{s} \beta_{si} \Delta \ln S_{t-i} + \sum_{i=1}^{f} \beta_{fi} \Delta \ln F_{t-i} + \varepsilon_t \]  

Here, \( h \) is the regression coefficient of \( \Delta \ln F_t \) and the hedge ratio we ask for.

2.2 Generalized autoregressive conditional autoregressive model with error correction (ECM-GARCH). Since Engle (1982) and Bollerslev (1986) have been proposed in the autoregressive conditional heteroscedasticity model (arch) and generalized autoregressive conditional heteroskedasticity model (GARCH), GARCH model is widely used in the analysis of financial time series. Compared with the ARCH model, the GARCH model takes into account the delay of the conditional variance, so the estimation of the GARCH model will be more accurate. Kroner (1995) proposed the ECM-GARCH model which is combination of ECM and GARCH model. It takes into account not only the co integration relationship exists between the futures market price and spot market price, but also consider the concentration between two markets. Taking into account the time variability and clustering of the SSE 50 index futures, we will use the ECM-GARCH model to calculate the hedging ratio of the portfolio. 

Mean equation expression is as follows:

\[ \Delta S_t = \alpha_0 + \alpha_1 \Delta F_t + \sum_{j=1}^{m} \gamma_j \Delta S_{t-j} + \sum_{j=1}^{n} \theta_j \Delta F_{t-j} + \alpha_A ECM_{t-1} + \varepsilon_{st} \]  

\[ \Delta F_t = \beta_0 + \beta_1 \Delta S_t + \sum_{j=1}^{m} \phi_j \Delta F_{t-j} + \sum_{j=1}^{n} \theta_j \Delta S_{t-j} + \beta_A ECM_{t-1} + \varepsilon_{ft} \]  

The conditional variance equation and conditional covariance equation can be expressed as follows:

\[ h_{ss,t} = C_1 + a_1 \varepsilon_{s,t-1}^2 + b_1 h_{ss,t-1} \]  

\[ h_{ss,t} = C_2 + a_2 \varepsilon_{f,t-1}^2 + b_1 h_{ff,t-1} \]  

\[ h_{sf,t} = \rho_{sf} \sqrt{h_{ss,t-1}} \sqrt{h_{ff,t-1}} \]  

Here, \( ECM_{t-1} = \theta + S_{t-1} - \lambda F_{t-1} \). So we can get the optimal hedge ratio:
3. DATA Selection and Statistical Analysis

3.1 Data selection and processing. This paper selects the daily closing price of 116 trading days from April 16, 2015, which is the first day of SSE 50 index futures, to September 30, 2015 (the last trading day of the second quarter).

The reason to choose this period of data is according to the SSE 50 Index futures contract rules. It’s trading months divided into four types: month continuous, next month continuous, quarter continuous and next quarter continuous. Thus, if carried out hedging operations in the first day of SSE 50 index futures, investors’ last trading day are May 17 under month continuous type and June 17 under the next month continuous type. The next two seasons are June and September.

This paper calculated the hedging effectiveness if investors keep their position to the end of the second season firstly, and then calculate the hedging effectiveness if investors keep their position one month or two months. Sample data comes from the Wind database.

In this article, main analysis object are SSE 50 Index Spot and futures contract after a quarter. We select the continuous contract data of stock index futures contract and separately does logarithm of treatment on the price of spot and futures. On the one hand, it can reduce the variance of the data. On the other hand, it will not cause a great impact on the final regression results.

In order to eliminate the volatility of the two sequences of stocks and futures prices, this article does natural logarithm on the data. The return rate takes a logarithmic form of the ratio between the closing price of the day and the previous day. The formula is as follows:

$$R_t = \ln(P_t / P_{t-1})$$

Here, $P_t$ is the closing price of the $t$ day after matching. $P_{t-1}$ is the closing price of the day before the $t$ day after matching.

Daily returns of futures market is $\Delta F_t = \ln F_t - \ln F_{t-1}$, Daily rate of return on the spot market is $\Delta S_t = \ln S_t - \ln S_{t-1}$.

3.2 Statistical description. Suppose the logarithm price series of the spot and futures are LnSpot, LnFutures, returns series were DLnS, DLnF. The log price and yield of each sequence descriptive statistical results is shown in table 1. We can find that, regardless of the logarithmic price or yield sequences are mostly have leptokurtosis which is the characteristic of financial time series. Leptokurtosis reflects the positive correlation of financial volatility, which indicates that the financial market has a positive feedback effect. And the negative bias means that there are number of trading days in which daily return is less than the average daily return.

