Impulsive Control and Synchronization of Time-delay Chaotic System

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Abstract. With the increasingly perfecting and development of chaos theory, this paper considers the impulsive control and synchronization under the influence of time-delay chaotic system. It will utilize Lyapunov—krasovskii function and linear matrix inequality to research the asymptotic stability of chaotic deviation dynamical system between receiving system and sending system, and several new criteria will be obtained to ensure the impulsive control and synchronization of receiving and sending system.

Introduction

The research of chaos theory is a challenging and cross-subjects project with great potential applications and broad developing prospect. Especially since the 1990s, the breaking progress of chaos synchronization and control promote the research of chaos theory and also bring some opportunities and bright future of chaos application.

With the increasingly perfecting and development of chaos theory application, One new area comes out, that is the research of chaotic system under the Impulsive control and synchronization (e.g[1]-[7]). The Pulse design is established through Discrete time sequence driving system's state variable sample, and results the discrete of receiving system in the time sequence. After limited time, the path of two chaotic system will approach together, therefore these two chaotic system synchronise. However, in the Communication security system, the transmission lag in the application of the impulsive control and synchronization is unavoidable. It is necessary to analysis the robustness of Impulsive control and synchronization under the time-delay chaotic system. Therefore, this paper will consider the chaotic system with time-delay influences:

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$$\dot{y}(t) = Ay(t) + By(t - \tau) + \sum_{i=1}^n f_i(y(t), t) + g(y(t - \tau), t) + u(t) \quad (1)$$

Where $y(\cdot) \in R^n$ is state variable. $u(\cdot) \in R^n$ is control feedback in put variable, $A, B \in R^{n \times n}$ is constant matrix and $f_i, g \in R^n$ is nonlinear continuous function which ensure solution exists and is unique. $\tau > 0$ is time-delay constant. This paper assume system (1) satisfies initial condition

$$y(t) = \varphi(t) \quad t \in [t_0 - \tau, t_0] \quad (2)$$

Where $\varphi(t)$ in $[t_0 - \tau, t_0]$ is bounded continuous

Assumesystem has unstable equilibrium or unstable periodic orbit $\tilde{y}(t)$, the purpose of feedback control is to ensure that $\tilde{y}(t)$ is asymptotic convergence.

This kind of control is a standard feedback control way, control input vector is:

$$u(t) = K(\tilde{y}(t) - y(t))$$

where K at here is adjustable matrix coefficient of controller.

And then, it apply the pulse control system in Chaotic system (1):

$$\begin{cases} \dot{y}(t) = Ay(t) + By(t-\tau) + \sum_{i=1}^n f_i(y(t), t) + g(y(t-\tau), t) + u(t) & t \neq t_i \\ \Delta y(t) = y(t^+) - y(t^-) = D_i y(t^-) & t = t_i \end{cases} \quad (3)$$

where $y(t_i^+) = \lim_{t \rightarrow t_i^+} y(t)$, $y(t_i^-) = \lim_{t \rightarrow t_i^-} y(t)$. $D_0 = 0$.

Pulse time satisfy: $t_0 < t_1 < t_2 < \dots < t_n < t_{n+1} < \dots$, and when $n \rightarrow \infty, t \rightarrow \infty$.

Assume $y(t_i^-) = y(t_i)$ ($i = 1, 2, \dots$). The receiving end of driving system (1) is

$$\tilde{y}(t) = A\tilde{y}(t) + B\tilde{y}(t-\tau) + \sum_{i=1}^n f_i(\tilde{y}(t), t) + g(\tilde{y}(t-\tau), t) \quad (4)$$

$\tilde{y}(t) = \tilde{\varphi}(t)$, $t \in [t_0 - \tau, t_0]$ is initial condition. where $\tilde{\varphi}(t)$ in $[t_0 - \tau, t_0]$ is bounded continuous.

In the discontinuity point t_i ($i = 1, 2, \dots$) the status which delivers from driving system will occur jump discontinuity. In this situation, the impulse model of driving system is:

$$\begin{cases} \tilde{y}(t) = A\tilde{y}(t) + B\tilde{y}(t-\tau) + \sum_{i=1}^n f_i(\tilde{y}(t), t) + g(\tilde{y}(t-\tau), t) & t \neq t_i \\ \Delta \tilde{y}(t) = \tilde{y}(t^+) - \tilde{y}(t^-) = D_i \tilde{y}(t^-) & t = t_i \end{cases} \quad (5)$$

Where $\tilde{y}(t_i^+) = \lim_{t \rightarrow t_i^+} \tilde{y}(t)$, $\tilde{y}(t_i^-) = \lim_{t \rightarrow t_i^-} \tilde{y}(t)$. $D_0 = 0$, and $\tilde{y}(t)$ assume is leftcontinuous, then $y(t_i^-) = y(t_i)$ ($i = 1, 2, \dots$).

According (3) and (5), it can obtain the error impulsive differential system is:

$$\begin{cases} \dot{e}(t) = Ae(t) + Be(t-\tau) + \sum_{i=1}^n F_i(e(t), t) + G(e(t-\tau), t) - Ke(t) & t \neq t_i \\ \Delta e(t) = e(t^+) - e(t^-) = D_i e(t^-) & t = t_i \end{cases} \quad (6)$$

Where

$$e(t) = y(t) - \tilde{y}(t), F_i[e(t), t] = f_i[y(t), t] - f_i[\tilde{y}(t), t], G[e(t-\tau), t] = g[y(t-\tau), t] - g[\tilde{y}(t-\tau), t].$$

The initial condition of error system (6) satisfy:

$$e(t) = \varphi(t) - \tilde{\varphi}(t) = \phi(t) \quad t \in [t_0 - \tau, t_0] \quad (7)$$

since $\varphi(t), \tilde{\varphi}(t)$ is bounded continuous, therefore is bounded,

$$\|\phi\|_\tau = \|\varphi - \tilde{\varphi}\| = \sup_{t \in [t_0 - \tau, t_0]} \{\|\varphi(t) - \tilde{\varphi}(t)\|\}$$

For the error system (6), due to the $F_i(0, t) \equiv 0, G(0, t) \equiv 0$, therefore it utilize Taylor expansion:

$$\sum_{i=1}^n F_i(e(t), t) + G(e(t-\tau), t) = \sum_{i=1}^n A_i(t)e(t) + o(e(t)) + A_0(t)e(t-\tau) + o(e(t-\tau)) \quad (8)$$

$$\text{Here } A_i(t) = \frac{\partial F_i[e(t), t]}{\partial e(t)} \Big|_{e(t)=0}, A_0(t) = \frac{\partial G[e(t-\tau), t]}{\partial e(t-\tau)} \Big|_{e(t-\tau)=0}.$$

Let

$$o(e(t)) + o(e(t-\tau)) = H[e(t), e(t-\tau), t] = o(\|e(t)\| + \|e(t-\tau)\|) \quad (9)$$

According to the infinitesimal of higher order conception, for all $\alpha_i > 0$ ($i = 1, 2$), all exist $M > 0$, when $\|e(t)\| \leq M$ and $\|e(t-\tau)\| \leq M$, then:

$$H[e(t), e(t-\tau), t] \leq \alpha_1 \|e(t)\| + \alpha_2 \|e(t-\tau)\| \quad (10)$$

Assume $A_i(t) = A_{i1} + A_{i2}(t)$ ($t = 0, 1, 2, \dots$), where A_{i1} is constant matrix, $A_{i2}(t)$ is time-varying uncertain matrix, which satisfy:

$$\|A_{i2}(t)\| \leq \beta_i \quad (11)$$

where β_i ($i = 0, 1, 2, \dots$) is constant.

According to the above assumption, system(6) can rewrite as:

$$\begin{cases} \dot{e}(t) = Ee(t) + Fe(t-\tau) + \sum_{i=1}^n A_{i2}(t)e(t) + A_{02}(t)e(t-\tau) \\ \quad + H[e(t), e(t-\tau), t] & t \neq t_i \\ \Delta e(t) = e(t^+) - e(t^-) = D_i e(t_i^-) & t = t_i \\ e(t) = \phi(t) & t \in [t_0 - \tau, t_0] \end{cases} \quad (12)$$

where $E = A - k + \sum_{i=1}^n A_{i1}$, $F = B + A_{01}$, $D_0 = 0$.

Due to $\left[F \int_{t-\tau}^t e(s) ds \right]' = Fe(t) - Fe(t-\tau)$, then system (12) can be changed as:

$$\begin{cases} \left[e(t) + F \int_{t-\tau}^t e(s) ds \right]' = (E + F)e(t) + \sum_{i=1}^n A_{i2}(t)e(t) + A_{02}(t)e(t-\tau) \\ \quad + H[e(t), e(t-\tau), t] & t \neq t_i \\ \Delta e(t) = e(t^+) - e(t^-) = D_i e(t_i^-) & t = t_i \\ e(t) = \phi(t) & t \in [t_0 - \tau, t_0] \end{cases} \quad (13)$$

For the sake of convenient, it define $L: C([- \tau, 0], R^n) \rightarrow R^n$ as:

$$L(e_t) = e(t) + F \int_{t-\tau}^t e(s) ds \quad (14)$$

Before this paper put forward the main conclusion, there introduce three lemmas:

Lemmas 1: For the vector $a, b \in R^n$ and symmetric positivematrix Q , there is:

$$2a^T b \leq a^T Q^{-1} a + b^T Q b$$

Lemmas 2: For any positive matrix $N \in R^{n \times n}$, it exists $\gamma > 0$, and vector function $\omega: [0, \gamma] \rightarrow R^n$,

there is: $\left(\int_0^\gamma \omega(s) ds \right)^T N \left(\int_0^\gamma \omega(s) ds \right) \leq \gamma \int_0^\gamma \omega^T(s) N \omega(s) ds$

Lemmas 3: (Schur Complement) exists constant matrix Z_1, Z_2, Z_3 , where $Z_1 = Z_1^T, Z_2 = Z_2^T$, there is:

$$\begin{pmatrix} Z_1 & Z_3^T \\ Z_3 & Z_2 \end{pmatrix} < 0, \text{ if and only if } Z_1 < 0, Z_2 - Z_3 Z_1^{-1} Z_3^T < 0; \text{ or } Z_2 < 0, Z_2 - Z_3 Z_2^{-1} Z_3^T < 0.$$

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Theorem1: For system (13), if it satisfy $\tau \|F\| < 1$, and exist positivesymmetric matrix $P > 0, Q > 0$, which make the linear matrix inequality satisfy:

$$N = \begin{pmatrix} (E + F)^T P + P(E + F) + L_1 I + L_2 I + L_3 I + L_4 I + 4Q\tau & \tau(E + F)^T P F \\ \tau F^T P(E + F) & -\tau Q \end{pmatrix} < 0 \quad (15)$$

$$\|1 + D_i\| \sqrt{\frac{(1 + \tau \|F\|)^2 \lambda_{\max}(P) + \tau(L_1 + L_2) + 2\tau^2 \lambda_{\max} Q}{(1 - \tau \|F\|)^2 \lambda_{\min}(P)}} < 1 \quad (16)$$

Then the zero-solution of error sytem(13) is exponentially stable, that is mean the pulsedindex of system (1) and system (4) is synchronous, where L_1, L_2, L_3, L_4 , can be viewed respectivelyas:

$$L_1 = \tau \beta_0^2 \|PFQ^{-1}FP\| + \beta_0 \|P\|, \quad L_2 = (\alpha_1 + \alpha_2) \alpha_2 \|PFQ^{-1}F^T P\| + \alpha_2 \|P\|$$

$$L_3 = 2 \sum_{i=1}^n \beta_i \|P\| + \tau \sum_{i=1}^n \beta_i^2 \|PFQ^{-1}F^T P\| + \beta_0 \|P\|$$

$$L_4 = (\alpha_1 + \alpha_2)\alpha_1 \|PFQ^{-1}F^T P\| + \alpha_2 \|P\| + 2\alpha_1 \|P\|$$

Proof:

According to the definition's condition (15), for $\varepsilon > 0$, let:

$$N + (e^{\varepsilon\tau} - 1) \begin{pmatrix} L_1 I + L_2 I + 4Q\tau & 0 \\ 0 & 0 \end{pmatrix} + \varepsilon e^{\varepsilon\tau} \begin{pmatrix} \tau Q + \tau^2 F^T P F & 0 \\ 0 & 0 \end{pmatrix} + \varepsilon \begin{pmatrix} P & P F \\ F^T P & Q^{-1} \end{pmatrix} \leq 0 \quad (17)$$

Let Lyapunov-Krasovskii function as :

$$V(e_t) = V_1(e_t) + V_2(e_t) + V_3(e_t), \quad V_1(e_t) = L^T(e_t) P L(e_t) e^{\varepsilon t},$$

$$V_2(e_t) = \int_{-\tau}^0 \int_{t+s}^t e^T(\rho) [(4 + \varepsilon)Q + \varepsilon \tau F^T P F] e(\rho) e^{\varepsilon(\rho+\tau)} d\rho ds, \quad V_3(e_t) = (L_1 + L_2) \int_{t-\tau}^t \|e(s)\|^2 e^{\varepsilon(s+\tau)} ds.$$

Through system (13), to make a derivative of Lyapunov-Krasovskii function $V_1(e_t)$, obtain:

$$\begin{aligned} \frac{dV_1(e_t)}{dt} &= 2e^{\varepsilon t} L^T(e_t) P \frac{dL(e_t)}{dt} + \varepsilon e^{\varepsilon t} L^T(e_t) P L(e_t) \\ &= e^{\varepsilon t} \{ e^T(t) [P(F + E + \sum_{i=1}^n A_{i2}(t))] + (F + E + \sum_{i=1}^n A_{i2}(t))^T P \} e(t) \\ &\quad + 2e^T(t) P A_{02}(t) e(t - \tau) + 2e^T(t) (E + F)^T P F \int_{t-\tau}^t e(s) ds \\ &\quad + 2e^T(t - \tau) A_{02}^T(t) P F \int_{t-\tau}^t e(s) ds + 2e^T(t) \left(\sum_{i=1}^n A_{i2}(t) \right)^T P F \int_{t-\tau}^t e(s) ds \\ &\quad + 2e^T(t) P H[e(t), e(t - \tau), t] + 2H^T[e(t), e(t - \tau), t] P F \int_{t-\tau}^t e(s) ds \\ &\quad + \varepsilon [e^T(t) P e(t) + 2e^T(t) P F \int_{t-\tau}^t e(s) ds + \int_{t-\tau}^t e^T(s) ds F^T P F \int_{t-\tau}^t e(s) ds] \} \end{aligned}$$

To apply Lemmas 1 and system (11), it can be obtained:

$$\begin{aligned} \frac{dV_1(e_t)}{dt} &\leq e^{\varepsilon t} \{ e^T(t) [P(F + E + \sum_{i=1}^n \beta_i I)] + (F + E + \sum_{i=1}^n \beta_i I)^T P \} e(t) \\ &\quad + 2e^T(t) P A_{02}(t) e(t - \tau) + 2e^T(t) (E + F)^T P F Q^{-1} F^T P (E + F) e(t) \\ &\quad + \tau \beta_0^2 e^T(t - \tau) P F Q^{-1} F^T P e(t - \tau) + 4 \int_{t-\tau}^t e^T(s) Q e(s) ds + \tau \sum_{i=1}^n \beta_i^2 e^T(t) P F Q^{-1} F^T P e(t) \\ &\quad + 2e^T(t) P H[e(t), e(t - \tau), t] + 2H^T[e(t), e(t - \tau), t] P F Q^{-1} F^T P H[e(t), e(t - \tau), t] \\ &\quad + \varepsilon [e^T(t) P e(t) + 2e^T(t) P F \int_{t-\tau}^t e(s) ds + \int_{t-\tau}^t e^T(s) ds F^T P F \int_{t-\tau}^t e(s) ds] \} \end{aligned}$$

When $\|\phi\| < M$, there exist $l > 0$, which make $\forall t \in [t_0, t_0 + l)$ has $\|e(t)\| \leq M, \|e(t - \tau)\| \leq M$, now satisfy system(10), and then apply Lemma 2, here it is:

$$\begin{aligned} \frac{dV_1(e_t)}{dt} &\leq e^{\varepsilon t} \{ e^T(t) [P(F + E + \sum_{i=1}^n \beta_i I)] + (F + E + \sum_{i=1}^n \beta_i I)^T P \} e(t) \\ &\quad + 4 \int_{t-\tau}^t e^T(s) Q e(s) ds + \tau e^T(t) (E + F)^T P F Q^{-1} F^T P (E + F) e(t) \\ &\quad + \tau \sum_{i=1}^n \beta_i^2 \|PFQ^{-1}F^T P\| \|e(t)\|^2 + \tau \beta_0^2 \|PFQ^{-1}F^T P\| \|e(t - \tau)\|^2 \\ &\quad + 2\beta_0 \|P\| \|e(t)\| \|e(t - \tau)\| + 2\alpha_1 \|P\| \|e(t)\|^2 \end{aligned}$$

$$\begin{aligned}
 & + 2\alpha_2 \|P\| \|e(t)\| \|e(t-\tau)\| + \|PFQ^{-1}F^T PH\| (\alpha_1 \|e(t)\| + \alpha_2 \|e(t-\tau)\|)^2 \\
 & + \varepsilon[e^T(t)Pe(t) + e^T(t)PFQ^{-1}F^T P e(t) + \int_{t-\tau}^t e^T(s)Qe(s)ds + \int_{t-\tau}^t e^T(s)F^T PFe(s)ds]
 \end{aligned}$$

Therefore, to make a derivative of function $V_1(e_t)$ with system (13), it obtain:

$$\begin{aligned}
 \frac{dV_1(e_t)}{dt} & \leq e^{\varepsilon t} \{e^T(t)[P(F+E) + (F+E)^T P + (E+F)^T PFQ^{-1}F^T P(E+F) \\
 & + \varepsilon P + \varepsilon PFQ^{-1}F^T P + L_3 I + L_4 I]e(t) + \varepsilon \int_{t-\tau}^t e^T(s)F^T P e(s)ds \\
 & + (4+\varepsilon) \int_{t-\tau}^t e^T(s)Qe(s)ds + (L_1 + L_2) \|e(t-\tau)\| \}
 \end{aligned} \quad (18)$$

For

$$\frac{dV_2(e_t)}{dt} \leq e^{\varepsilon t} \tau e^T(t)[(4+\varepsilon)Q + \varepsilon \tau F^T PF]e(t) - e^{\varepsilon t} \int_{t-\tau}^t e^T(s)[(4+\varepsilon)Q + \varepsilon \tau F^T PF]e(s)ds \quad (19)$$

$$\frac{dV_3(e_t)}{dt} = (L_1 + L_2) \|e(t)\|^2 e^{\varepsilon(t+\tau)} - (L_1 + L_2) \|e(t-\tau)\|^2 e^{\varepsilon t} \quad (20)$$

Accord to system (18), (19), (20), there is :

$$\begin{aligned}
 \frac{dV(e_t)}{dt} & \leq e^{\varepsilon t} \{e^T(t)(P(E+F) + (E+F)^T P + \tau(E+F)^T PFQ^{-1}F^T P(E+F)) \\
 & + (L_1 I + L_2 I)e^{\varepsilon t} + L_3 I + L_4 I + \tau(4+\varepsilon)e^{\varepsilon t} Q + \varepsilon \tau^2 e^{\varepsilon t} F^T PF + \varepsilon P + PFQ^{-1}F^T P)e(t)\}
 \end{aligned}$$

For $\forall t \in [t_0, t_0 + l]$, according to the inequation (17), there is :

$$V'(e_t) \leq 0 \quad (21)$$

for $\forall t \in [t_0, t_1]$, there is :

$$\begin{aligned}
 V(e_t) & \leq V(e_{t_0}) \leq \lambda_{\max}(P)(1-\tau \|F\|)^2 \|\phi\|_{\tau}^2 e^{\varepsilon t_0} + \tau(L_1 + L_2)e^{\varepsilon t} e^{\varepsilon t_0} \|\phi\|_{\tau}^2 \\
 & + \lambda_{\max}[(4+\varepsilon)Q + \varepsilon \tau F^T PF] \frac{\tau^2}{2} e^{\varepsilon t} e^{\varepsilon t_0} \|\phi\|_{\tau}^2 = C e^{\varepsilon t_0} \|\phi\|_{\tau}^2
 \end{aligned}$$

where

$$C = \lambda_{\max}(P)(1-\tau \|F\|)^2 \|\phi\|_{\tau}^2 e^{\varepsilon t_0} + \tau(L_1 + L_2)e^{\varepsilon t} e^{\varepsilon t_0} + \lambda_{\max}[(4+\varepsilon)Q + \varepsilon \tau F^T PF] \frac{\tau^2}{2} e^{\varepsilon t}.$$

From literature (8), it is:

$$(1-\tau \|F\|)^2 \|e(t)\|^2 \leq \|L(e_t)\|^2,$$

and then

$$(1-\tau \|F\|)^2 \lambda_{\min}(P) \|e(t)\|^2 e^{\varepsilon t} \leq L^T(e_t)PL(e_t)e^{\varepsilon t} \leq V(e_t) \leq C \|\phi\|_{\tau}^2 e^{\varepsilon t_0}.$$

That is

$$\|e(t)\| \leq \sqrt{\frac{C}{(1-\tau \|F\|)^2 \lambda_{\min}(P)}} \|\phi\|_{\tau} e^{-\frac{\varepsilon}{2}(t-t_0)} = \Phi \|\phi\|_{\tau} e^{-\frac{\varepsilon}{2}(t-t_0)} \quad t \in [t_0, t_1],$$

where

$$\Phi = \sqrt{\frac{C}{(1-\tau \|F\|)^2 \lambda_{\min}(P)}},$$

according to the system (13), it can deduct that:

$$\|e(t_1^+)\| \leq \|(1+D_1)e(t_1)\| \leq (1+D_1) \Phi \|\phi\|_{\tau} e^{-\frac{\varepsilon}{2}(t_1-t_0)},$$

according to system (21), $\forall t \in [t_0, t_1]$, there is

$$V(e_t) \leq V(e_{t_1^+}) \leq C \|e(t_1^+)\|^2 e^{\varepsilon t_1} \leq C \|(1+D_1)\|^2 \Phi \|\phi\|_{\tau}^2 e^{\varepsilon t} e^{-\varepsilon(t_1-t_0)} e^{\varepsilon t_1},$$

for

$$(1 - \tau \|F\|)^2 \lambda_{\min}(P) \|e(t)\|^2 e^{\varepsilon t} \leq L^T(e_t) PL(e_t) e^{\varepsilon t} \leq V(e_t) \leq C \|(1 + D_1)\|^2 \Phi \|\phi\|_\tau^2 e^{\varepsilon \tau} e^{-\varepsilon(t_1 - t_0)} e^{\varepsilon t_1},$$

$$\text{so } \|e(t)\| \leq \|(1 + D_1)\| \Phi^2 \|\phi\|_\tau e^{\frac{\varepsilon \tau}{2}} e^{-\frac{\varepsilon}{2}(t - t_0)} \quad t \in (t_1, t_2].$$

by utilizing mathematical induction $\forall t \in (t_k, t_{k+1}]$, there is:

$$\|e(t_{k+1}^+)\| \leq \prod_{i=1}^{k+1} \|(1 + D_i)\| \Phi^{k+1} \|\phi\|_\tau e^{\frac{k\varepsilon\tau}{2}} e^{-\frac{\varepsilon}{2}(t_{k+1} - t_0)},$$

when $\varepsilon > 0$, according to the condition (16) can be known:

$$\|1 + D_i\| \sqrt{\frac{C}{(1 - \tau \|F\|)^2 \lambda_{\min}(P)}} e^{\frac{\varepsilon \tau}{2}} = \|1 + D_i\| \Phi e^{\frac{\varepsilon \tau}{2}} \leq 1,$$

if $d = \max\{1, \Phi\}$, $\forall t \geq t_0$ all have:

$$\|e(t)\| \leq d \|\phi\| e^{-\frac{\varepsilon}{2}(t - t_0)} \quad (22)$$

When $\|\phi\| \leq \frac{M}{d}$, it is easy to get $\forall t \in [t_0, \infty)$, and all $\|e(t)\| \leq M$, $\|e(t - \tau)\| \leq M$, therefore,

for $\forall t \in [t_0, \infty)$, inequation (22) is established. therefore, the zero solution of error system (13) is exponentially stable, thus the pulsed index of system (1) and system (4) is synchronous.

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