<table>
<thead>
<tr>
<th>Tab.1 Descriptive statistics of return rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence name</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>LnSpot</td>
</tr>
<tr>
<td>LnFutures</td>
</tr>
<tr>
<td>DLnS</td>
</tr>
<tr>
<td>DLnF</td>
</tr>
</tbody>
</table>

Note: The JB statistic is the test statistic of the normal distribution, and the original assumption is that the sequence follows the normal distribution. The original hypothesis also includes a sequence that there is no autocorrelation of the sequence. *** represent at 1%, 5%, 10% level respectively to reject the null hypothesis.
4. Empirical process and results

4.1 An empirical analysis of hedging ratio based on B-VAR model. Firstly, this article calculates the lowest risk hedging ratio of hedging contracts IH03. Before establishing the B-VAR model, it is needed to examine the stability of the variables. If the sequence is stationary, the B-VAR model can be established. The standard method for checking the sequence stability is the unit root test, the specific method has the ADF test and the PP test. Table 2 brings together ADF test results of stock daily return series, futures daily return series, spot daily return series with a first-order difference, and futures daily return series with a first-order difference.

<table>
<thead>
<tr>
<th>Sequence name</th>
<th>ADF</th>
<th>PP</th>
<th>Test result</th>
</tr>
</thead>
<tbody>
<tr>
<td>LnSpot</td>
<td>-0.502 (-3.4891)</td>
<td>-0.6708 (-3.4881)</td>
<td>Unstable</td>
</tr>
<tr>
<td>LnFutures</td>
<td>-0.4975 (-3.4891)</td>
<td>-0.6488 (-3.4881)</td>
<td>Unstable</td>
</tr>
<tr>
<td>DlnS</td>
<td>-8.9426 (-3.4891)</td>
<td>-9.929926 (-3.4885)</td>
<td>Stable</td>
</tr>
<tr>
<td>DlnF</td>
<td>-7.9846 (-3.4897)</td>
<td>-10.2216 (-3.4886)</td>
<td>Stable</td>
</tr>
</tbody>
</table>

Note: the ADF test value and the data in brackets after the value of the PP test indicated a significant level of 1%.

Therefore, the establishment of a two variable autoregressive model is as follows.

\[ \Delta S_t = \alpha + h \Delta F_t + \sum_{i=1}^{N} \beta_i \Delta S_{t-i} + \sum_{j=1}^{M} \delta_j \Delta F_{t-j} + \epsilon_t \]  

\( \Delta \) is the amount of change in the spot price of the day \( t \). \( \alpha \) is the intercept of the regression function. \( h \) is the slope of the regression function. In the other word, it’s the minimum risk hedge ratio. \( F_t \) is the amount of change in futures prices of the day \( t \). \( \beta_i \) and \( \delta_j \) are regression coefficients. \( \epsilon_t \) is the random error term. \( N \) and \( M \) are natural numbers, \( N \neq M \).

This article gets the autoregressive order according to graphs of DlnS and DlnF’s self correlation graph and partial correlation graph. Try to introduce (1, 1), (1, 2), (2, 1), (2, 2) four lag order combination. By comparing the SC (Schwarz Criterion) value, Akaike (Akaike Info Criterion) value and the F statistic of the different models of the lag, we find that the introduction of the higher order lag does not significantly improve the model. Taking into account the model design should be as simple as possible, so select hysteresis (2,2) order to establish B-VAR model is as follows.

\[ \Delta S_t = \alpha + h \Delta F_t + \beta \Delta S_{t-1} + \gamma \Delta F_{t-1} + \eta \Delta S_{t-2} + \lambda \Delta F_{t-2} + \epsilon_t \]  

Using historical data for regression analysis, the results are obtained:

\[ \Delta S_t = -0.000276 + 0.809576 \Delta F_t + 0.088029 \Delta S_{t-1} + 0.001972 \Delta F_{t-1} + 0.117860 \Delta S_{t-2} + 0.089004 \Delta F_{t-2} \]  

The estimated minimum risk hedge ratio of B-VAR model is \( h = 0.8096 \).

Using the same method to calculate the minimum risk hedge ratio of IH00 and IH01 data, the resulting model is as follows:

\[ \Delta S_t = -0.000602 + 0.829489 \Delta F_t - 0.1058 \Delta S_{t-1} + 0.162018 \Delta F_{t-1} - 0.16885 \Delta S_{t-2} + 0.119513 \Delta F_{t-2} \]  

\[ \Delta S_t = -0.000478 + 0.813034 \Delta F_t - 0.009084 \Delta S_{t-1} + 0.081907 \Delta F_{t-1} - 0.146422 \Delta S_{t-2} + 0.103804 \Delta F_{t-2} \]  

4.2 An empirical analysis of hedging ratio based on ECM-GARCH model. Firstly, this article calculates the lowest risk hedging ratio of hedging contracts which have been positioning for half a year. Theoretically, the futures price and the spot price should be a cointegration relationship, because they are price of the same asset in different time points. In the long term, they have balanced relationships. Therefore, this paper carries out the cointegration testing on two logarithmic price series—LnSpot and LnFutures.
The test can be divided from the object into two kinds: a kind is the Engle Granger two-step test, but it has some limitations, because the test is limited to only one co integration relationship; another is co integration test which based on regression coefficient, it’s called the Johansen co integration test which is mainly used to test the multivariate co integration relationship.

### Tab.3 Johansen Co integration test results

<table>
<thead>
<tr>
<th>Hypothesized No.of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.195991</td>
<td>26.87283</td>
<td>20.26184</td>
<td>0.0053</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.019476</td>
<td>2.222513</td>
<td>1.9164546</td>
<td>0.7331</td>
</tr>
</tbody>
</table>

From the table 3, we can find that, at 95% of the confidence level, there are only 1 co integration relationship. The expression of co integration relation can be obtained from table 3. Make it equal to ECM, then we can get: $\text{ecm}_t = \ln S_t - 0.7702\ln F_t - 1.8351$

Carry out ADF unit root test on ECM sequence, we can get the T statistics which is as shown in Table 4.

### Tab.4 Mean spillover effect of the second period

<table>
<thead>
<tr>
<th>Augmented Dickey-Fuller Test statistic</th>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% level</td>
<td>-5.270772</td>
<td>0.0000</td>
</tr>
<tr>
<td>5% level</td>
<td>-3.488585</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-2.886959</td>
<td></td>
</tr>
<tr>
<td>2.580402</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The original hypothesis of ARCH effect test is that sequences which composed of squared residuals have no autocorrelation. The results show that the square of the residual error rejects the original hypothesis at the 10% significance level, indicating the existence of ARCH phenomenon. It’s suitable for the establishment of GARCH model.

Therefore, the generalized autoregressive conditional variance model with error correction is established as follows:

$$\Delta S_t = \alpha + \eta F_t + \beta_1 \Delta S_{t-1} + \gamma_1 \Delta F_{t-1} + \eta_2 \Delta S_{t-2} + \lambda_2 \Delta F_{t-2} + \mu \ln S_t - 0.7702 \ln F_t - 1.8351 + \epsilon_t$$ (17)

After getting the lag order (2,2), the regression analysis of the historical data is carried out, and results of the model estimation are obtained:

$$\Delta S_t = -0.0007 + 0.8754 \Delta F_t - 0.4879 \Delta S_{t-1} + 0.4386 \Delta F_{t-1} - 0.2543 \Delta S_{t-2} + 0.2127 \Delta F_{t-2} + 0.4994 \ln S_t - 0.7702 \ln F_t - 1.8351$$ (18)

The estimated minimum risk hedge ratio of ECM-GARCH model is $h=0.8754$.

Using the same method to calculate the minimum risk hedge ratio of IH00 and IH01 data, the resulting model is as follows:

$$\Delta S_t = -0.000553 + 0.872375 \Delta F_t - 0.648629 \Delta S_{t-1} + 0.586762 \Delta F_{t-1} - 0.303112 \Delta S_{t-2} + 0.272101 \Delta F_{t-2} + 0.799657 \ln S_t - 0.919059 \ln F_t - 0.650065$$ (19)

$$\Delta S_t = -0.00203 + 0.868534 \Delta F_t - 0.500065 \Delta S_{t-1} + 0.471791 \Delta F_{t-1} - 0.218669 \Delta S_{t-2} + 0.192037 \Delta F_{t-2} + 0.325446 \ln S_t - 0.869714 \ln F_t - 1.048829$$ (20)

### 4.3 Analysis of empirical results based on the Ederington model

We have calculated the optimal hedge ratio. In order to compare the hedging efficiency of two models, we will use the hedging efficiency formula:

$$HE = \frac{2h \text{cov}(\Delta S_t, \Delta F_t) - h^2 \text{var}(\Delta F_t)}{\text{var}(\Delta S_t)}$$ (21)
The minimum risk hedging effect of the SSE 50 index futures calculated by the B-V AR and ECM model can be seen in table 5. It can be seen that the effectiveness of stocks in the portfolio of risk aversion calculated by the two methods is different. The portfolio hedging rate using B-V AR model is more than 80%. However, the portfolio hedging rate using ECM-GARCH model is more than 85%.

According to the "Accounting Standards for Enterprises No. 24 - Hedging", the identified range of highly effective hedging rate is 80% to 125%. This proves that the effects of the hedge portfolio is highly effective. This shows that using SSE 50 index futures to hedge can achieve the desired purpose.

5. Conclusion and Prospect

This paper makes an empirical analysis of the hedging efficiency of SSE 50 stock index futures using model analysis technique. The study hedging on the SSE 50 index futures and calculate hedge ratio using the bivariate regression model and generalized autoregressive conditional heteroscedastic model with error correction respectively. The calculation of the hedging effectiveness is based on Ederington model. The results show that the hedging efficiency of the ECM-GARCH model is higher than that of the B-V AR model, and the hedge ratio of the ECM-GARCH model is more than 85%. This can prove that, investors can achieve the purpose of hedging using the SSE 50 index futures. The SSE 50 index futures can play a role in the market to avoid the risk of spot market system, protect the role of investors, and promote the healthy development of China's securities market.

References:


